Spin Hydrodynamics from Quantum Kinetic Theory

David Wagner
in collaboration with
Nora Weickgenannt, Enrico Speranza, and Dirk Rischke

based mainly on

NW, ES, X.-L. Sheng, Q. Wang, DHR, Phys. Rev. D 104 1, 016022 (2021)
NW, DW, ES, DHR, Phys. Rev. D 106 9, 096014 (2022)
NW, DW, ES, Phys. Rev. D 105 11,116026 (2022)
DW, NW, ES, 2207.01111 (2022)
DW, NW, DHR, Phys. Rev. D 106 11, 116021 (2022)

Emergent Topics in Relativistic Hydrodynamics | Feb 2-5, 2023

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CRC-TR 211 Strong-interaction matter under extreme conditions
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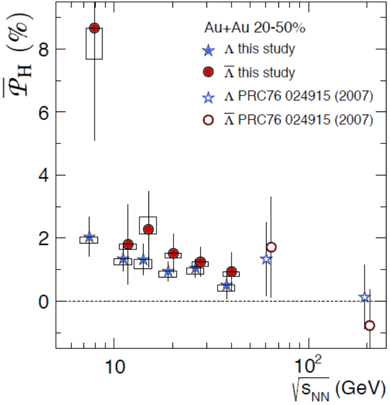
EL EM EN TS

Global Λ-polarization

- Global polarization: polarization of Λ-hyperons along angular-momentum direction

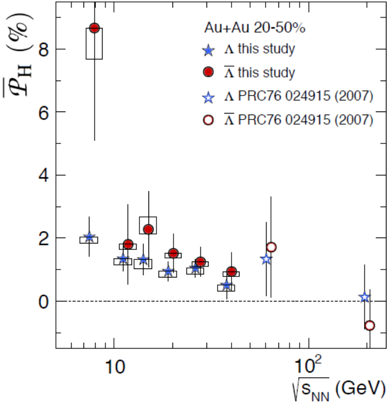


L. Adamczyk et al. (STAR), Nature 548 (2017) 62

Global Λ-polarization

- Global polarization: polarization of Λ-hyperons along angular-momentum direction
  - Can be well explained by considering **local equilibrium** on freeze-out hypersurface

$$S^\mu_\varpi = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0(1-f_0)\varpi_{\alpha\beta}}{8m \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$



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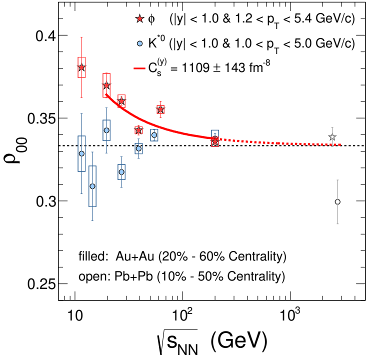
$$\varpi_{\mu\nu} := -\tfrac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu),\ \beta^\mu := u^\mu/T,\ f_0 = [\exp(u^\mu k_\mu/T)+1]^{-1}$$

Alignment of ϕ-mesons

- Spin-1 particles feature tensor polarization ($\hat{=}$ alignment)



STAR collaboration, arXiv:2204.02302 (2022)

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  - Some theoretical developments, but no definitive answer yet

X.-L. Xia, H. Li, X-G. Huang, H.-Z. Huang, PLB 817 (2021) 136325
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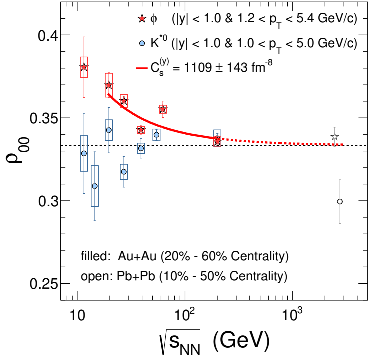

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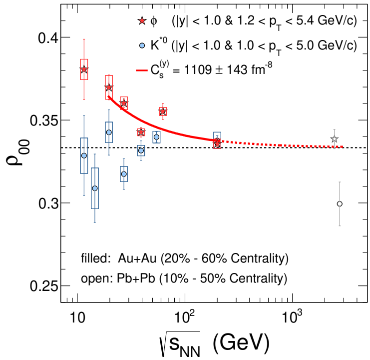
  - Can **spin-1 hydrodynamics** help explain this?



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- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - $\rightarrow$ Should conserve energy-momentum and total angular momentum

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$$\partial_\lambda J^{\lambda\mu\nu} =: \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0 \tag{1b}$$

$A^{[\mu} B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

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- Additional information about dissipative quantities has to be provided
  - $\rightarrow$ Use **kinetic theory with spin** as effective microscopic model
- Rest of the presentation:
  - Construct such a kinetic theory
  - Perform hydrodynamic limit
  - Obtain expressions for observables

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

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  - → Use **Wigner-function formalism**

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Wigner function (Spin 1)

$$W^{\mu\nu}(x,k) := -\frac{2}{(2\pi\hbar)^4\hbar}\int \mathrm{d}^4 v e^{-ik\cdot y/\hbar}\left\langle : V^{\dagger\mu}(x+y/2)V^{\nu}(x-y/2) : \right\rangle$$

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- Equations of motion follow from field equations
  - Determined by Lagrangian $\mathcal{L}_0 + \mathcal{L}_{\text{int}}$

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- Equations of motion follow from field equations
  - Determined by Lagrangian $\mathcal{L}_0 + \mathcal{L}_{\text{int}}$
- Independent components: scalar f_K, axial vector G^μ and traceless symmetric tensor $F_K^{\mu\nu}$

$f_K := (1/3)K_{\mu\nu}W^{\mu\nu}$, $G^\mu := -(i/2m)\epsilon^{\mu\nu\alpha\beta}k_\nu W_{\alpha\beta}$, $F_K^{\mu\nu} := K^{\mu\nu}_{\alpha\beta}W^{\alpha\beta}$
$K^{\mu\nu} := g^{\mu\nu} - k^\mu k^\nu/m^2$, $K^{\mu\nu}_{\alpha\beta} := (K^\mu_\alpha K^\nu_\beta + K^\mu_\beta K^\nu_\alpha)/2 - 1/3K^{\mu\nu}K_{\alpha\beta}$

Extending phase space

Boltzmann equations

- Not one, but nine equations in (x, k)-phase space

$$k \cdot \partial f_K(x,k) = \mathcal{C}_K \;, \quad k \cdot \partial G^\mu(x,k) = \mathcal{C}_G^\mu \;, \quad k \cdot \partial F_K^{\mu\nu}(x,k) = \mathcal{C}_K^{\mu\nu}$$

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- Measure $\mathrm{d}S := \frac{3m}{2\sigma\pi}\mathrm{d}^4\mathfrak{s}\,\delta[\mathfrak{s}^2+\sigma^2]\delta(k\cdot\mathfrak{s})$

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Boltzmann equation in extended phase space

$$\mathfrak{f}(x, k, \mathfrak{s}) := f_K - \mathfrak{s}_\mu G^\mu + \frac{5}{8}\mathfrak{s}_\mu \mathfrak{s}_\nu F_K^{\mu\nu} \tag{2}$$

- Only on-shell parts $\mathfrak{f}(x, k, \mathfrak{s}) = \delta(k^2 - m^2) f(x, k, \mathfrak{s})$ contribute

$$k \cdot \partial f(x, k, \mathfrak{s}) = \mathfrak{C}[f] \tag{3}$$

$\mathfrak{C} := \mathcal{C}_{\mathcal{F}} - \mathfrak{s}_\mu \mathcal{C}_G^\mu + \frac{5}{8}\mathfrak{s}_\mu \mathfrak{s}_\nu \mathcal{C}_K^{\mu\nu}$

Collisions and equilibrium

Collision kernel

$$\mathfrak{C}[f] = \frac{1}{2}\int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma_2 \mathrm{d}\Gamma' \mathrm{d}\bar{S}(k)\mathcal{W}\left[f(x+\Delta_1,k_1,\mathfrak{s}_1)f(x+\Delta_2,k_2,\mathfrak{s}_2)\right.$$
$$\left. - f(x+\Delta,k,\bar{\mathfrak{s}})f(x+\Delta',k',\mathfrak{s}')\right] \quad (4)$$

$\mathrm{d}\Gamma := 2\mathrm{d}^4k\delta(k^2-m^2)\mathrm{d}S(k)$

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Local-equilibrium distribution function

$$f_{\mathsf{eq}}(x,k,\mathfrak{s}) = \exp\left(-\beta_0 E_{\mathbf{k}} + \frac{\hbar}{2}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right) \qquad (5)$$

$\mathrm{d}\Gamma := 2\mathrm{d}^4k\delta(k^2-m^2)\mathrm{d}S(k)$, $\quad \Sigma_{\mathfrak{s}}^{\mu\nu} := -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha\mathfrak{s}_\beta$, $E_{\mathbf{k}} := u^\mu k_\mu$

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- Lagrange multipliers β_0, u^μ and $\Omega_{\mu\nu}$ determine ideal spin hydrodynamics

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Moment expansion

- Split distribution function $f = f_{\mathsf{eq}} + \delta f$

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Irreducible moments

$$\rho_r^{\mu_1\cdots\mu_\ell}(x) \quad := \quad \int \mathrm{d}\Gamma\, E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{6a}$$

$k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

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$$\tau_r^{\mu,\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{6b}$$

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- Equations of motion can be derived from Boltzmann equation
- Knowing the evolution of all moments is equivalent to solving the Boltzmann equation

---

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Polarization observables in kinetic theory

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Vector Polarization (Pauli-Lubanski Pseudovector)

$$S^\mu(k) \coloneqq \mathrm{Tr}\left[\hat{S}^\mu\,\hat{\rho}(k)\right] = \frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k)\mathfrak{s}^\mu f(x,k,\mathfrak{s}) \qquad (7)$$

$N(k) \coloneqq \int \mathrm{d}\Sigma_\gamma k^\gamma \int \mathrm{d}S(k) f(x,k,\mathfrak{s})$, $\qquad \hat{S}^\mu \coloneqq -(1/2m)\epsilon^{\mu\nu\alpha\beta}\hat{J}_{\nu\alpha}\hat{P}_\beta$

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Tensor Polarization

$$\rho_{00}(k) \quad = \quad \frac{1}{3} - \sqrt{\frac{2}{3}} \epsilon_\mu^{(0)}(k) \epsilon_\nu^{(0)}(k) \Theta^{\mu\nu}(k) \tag{8a}$$

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$$\Theta^{\mu\nu}(k) := \frac{1}{2}\sqrt{\frac{3}{2}}\mathrm{Tr}\left[\left(\hat{S}^{(\mu}\hat{S}^{\nu)} + \frac{4}{3}K^{\mu\nu}\right)\hat{\rho}(k)\right]$$

$$= \frac{1}{2}\sqrt{\frac{3}{2}}\frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k) K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta f(x,k,\mathfrak{s}) \qquad (8b)$$

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- Which moments are contained in **conserved quantities** and **polarization observables**?

Needed moments

$$\Pi := -\frac{m^2}{3}\rho_0 \,, \quad \pi^{\mu\nu} := \rho_0^{\mu\nu} \quad (T^{\mu\nu}) \tag{9a}$$

$$\mathfrak{p}^{\mu} := \tau_0^{\langle\mu\rangle} \,, \quad \mathfrak{z}^{\mu\nu} := \tau_1^{(\langle\mu\rangle,\langle\nu\rangle)} \,, \quad \mathfrak{q}^{\lambda\mu\nu} := \tau_0^{\langle\lambda\rangle,\mu\nu} \quad (J^{\lambda\mu\nu}) \tag{9b}$$

$$\psi_1^{\mu\nu} \,, \quad \psi_0^{\mu\nu,\lambda} \quad (\Theta^{\mu\nu}) \tag{9c}$$

Results I: Dissipative Spin Hydro

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Dissipative Hydro: Evolution equations

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \text{h.o.t.} \tag{10a}$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \text{h.o.t.} \tag{10b}$$

$$\tau_{\mathfrak{p}} \dot{\mathfrak{p}}^{\langle\mu\rangle} + \mathfrak{p}^{\langle\mu\rangle} = \mathfrak{e}^{(0)}(\tilde{\Omega}^{\mu\nu} - \tilde{\varpi}^{\mu\nu})u_\nu + \text{h.o.t.} \tag{10c}$$

$$\tau_{\mathfrak{z}} \dot{\mathfrak{z}}^{\langle\mu\rangle\langle\nu\rangle} + \mathfrak{z}^{\langle\mu\rangle\langle\nu\rangle} = \text{h.o.t.} \tag{10d}$$

$$\tau_{\mathfrak{q}} \dot{\mathfrak{q}}^{\langle\lambda\rangle\langle\mu\nu\rangle} + \mathfrak{q}^{\langle\lambda\rangle\langle\mu\nu\rangle} = \mathfrak{d}^{(2)}\beta_0 \sigma_\alpha{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\beta + \text{h.o.t.} \tag{10e}$$

$$\tau_{\psi_1} \dot{\psi}_1^{\langle\mu\nu\rangle} + \psi_1^{\langle\mu\nu\rangle} = \xi\beta_0\pi^{\mu\nu} + \text{h.o.t.} \tag{10f}$$

$$\tau_{\psi_0} \dot{\psi}_0^{\langle\mu\nu\rangle,\lambda} + \psi_0^{\langle\mu\nu\rangle,\lambda} = \text{h.o.t.} \tag{10g}$$

$\varpi^{\mu\nu} := -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$, $\tilde{A}^{\mu\nu} := \epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta}$

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$$\tau_\mathfrak{z} \dot{\mathfrak{z}}^{\langle\mu\rangle\langle\nu\rangle} + \mathfrak{z}^{\langle\mu\rangle\langle\nu\rangle} = \text{h.o.t.} \tag{10d}$$

$$\tau_\mathfrak{q} \dot{\mathfrak{q}}^{\langle\lambda\rangle\langle\mu\nu\rangle} + \mathfrak{q}^{\langle\lambda\rangle\langle\mu\nu\rangle} = \mathfrak{d}^{(2)}\beta_0\sigma_\alpha{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\beta + \text{h.o.t.} \tag{10e}$$

$$\tau_{\psi_1} \dot{\psi}_1^{\langle\mu\nu\rangle} + \psi_1^{\langle\mu\nu\rangle} = \xi\beta_0\pi^{\mu\nu} + \text{h.o.t.} \tag{10f}$$

$$\tau_{\psi_0} \dot{\psi}_0^{\langle\mu\nu\rangle,\lambda} + \psi_0^{\langle\mu\nu\rangle,\lambda} = \text{h.o.t.} \tag{10g}$$

- Evaluate polarization and alignment in the **Navier-Stokes limit**

$\varpi^{\mu\nu} := -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$, $\tilde{A}^{\mu\nu} := \epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta}$

Results II: Alignment

- Moments of spin-rank 2:

$$\psi_1^{\langle\mu\nu\rangle} \simeq \xi\beta_0\pi^{\mu\nu}\ , \quad \psi_0^{\langle\mu\nu\rangle,\lambda} \simeq 0 \tag{11}$$

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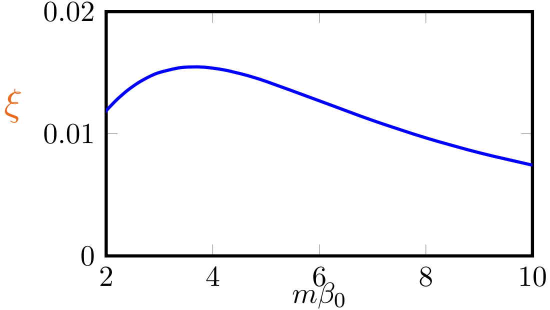

Results II: Alignment

Alignment: Explicit expression

$$\rho_{00}(k) = \frac{1}{3} - \frac{4}{15}\frac{\int \mathrm{d}\Sigma_\alpha k^\alpha \xi\, \beta_0 f_{0\mathbf{k}} \mathcal{H}^{(2,0)}_{\mathbf{k}1} \epsilon^{(0)}_\alpha \epsilon^{(0)}_\beta K^{\alpha\beta}_{\mu\nu} \Xi^{\mu\nu}_{\rho\sigma} \boldsymbol{\pi}^{\rho\sigma}}{\int \mathrm{d}\Sigma_\lambda k^\lambda f_{0\mathbf{k}} \left(1 - 3\mathcal{H}^{(0,0)}_{\mathbf{k}0} \Pi/m^2 + \mathcal{H}^{(0,2)}_{\mathbf{k}0} \pi^{\mu\nu} k_\mu k_\nu\right)}$$

- Main contribution from projected **shear-stress tensor**
- Only remnant of collisions is ξ
  - $\mathcal{H}^{(j,\ell)}_{\mathbf{k}n}$: Thermodynamic integrals

---

$$f_{0\mathbf{k}} := \exp(-\beta_0 E_\mathbf{k})$$

$$\Xi_{\mu\nu} := \Delta_{\mu\nu} + \frac{k_{\langle\mu\rangle} k_{\langle\nu\rangle}}{E^2_\mathbf{k}}\, , \; \Xi_{\mu\nu,\alpha\beta} := \frac{1}{2}\left(\Xi_{\mu\alpha}\Xi_{\nu\beta} + \Xi_{\mu\beta}\Xi_{\nu\alpha}\right) - \frac{1}{\Xi^2}\Xi_{\mu\gamma}\Xi_\nu{}^\gamma \Xi_{\delta\alpha}\Xi^\delta{}_\beta$$

$$K^{\mu\nu} := g^{\mu\nu} - k^\mu k^\nu/m^2\, , \; K^{\mu\nu}_{\alpha\beta} := (K^\mu_\alpha K^\nu_\beta + K^\mu_\beta K^\nu_\alpha)/2 - 1/3 K^{\mu\nu} K_{\alpha\beta}$$

Results III: Polarization

- Polarization is determined by the Pauli-Lubanski (pseudo)vector

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Pauli-Lubanski pseudovector (spin 1/2)

$$S^\mu(k) = \frac{1}{2\mathcal{N}} \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) \mathfrak{s}^\mu f(x,k,\mathfrak{s}) \tag{12a}$$

$$\simeq \int \mathrm{d}\Sigma_\lambda k^\lambda \frac{f_0}{2\mathcal{N}} \left\{ -\frac{\hbar}{2m} \tilde{\Omega}^{\mu\nu} k_\nu + \left(\delta^\mu_\nu - \frac{u^\mu k_{\langle\nu\rangle}}{E_{\mathbf{k}}} \right) \times \left[\mathfrak{e}\chi_{\mathfrak{p}} \left(\tilde{\Omega}^{\nu\rho} - \tilde{\varpi}^{\nu\rho} \right) u_\rho - \chi_{\mathfrak{q}} \mathfrak{d} \beta_0 \sigma_\rho{}^{\langle\alpha} \epsilon^{\beta\rangle\nu\sigma\rho} u_\sigma k_{\langle\alpha} k_{\beta\rangle} \right] \right\} \tag{12b}$$

$\mathcal{N} := \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) f(x,k,\mathfrak{s})$

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- Contains novel contributions from fluid shear

$\mathcal{N} := \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) f(x,k,\mathfrak{s})$

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Summary

- Developed dissipative spin hydrodynamics from kinetic theory
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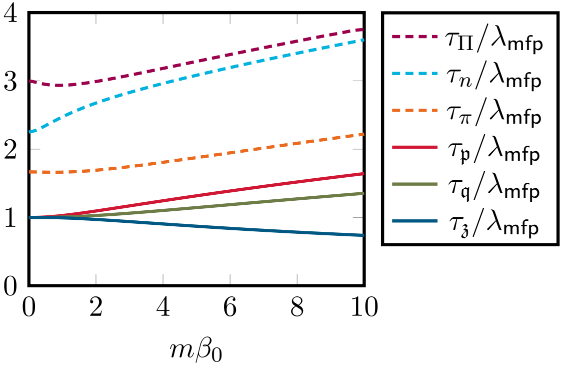
  - Employed method of moments to extract hydrodynamic limit
    - $\rightarrow$ Introduce multiple sets of moments dependent on spin
    - $\rightarrow$ Follow standard procedure to obtain equations of motion
    - $\rightarrow$ Truncate such that the evolution of $S^{\lambda\mu\nu}$ can be described
- Connected polarization and alignment to fluid quantities in the Navier-Stokes limit

Future perspectives

- Evaluate expressions for polarization and alignment with hydrodynamic simulations
- Numerically implement full spin hydrodynamics
- Move towards Spin-Magnetohydrodynamics from Quantum Kinetic Theory

Appendix

Relevant time scales: An estimation



- Simplest interaction: constant cross section
- Spin-related relaxation times shorter than standard dissipative time scales, but not much