Hybrid Hydrodynamic Attractors and Phenomenology

Ayan Mukhopadhyay
IIT Madras

Emerging Topics in
Relativistic Hydrodynamics, Chirality, Vorticity and Magnetic Fields

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Based on

- T. Mitra, S. Mondkar, AM, A. Rebhan and A. Soloviev, arXiv:2211.05480
- T. Mitra, S. Mondkar, AM, A. Rebhan and A. Soloviev, Phys. Rev. Res. 2, no.4, 043320 (2020)

Setup developed in

- E. Iancu and AM, JHEP **1506**, 003 (2015)
- AM, F. Preis, S. Stricker and A. Rebhan, JHEP**1605**,141(2016),
- S. Banerjee, N. Gaddam and A. Mukhopadhyay Phys. Rev. D **95**, no. 6, 066017 (2017),
- A. Kurkela, AM, F. Preis, A. Rebhan and A. Soloviev, JHEP 1808 (2018) 054
- C. Ecker, AM, F. Preis, A. Rebhan and A. Soloviev, JHEP 1808 (2018) 074
- S. Mondkar, AM, A. Rebhan and A. Soloviev, JHEP **11**, 080 (2021)

Motivation

Describing evolution of quark-gluon plasma in HIC requires a non-perturbative approach.

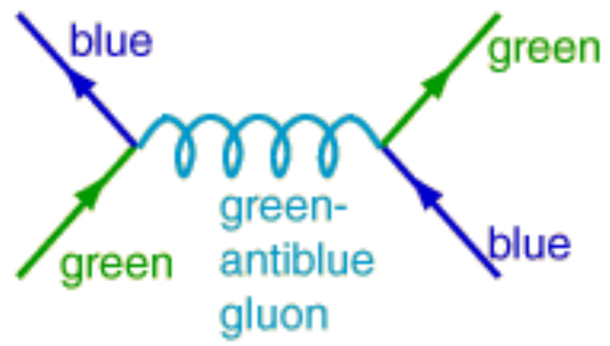
At early stages, perturbative approaches based on glasma effective theory is relevant. At later stages, a strongly interacting droplet is formed eventually admitting hydrodynamic description

Theoretical results suggest that there exists a “slowly evolving manifold” to which the dynamics monitored by coarse-grained measurements is attracted irrespective of initial conditions. This manifold can exist even up to the “tip of the for light cone” and is called the ATTRACTOR.

This attractor is a trans-series resummation of the hydrodynamic derivative expansion with precise Stokes data which gives slowest evolution.

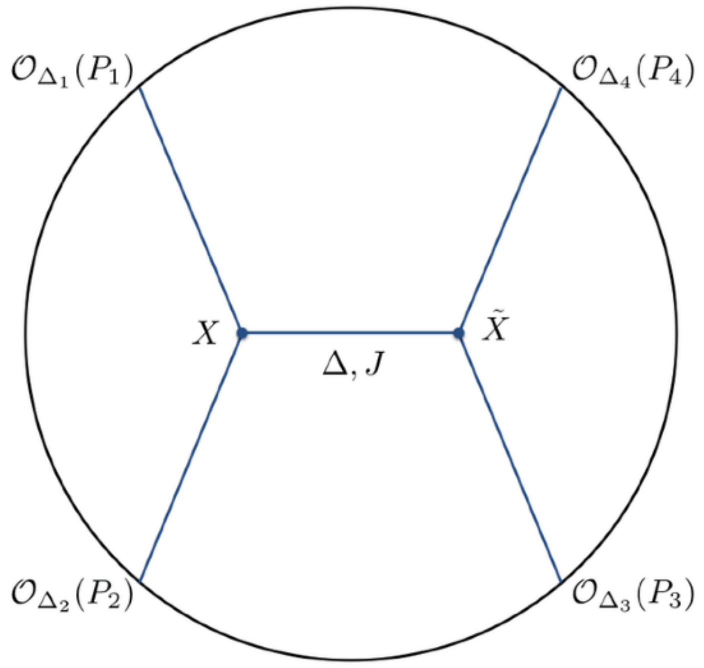
We need an approach which can **incorporate both perturbative and non-perturbative sectors throughout the entire evolution** to understand the QCD attractor in HIC.

The matching of hydrodynamic response to initial conditions on attractor surface, as defined via tuning of Stokes data, will be erroneous if we work with only one picture at either or both early and late time.



Feynman diagrams

We will describe the hybrid attractor in such a non-perturbative approach and find many results which give fresh perspectives on hydrodynamization in HIC.



itten diagrams

Setup

The Wilsonian effective action at any scale consists of the following pieces:

$S =$ perturbative terms from Feynman diagrams

$\tilde{S} =$ non-perturbative terms from the on-shell action of a holographic classical gravity theory

$S_{int} =$ interactions between two sectors

At any scale, S should determine $\tilde{S}$ and S_{int}.

What is S_{int}?

It is not terms of the form $O\tilde{O}$ or $t_{\mu\nu}\tilde{t}^{\mu\nu}$. This will be inconsistent with Wilsonian RG and also renormalizabilty.

The two sectors are coupled by promoting the marginal and relevant couplings of each sector to auxiliary fields.
S_{int} couples these auxiliary fields ultra-locally so that they are not new degrees of freedom.

The marginal and relevant couplings promoted to auxiliary fields include the background metric of each sectors also.

Although the two sectors now live in two different effective metrics in the same topological space, the full energy-momentum tensor is conserved in physical background metric which is flat for practical purposes.

The Wilsonian effective action is then

$$S[\psi, \tilde{\psi}, G_{\mu\nu}] = S[\psi, g_{\mu\nu}] + \tilde{S}[\tilde{\psi}, \tilde{g}_{\mu\nu}] + S_{\mathrm{int}}[g_{\mu\nu}, \tilde{g}_{\mu\nu}, G_{\mu\nu}],$$
$$S_{\mathrm{int}} = \frac{1}{2\gamma}\int d^d x\sqrt{-G}(g_{\mu\alpha} - G_{\mu\alpha})G^{\alpha\beta}(\tilde{g}_{\nu\beta} - G_{\nu\beta})G^{\mu\nu}$$
$$+ \frac{1}{2\gamma}\frac{\gamma'}{d\gamma' - \gamma}\int d^d x\sqrt{-G}(g_{\mu\nu}G^{\mu\nu} - d)(\tilde{g}_{\alpha\beta}G^{\alpha\beta} - d).$$

$\psi, \tilde{\psi} \rightarrow$ fields of both sectors
$g_{\mu\nu}, \tilde{g}_{\mu\nu} \rightarrow$ effective metrics in which the two systems live
$G_{\mu\nu} \longrightarrow$ physical background metric ($= \eta_{\mu\nu}$ practically)
$\gamma, \gamma' \rightarrow$ inter-system couplings of mass dimension $-d$

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0, \quad \frac{\delta S}{\delta \tilde{g}_{\mu\nu}} = 0$$

determine the effective metrics in terms of the energy-momentum tensors of the respective complementary sectors:

$$g_{\mu\nu} = G_{\mu\nu} + \frac{\sqrt{-\tilde{g}}}{\sqrt{-G}} \left[\gamma\, \tilde{t}^{\alpha\beta} G_{\alpha\mu} G_{\beta\nu} + \gamma'\, t^{\alpha\beta} G_{\alpha\beta} G_{\mu\nu}\right]$$
$$\tilde{g}_{\mu\nu} = G_{\mu\nu} + \frac{\sqrt{-g}}{\sqrt{-G}} \left[\gamma\, t^{\alpha\beta} G_{\alpha\mu} G_{\beta\nu} + \gamma'\, t^{\alpha\beta} G_{\alpha\beta} G_{\mu\nu}\right]$$

Note:

$$t^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \quad \text{and} \quad \tilde{t}^{\mu\nu} = \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta \tilde{S}}{\delta \tilde{g}_{\mu\nu}}$$

The full energy-momentum tensor is $T^{\mu\nu} = \frac{2}{\sqrt{-G}} \frac{\delta S_{total}}{\delta G_{\mu\nu}}$

Explicitly:

$$T^{\mu}{}_{\nu} = \frac{1}{2}\left(\frac{\sqrt{-g}}{\sqrt{-G}} (t^{\mu}{}_{\nu} + t_{\nu}{}^{\mu}) + \frac{\sqrt{-\tilde{g}}}{\sqrt{-G}} (\tilde{t}^{\mu}{}_{\nu} + \tilde{t}_{\nu}{}^{\mu}) \right) + \Delta K \delta^{\mu}_{\nu},$$

$$\Delta K = -\frac{1}{2} \frac{\sqrt{-g}}{\sqrt{-G}} \frac{\sqrt{-\tilde{g}}}{\sqrt{-G}} \left[\gamma\, t^{\mu\nu} G_{\mu\alpha} t^{\alpha\beta} G_{\beta\nu} + \gamma'\, t^{\alpha\beta} G_{\alpha\beta} \tilde{t}^{\rho\sigma} G_{\rho\sigma} \right].$$

From diffeomorphism invariance we obtain that on-shell $\nabla_{\mu} t^{\mu\nu} = 0 \quad \text{and} \quad \tilde{\nabla}_{\mu} \tilde{t}^{\mu\nu} = 0,$

We can readily check that these imply that on-shell $\nabla^{(B)}_{\mu} T^{\mu}{}_{\nu} = 0.$

In general the full system should be solved iteratively with initial conditions kept fixed. When iterations converge, the full stress-tensor is conserved in physical background metric.

Has been tested for classical SYM and MIS coupled to holographic gravity etc. Iterations quickly converge to very high precision. [AM, F. Preis, A Rebhan, S. Stricker 2016, C Ecker, AM, F. Preis, A Rebhan, A Soloviev 2018, S. Mondkar, AM, A Rebhan, A Soloviev 2021, to appear soon] Has been generalised to quantum setups [To appear soon]

A concrete proposal for HIC simulations exists. The couplings $\gamma, \tilde{\gamma} = \mathcal{O}(Q_s^{-4})$ [AM and E. Iancu 2014]

To find hybrid hydrodynamic attractor, we should couple glasma and kinetic theory to gravity. Possible with current state-of-the-art techniques but very hard. Good to simplify.

Simple but crude formulation

To get to hydro, we simply need to parametric $t_{\mu\nu}$ and $\tilde{t}_{\mu\nu}$ in terms of macroscopic variables.

The full em-tensor is a polynomial of $t_{\mu\nu}$ and $\tilde{t}_{\mu\nu}$ and does not require any separate constitutive relation.

$$t^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \Pi^{\mu\nu},$$
$$\tilde{t}^{\mu\nu} = (\tilde{\varepsilon} + \tilde{P})\tilde{u}^\mu \tilde{u}^\nu + \tilde{P}\tilde{g}^{\mu\nu} + \tilde{\Pi}^{\mu\nu},$$

$$\nabla_\mu t^{\mu\nu} = 0 \quad \text{and} \quad \tilde{\nabla}_\mu \tilde{t}^{\mu\nu} = 0,$$

$$(\tau_\pi u^\mu \nabla_\mu + 1)\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu},$$
$$(\tilde{\tau}_\pi \tilde{u}^\mu \tilde{\nabla}_\mu + 1)\tilde{\Pi}^{\mu\nu} = -\tilde{\eta}\tilde{\sigma}^{\mu\nu},$$

and the coupling equations which determine the effective metrics.

More explicitly, for **Bjorken flow** we make the following Ansatze

$$G_{\mu\nu} = \mathrm{diag}(-1, 1, 1, \tau^2).$$

$$g_{\mu\nu}(\tau) = \mathrm{diag}(-a(\tau)^2, b(\tau)^2, b(\tau)^2, c(\tau)^2),$$
$$\tilde{g}_{\mu\nu}(\tau) = \mathrm{diag}(-\tilde{a}(\tau)^2, \tilde{b}(\tau)^2, \tilde{b}(\tau)^2, \tilde{c}(\tau)^2).$$

$$u^\mu = \left(\frac{1}{a(\tau)}, 0, 0, 0\right) \quad \text{and} \quad u^\mu = \left(\frac{1}{\tilde{a}(\tau)}, 0, 0, 0\right)$$

$$\Pi^{\mu\nu} = \mathrm{diag}\left(0, \frac{\phi(\tau)}{2}, \frac{\phi(\tau)}{2}, \frac{-\phi(\tau)}{\tau^2}\right)$$
$$\tilde{\Pi}^{\mu\nu} = \mathrm{diag}\left(0, \frac{\tilde{\phi}(\tau)}{2}, \frac{\tilde{\phi}(\tau)}{2}, \frac{-\tilde{\phi}(\tau)}{\tau^2}\right)$$

Variables: $a, b, c, \tilde{a}, \tilde{b}, \tilde{c}, \epsilon, \tilde{\epsilon}, \phi, \tilde{\phi}$ (total 10 variables)

We have 10 equations for the 10 variables.

The conservation of em-tensors give:

$$0 = \partial_\tau \varepsilon + \frac{4}{3}\varepsilon \partial_\tau \log(b^2 c) + \phi \partial_\tau \log(\frac{b}{c}),$$
$$0 = \partial_\tau \tilde{\varepsilon} + \frac{4}{3}\tilde{\varepsilon} \partial_\tau \log(\tilde{b}^2 \tilde{c}) + \tilde{\phi} \partial_\tau \log(\frac{\tilde{b}}{\tilde{c}}),$$

The conformal MIS equations give

$$0 = \tau_\pi \partial_\tau \phi + \frac{4}{3}\eta \partial_\tau \log \frac{b}{c} + \left[a + \frac{4}{3}\tau_\pi \log(b^2 c)\right] \phi,$$
$$0 = \tilde{\tau}_\pi \partial_\tau \tilde{\phi} + \frac{4}{3}\tilde{\eta} \partial_\tau \log \frac{\tilde{b}}{\tilde{c}} + \left[\tilde{a} + \frac{4}{3}\tilde{\tau}_\pi \log(\tilde{b}^2 \tilde{c})\right] \tilde{\phi},$$

Then we have six ALGEBRAIC coupling equations

$$
\begin{aligned}
1-a^2 &= \gamma\frac{\tilde{a}\tilde{b}^2\tilde{c}}{\tau}\Big[\frac{\tilde{\varepsilon}}{\tilde{a}^2}+r(-\frac{\tilde{\varepsilon}}{\tilde{a}^2}+\frac{\tilde{\phi}}{\tilde{b}^2}+\frac{2\tilde{\varepsilon}}{3\tilde{b}^2}+\tau^2(\frac{\tilde{\varepsilon}}{3\tilde{c}^2}-\frac{\tilde{\phi}}{\tilde{c}^2}))\Big],\\
b^2-1 &= \gamma\frac{\tilde{a}\tilde{b}^2\tilde{c}}{\tau}\Big[\frac{(\tilde{\phi}+\frac{2\tilde{\varepsilon}}{3})}{2\tilde{b}^2}-r(-\frac{\tilde{\varepsilon}}{\tilde{a}^2}+\frac{\tilde{\phi}}{\tilde{b}^2}+\frac{2\tilde{\varepsilon}}{3\tilde{b}^2}+\tau^2(\frac{\tilde{\varepsilon}}{3\tilde{c}^2}-\frac{\tilde{\phi}}{\tilde{c}^2}))\Big],\\
c^2-\tau^2 &= \gamma\frac{\tilde{a}\tilde{b}^2\tilde{c}}{\tau}\Big[\frac{\tau^4}{\tilde{c}^2}(\frac{\tilde{\varepsilon}}{3}-\tilde{\phi})-r\tau^2(-\frac{\tilde{\varepsilon}}{\tilde{a}^2}+\frac{\tilde{\phi}}{\tilde{b}^2}+\frac{2\tilde{\varepsilon}}{3\tilde{b}^2}+\tau^2(\frac{\tilde{\varepsilon}}{3\tilde{c}^2}-\frac{\tilde{\phi}}{\tilde{c}^2}))\Big],\\
1-\tilde{a}^2 &= \gamma\frac{ab^2c}{\tau}\Big[\frac{\varepsilon}{a^2}+r(-\frac{\varepsilon}{a^2}+\frac{\phi}{b^2}+\frac{2\varepsilon}{3b^2}+\tau^2(\frac{\varepsilon}{3c^2}-\frac{\phi}{c^2}))\Big],\\
\tilde{b}^2-1 &= \gamma\frac{ab^2c}{\tau}\Big[\frac{(\phi+\frac{2\varepsilon}{3})}{2b^2}-r(-\frac{\varepsilon}{a^2}+\frac{\phi}{b^2}+\frac{2\varepsilon}{3b^2}+\tau^2(\frac{\varepsilon}{3c^2}-\frac{\phi}{c^2}))\Big],\\
\tilde{c}^2-\tau^2 &= \gamma\frac{ab^2c}{\tau}\Big[\frac{\tau^4}{c^2}(\frac{\varepsilon}{3}-\phi)-r\tau^2(-\frac{\epsilon(\tau)}{a^2}+\frac{\phi}{b^2}+\frac{2\varepsilon}{3b^2}+\tau^2(\frac{\varepsilon}{3c^2}-\frac{\phi}{c^2}))\Big]
\end{aligned}
$$

We need initial conditions for $\epsilon, \tilde{\varepsilon}, \phi, \tilde{\phi}$

Then we solve 6 coupling equations and use four dynamical equations to propagate.

No need for iterations.

We assume that both sectors are weakly coupled and strongly coupled conformal theories respectively.

So,

$$
\eta = C_\eta \frac{\varepsilon + P}{\varepsilon^{1/4}}, \quad \text{and} \quad \tau_\pi = C_\tau \varepsilon^{-1/4},
$$
$$
\tilde{\eta} = \tilde{C}_\eta \frac{\tilde{\varepsilon} + \tilde{P}}{\tilde{\varepsilon}^{1/4}}, \quad \text{and} \quad \tilde{\tau}_\pi = \tilde{C}_\tau \tilde{\varepsilon}^{-1/4}.
$$

As for example, we choose

$$
\tilde{C}_\eta = \frac{1}{4\pi}, \quad \tilde{C}_\tau = \frac{2 - \log 2}{2\pi}, \quad C_\eta = 10\tilde{C}_\eta, \quad \text{and} \quad C_\tau = 5C_\eta.
$$

Attractor Surface

There exists a two-dimensional attractor surface in four-dimensional phase space.

This surface is ruled by slow evolution curves.

Any solution approaches one such curve on the attractor surface

$$\chi = \frac{\phi}{\epsilon + P}, \quad \tilde{\chi} = \frac{\tilde{\phi}}{\tilde{\epsilon} + \tilde{P}}$$

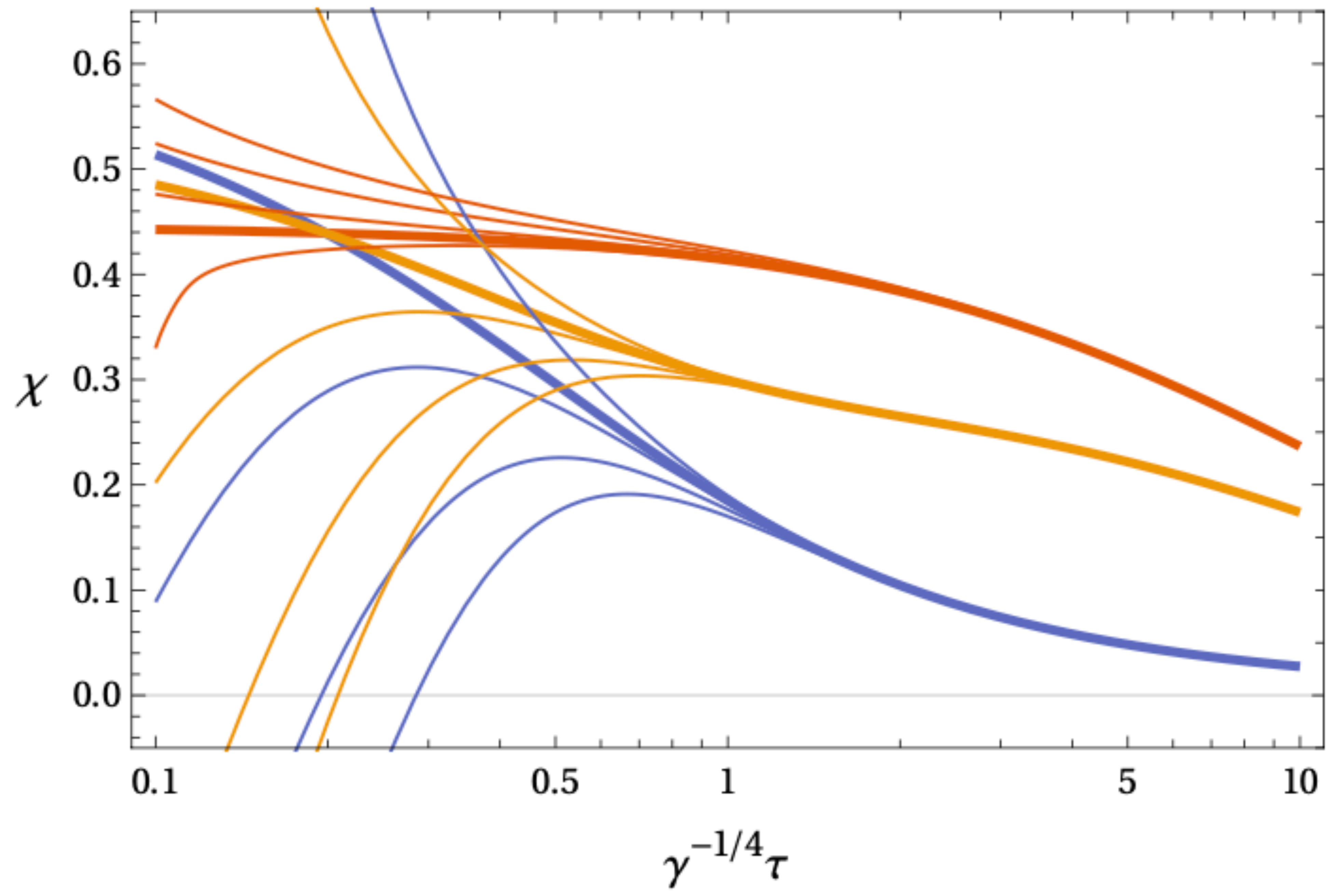


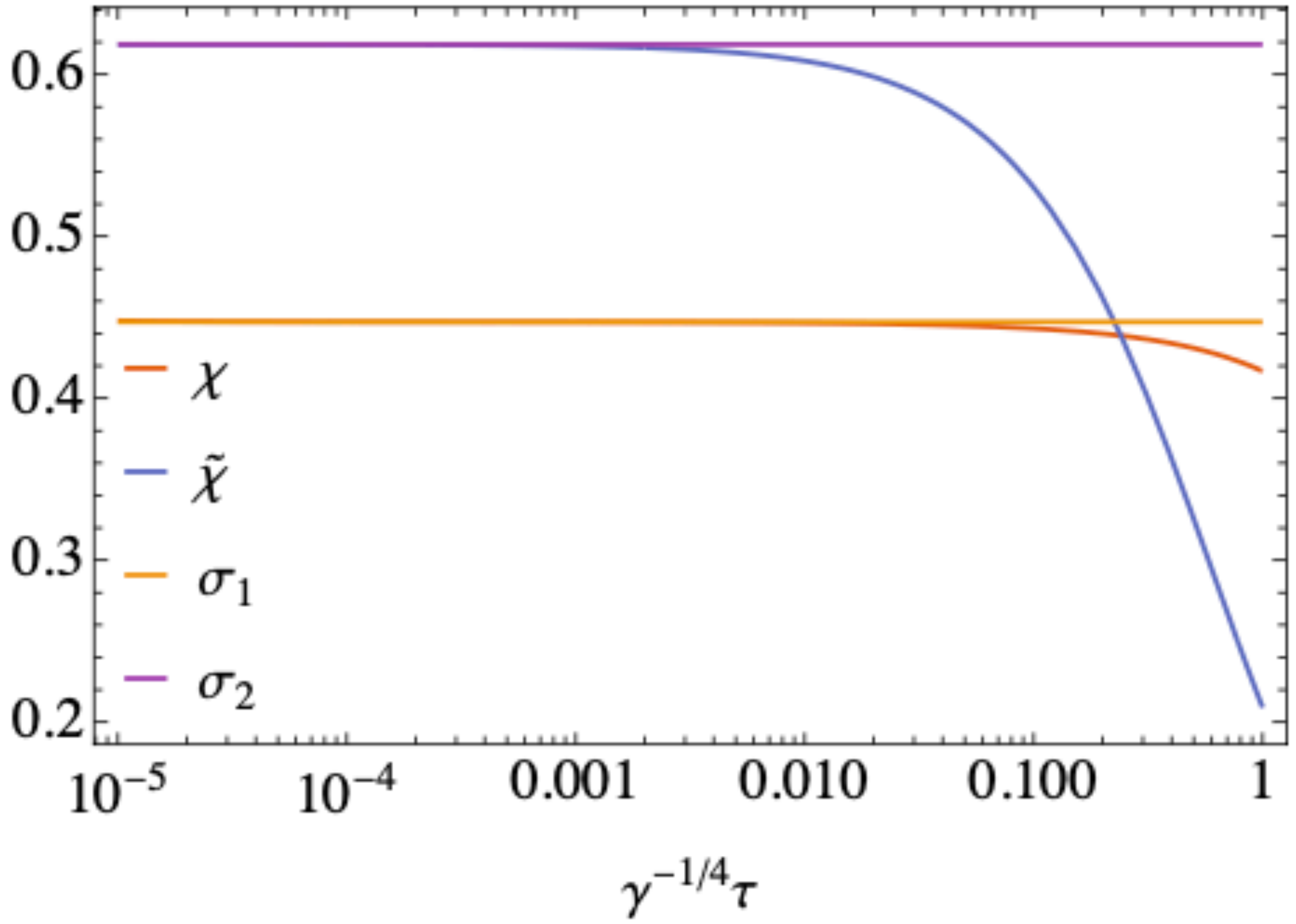
Figure 1. Anisotropies χ of one particular attractor solution (thick lines) and four neighboring trajectories (thin lines). The less viscous (strongly coupled) system is displayed by blue curves, the more viscous (more weakly coupled) system by red curves, the total system by orange curves. From [53], where more plots are given for this particular solution.

Early time

$$\lim_{\tau\to 0}\chi = \sigma_1, \quad \lim_{\tau\to 0}\tilde{\chi} = \sigma_2,$$

where

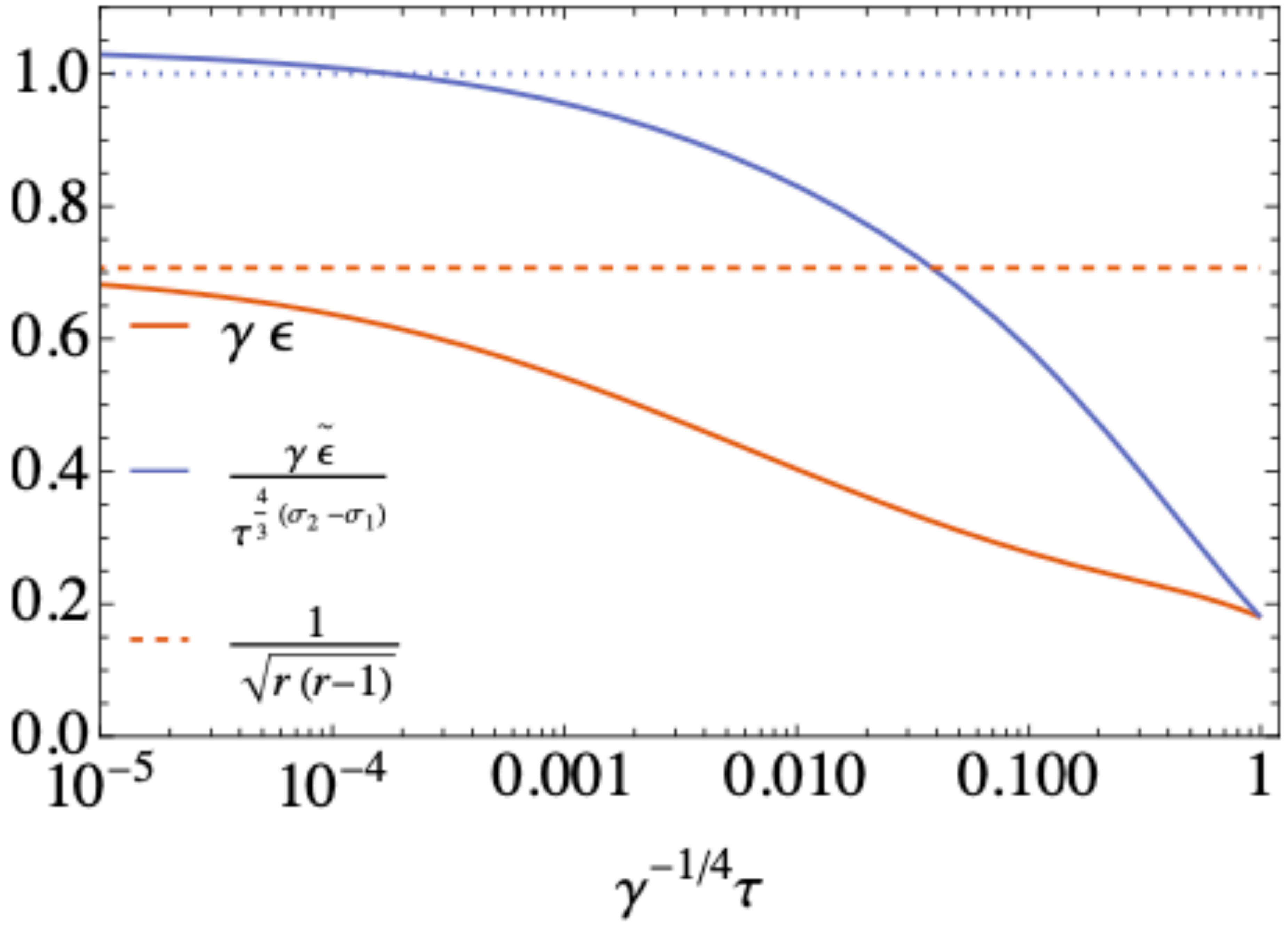
$$\sigma_1 = \sqrt{\frac{C_\eta}{C_\tau}}, \quad \sigma_2 = \sqrt{\frac{\tilde{C}_\eta}{\tilde{C}_\tau}}.$$



Furthermore,

$$\gamma\varepsilon \to \sqrt{\frac{r-1}{r}}, \qquad \gamma\tilde{\varepsilon} \to g_2 \tau^{\frac{4}{3}(\sigma_2-\sigma_1)},$$
$$\chi \to \sigma_1 + k_1 \tau^{\frac{1}{3}(\sigma_1+2)}, \qquad \tilde{\chi} \to \sigma_2 + k_2 \tau^{\frac{5}{3}\sigma_2 - \frac{4}{3}\sigma_1 + \frac{2}{3}}$$

$$r = -\gamma'/\gamma$$



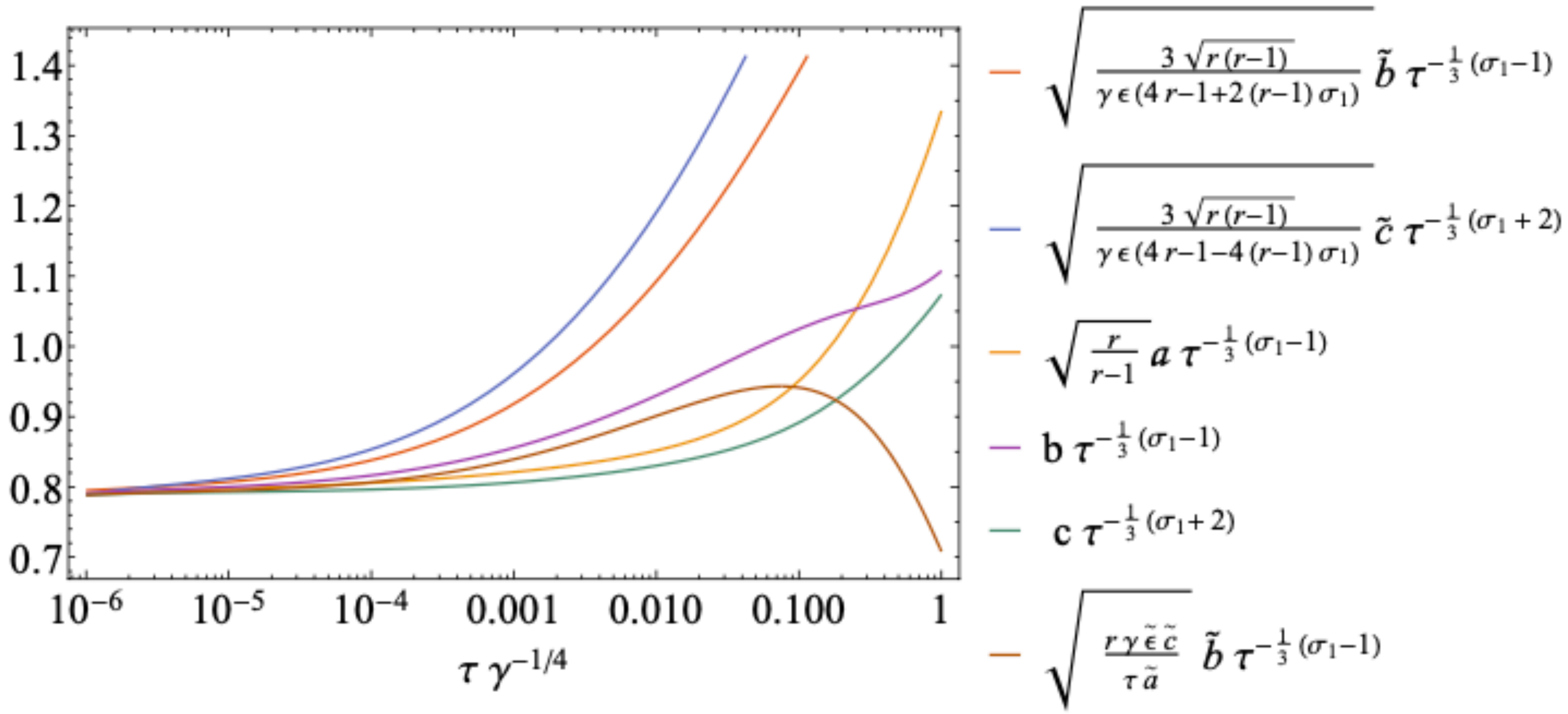
The full early time expansion us given two variables, namely g_2 and k_2 which parametrize the attractor surface

In the limit $\tau \to 0$

$$\frac{b}{\tau^{\frac{1}{3}(\sigma_1-1)}} = \frac{c}{\tau^{\frac{1}{3}(\sigma_1+2)}} = \sqrt{\frac{r}{r-1}}\frac{a}{\tau^{\frac{1}{3}(\sigma_1-1)}} = \frac{\tilde{b}}{\tau^{\frac{1}{3}(\sigma_1-1)}}\sqrt{r\gamma\tilde{\varepsilon}\frac{\tilde{c}}{\tau\tilde{a}}}$$

$$= \sqrt{\frac{3\sqrt{r(r-1)}}{\gamma\varepsilon(4r-1-2(r-1)\chi)}}\frac{\tilde{b}}{\tau^{\frac{1}{3}(\sigma_1-1)}} = \sqrt{\frac{3\sqrt{r(r-1)}}{\gamma\varepsilon(4r-1+4(r-1)\chi)}}\frac{\tilde{c}}{\tau^{\frac{1}{3}(\sigma_1+2)}}$$

Numeircal check is fantastic



Bottom up scenario is universal!

$$\mathcal{E}_1 = \varepsilon \frac{ab^2c}{\tau} \sim \tau^{\frac{4}{3}\sigma_1 - \frac{4}{3}}, \quad \mathcal{E}_2 = \varepsilon \frac{\tilde{a}\tilde{b}^2\tilde{c}}{\tau} \sim \tau^{\frac{8}{3}\sigma_2 - \frac{4}{3}\sigma_1 - \frac{4}{3}},$$

and therefore

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} \sim \tau^{\frac{8}{3}(\sigma_2 - \sigma_1)}.$$

Note, $\sigma_2 > \sigma_1$ since the perturbative sector is weakly self-interacting compared to the non-perturbative sector.

Therefore, at early time the perturbative sector always dominates the energy density (in physical background metric).

The exponent of dominance can be written simply in terns of transport and relaxation data!

Late time hydrodynamic expansion

At late time, the evolution can be described by hydrodynamic (gradient) expansion.

Each curve on attractor surface is parametrised by two variables which determine the hydro expansion, are dimensionless and time-reparametrization invariant.

$$\varepsilon = \epsilon_{1,pf} + \cdots , \quad \tilde{\varepsilon} = \epsilon_{2,pf} + \cdots , \quad \epsilon_{1,pf} = \epsilon_{10} \left(\frac{\tau}{\tau_0} \right)^{-4/3} , \quad \epsilon_{2,pf} = \epsilon_{20} \left(\frac{\tau}{\tau_0} \right)^{-4/3}$$

$$\alpha := \gamma(\epsilon_{20}\tau_0^{4/3})\sqrt{\epsilon_{10}\tau_0^{4/3}} = \gamma\epsilon_{20}\sqrt{\epsilon_{10}}\tau_0^2 ,$$

$$\beta := \gamma(\epsilon_{10}\tau_0^{4/3})\sqrt{\epsilon_{20}\tau_0^{4/3}} = \gamma\epsilon_{10}\sqrt{\epsilon_{20}}\tau_0^2 .$$

The full system can be described as a SINGLE fluid in the physical flat background metric.

The equation of state and transport coefficients depends on the attractor curve.

The hydrodynamic expansion of the total em-tensor (up to first order) can be described by the equations

$$\mathcal{E}' = -\frac{1}{\tau}\left(\mathcal{E} + \mathcal{P}(\mathcal{E})\right) + \frac{1}{\tau^2}\left(\zeta(\mathcal{E}) + \frac{2}{3}\eta(\mathcal{E})\right)$$

$$\mathcal{P}(\mathcal{E}) = \frac{\mathcal{E}}{3} + k\mathcal{E}^2 + \cdots$$

$$\eta(\mathcal{E}) = \kappa\mathcal{E}^{3/4} + \cdots, \quad \zeta(\mathcal{E}) = \tilde{\kappa}\mathcal{E}^{3/4} + \cdots.$$

$$T^\mu_\mu = 3\mathcal{P}(\mathcal{E}) - \mathcal{E} - \frac{3}{\tau}\zeta(\mathcal{E})$$

The equations of state and the transport coefficients of full system are given by k, κ and $\tilde{\kappa}$ which are determined by α and β

$$k = -\frac{1}{3}\gamma\frac{\epsilon_{10}\epsilon_{20}}{\mathcal{E}_0^2} = -\frac{1}{3}\gamma\frac{\epsilon_{10}\epsilon_{20}}{(\epsilon_{10}+\epsilon_{20})^2} = -\frac{1}{3}\gamma\frac{\alpha^2\beta^2}{(\alpha^2+\beta^2)^2},$$

$$\text{and } \tilde{\kappa} = 0,$$

$$C_{\eta,eff} = \frac{C_\eta \epsilon_{10}^{3/4} + \tilde{C}_\eta \epsilon_{20}^{3/4}}{(\epsilon_{10}+\epsilon_{20})^{3/4}} = \frac{C_\eta \beta^{3/4} + \tilde{C}_\eta \alpha^{3/4}}{(\alpha^2+\beta^2)^{3/4}}$$

$$\kappa = \frac{4}{3}C_{\eta,eff}$$

$$\kappa = \frac{4}{3}C_{\eta,eff}$$

Hydrodynamization

Hydrodynamization time is defined as the proper time after which the evolution of the anisotropic pressure is well described by first order hydrodynamics

$$\frac{|\Delta P_L|}{P} := \frac{|\phi - \phi_{1st}|}{P} < 0.1, \quad \text{for} \quad \tau > \tau_{hd},$$

It is different for the two sectors.

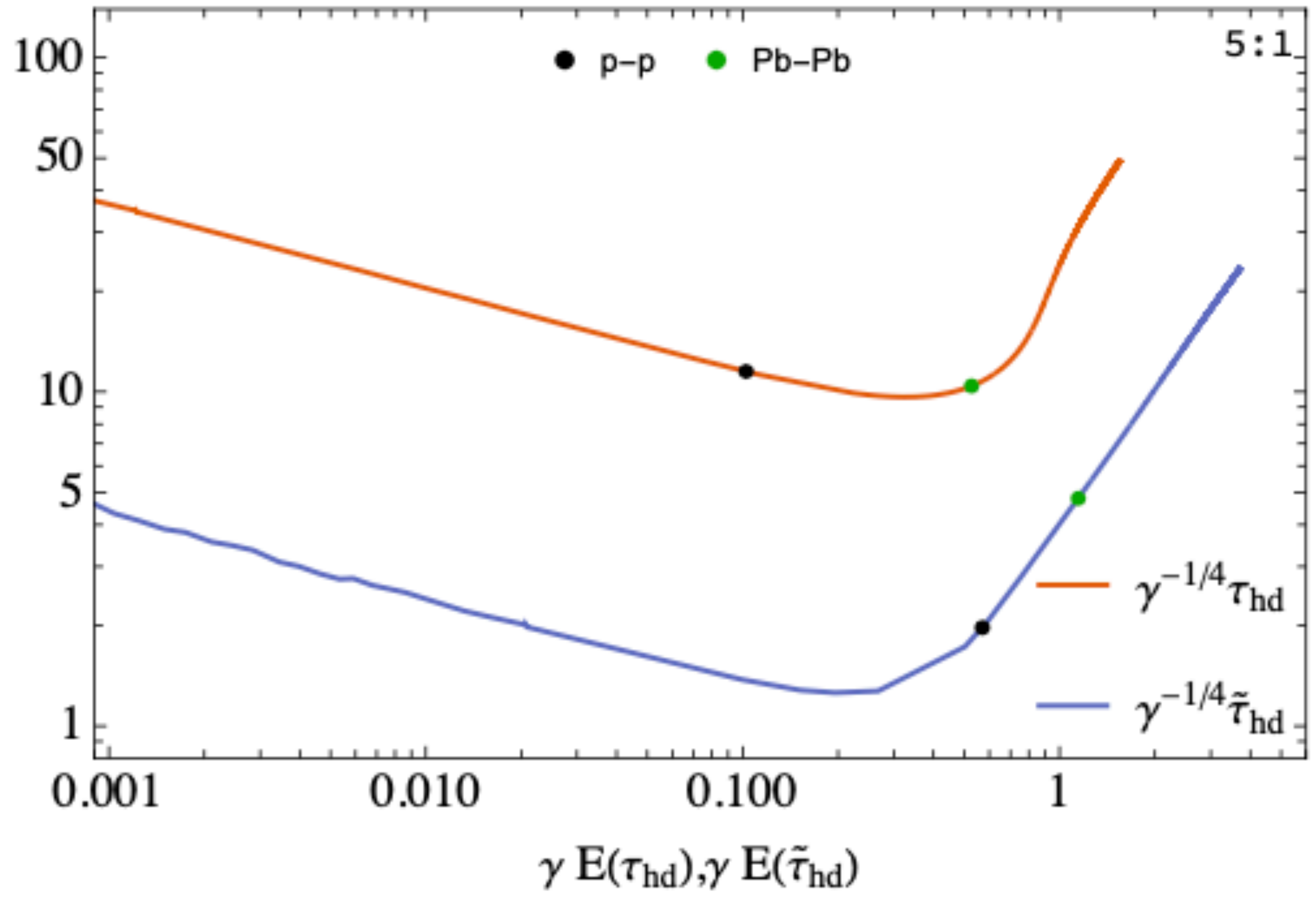
In both sectors there are three regimes

Conformal regime: $$0.0001\gamma^{-1} \lessapprox \mathcal{E}(\tau_{hd}), \mathcal{E}(\tilde{\tau}_{hd}) \lessapprox \gamma^{-1}$$

$$\tau_{hd} \sim p \times (4\pi C_\eta) \times \mathcal{E}(\tau_{hd})^{-1/4}$$

and

$$\tilde{\tau}_{hd} \sim \tilde{p} \times (4\pi \tilde{C}_\eta) \times \mathcal{E}(\tilde{\tau}_{hd})^{-1/4}$$

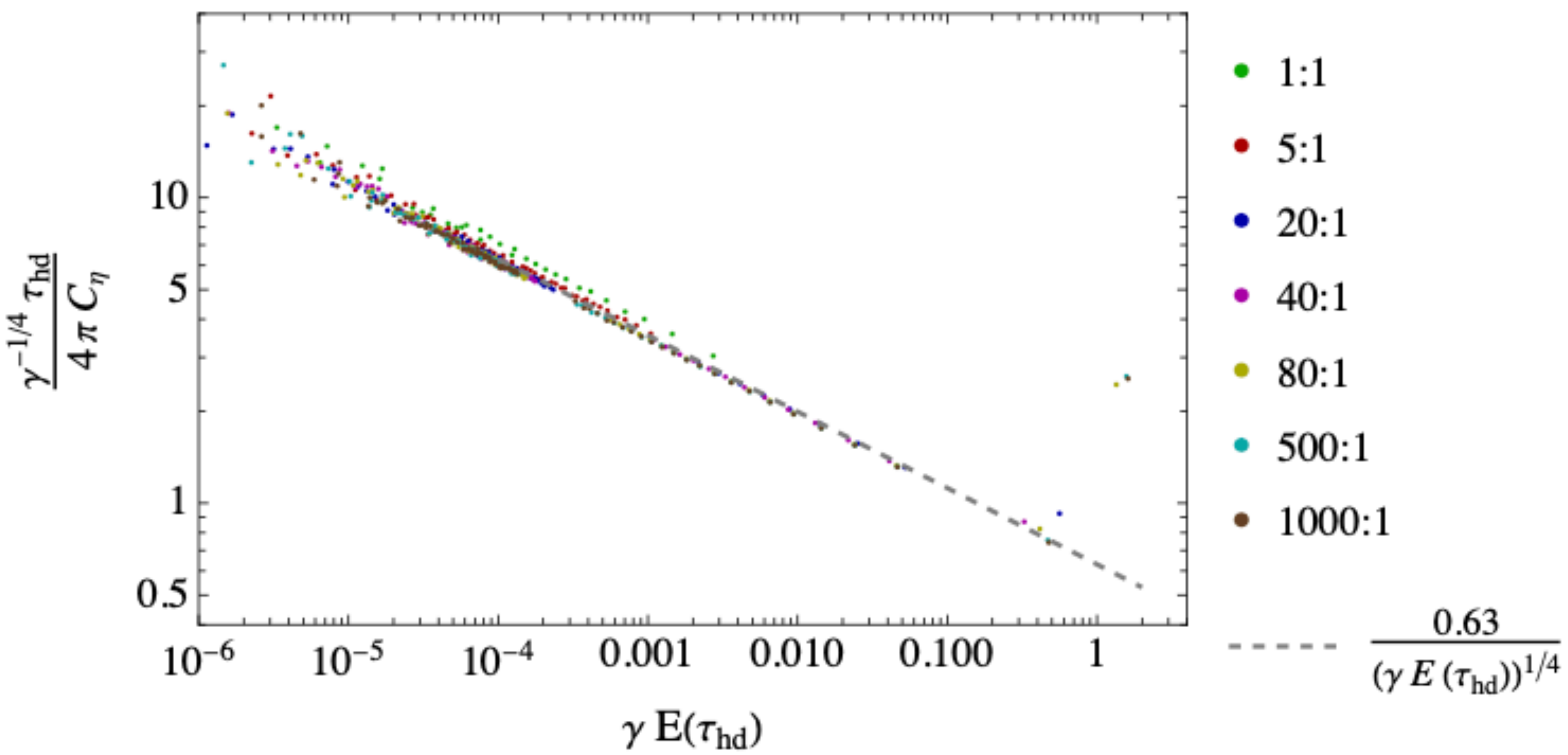


Intermediate regime: $\mathscr{E}(\tau_{hd}), \tilde{\mathscr{E}}(\tilde{\tau}_{hd}) \approx \gamma^{-1}$
Here hydrodynamization times reach minimal values

Large energy regime: $\mathscr{E}(\tau_{hd}), \tilde{\mathscr{E}}(\tilde{\tau}_{hd}) > \gamma^{-1}$
Here hydrodynamization times increase!

In the conformal regime, p and $\tilde{p}$ are determined by which sector is dominant around $\tau \approx 0.001\gamma^{1/4}$

For typical (and realistic) initial conditions, the perturbative sector is dominant

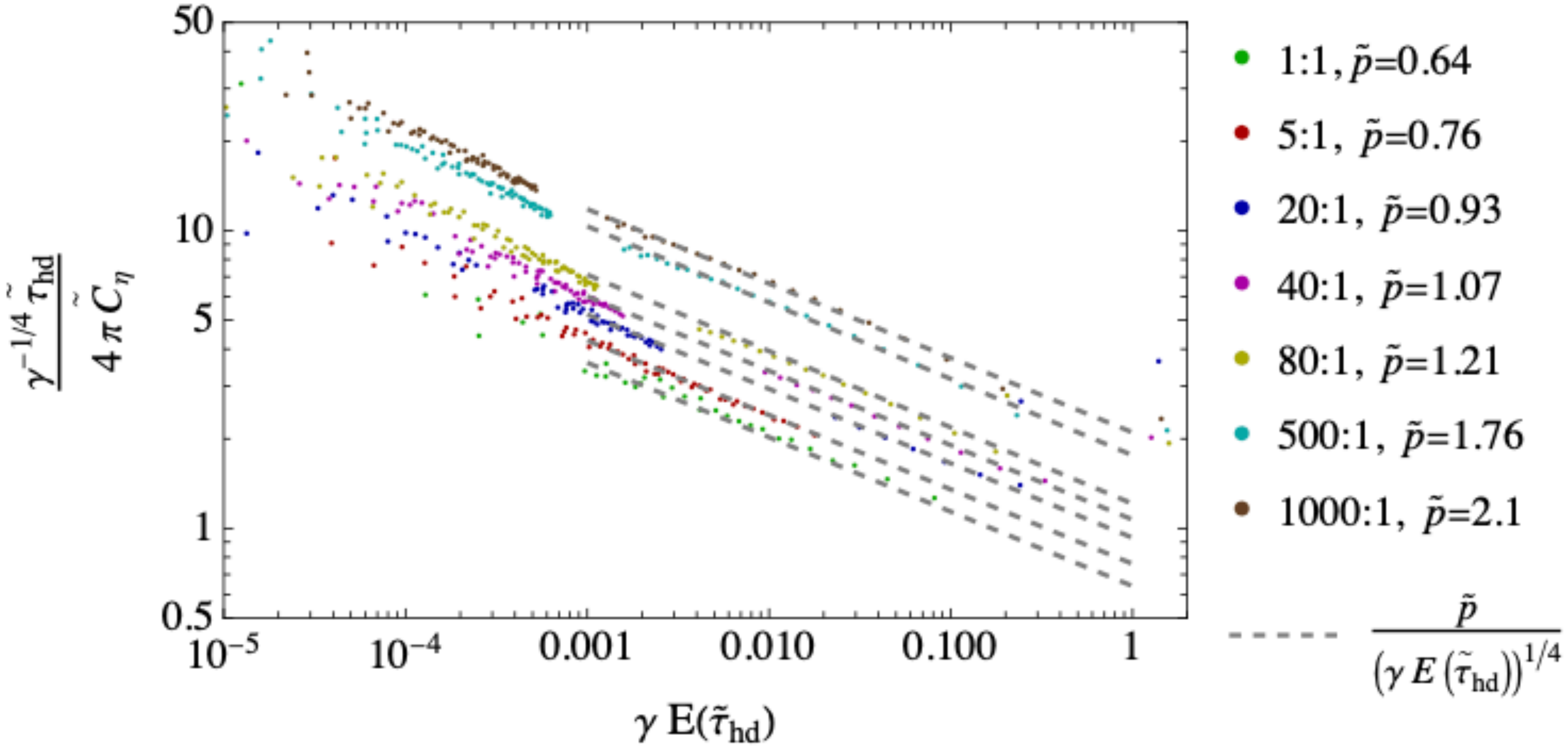


p is universal!

See plots for various rates of ϵ to $\tilde{\epsilon}$ at $\tau \approx 0.001\gamma^{1/4}$

Furthermore, $p = 0.63$ and independent of the couplings! It is thus the same as in usual conformal attractors!

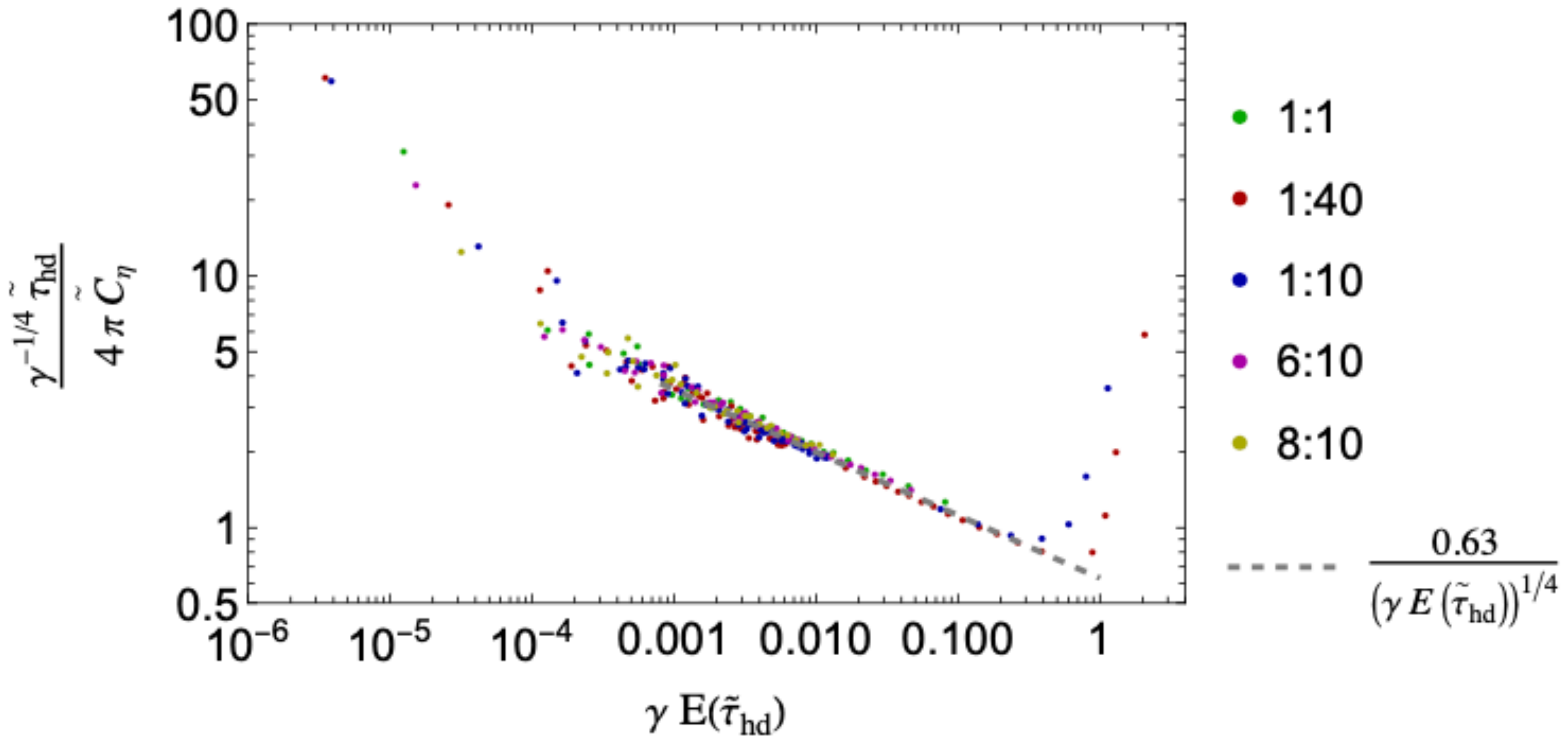
The value of $\tilde{p}$ increases with the ratio of ϵ to $\tilde{\epsilon}$ at $\tau \approx 0.001\gamma^{1/4}$. Thus interactions with the perturbative sector delays hydrodynamization of the strongly self-interacting sub-sector

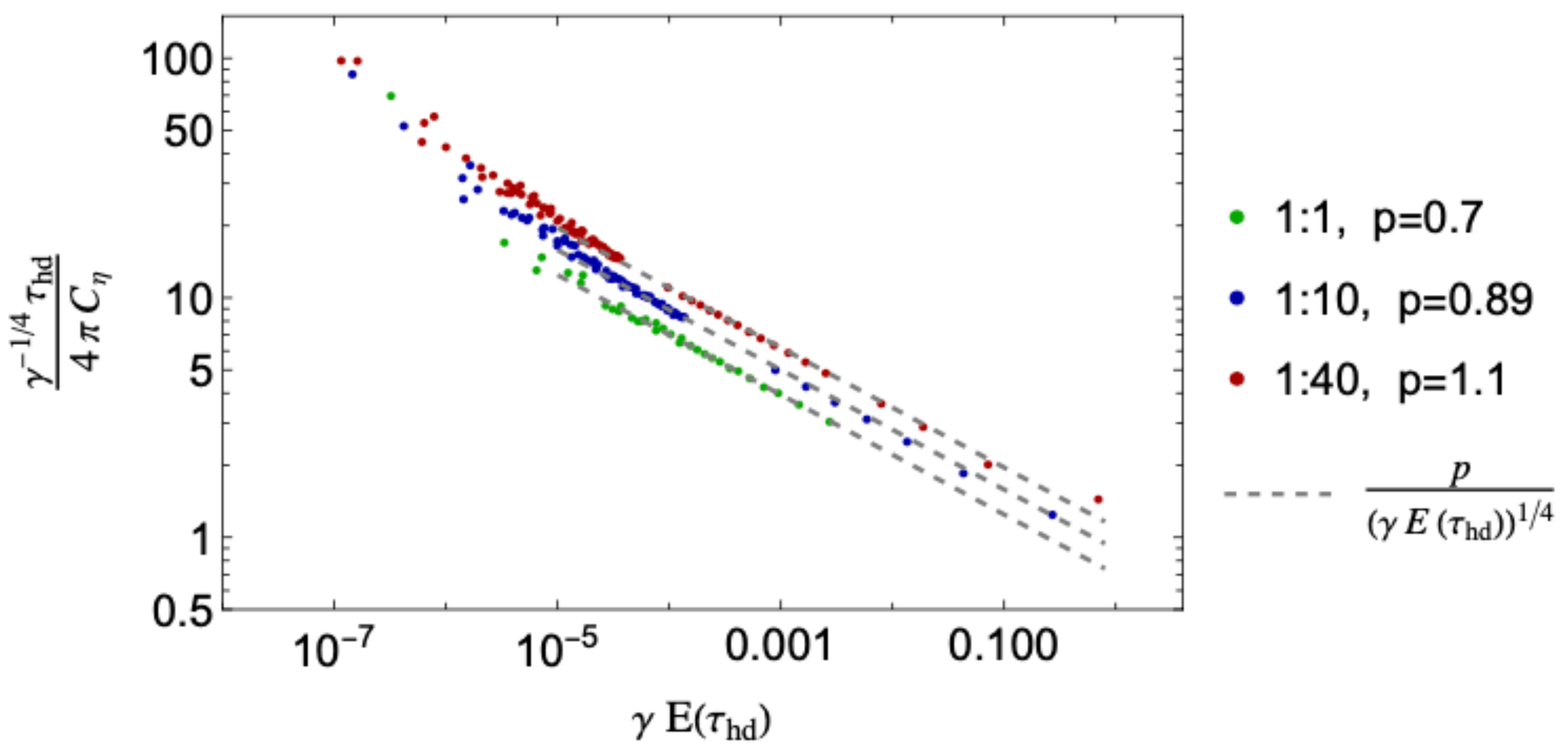


See plots for various ratios of ϵ to $\tilde{\epsilon}$ at $\tau \approx 0.001\gamma^{1/4}$

In the flipped case, the hydrodynamization time of the non-perturbative sector is unversal.

The hydrodynamization of perturbative sector is delayed. However, this is not relevant for phenomenology





Phenomenological Relevance: Small vs Large System Collisions

We identify $\gamma^{-1/4}$ with Q_s^{p-p}.

Realistic initial conditions can be obtained from glasma models.

They satisfy

$\mathscr{E}_{tot}(\tau = Q_s^{-1/4}) \approx Q_s^4$

To compare small vs large system collisions: $\boxed{Q^2_{s,Pb} = 2.5\, Q^2_{s,p}}$

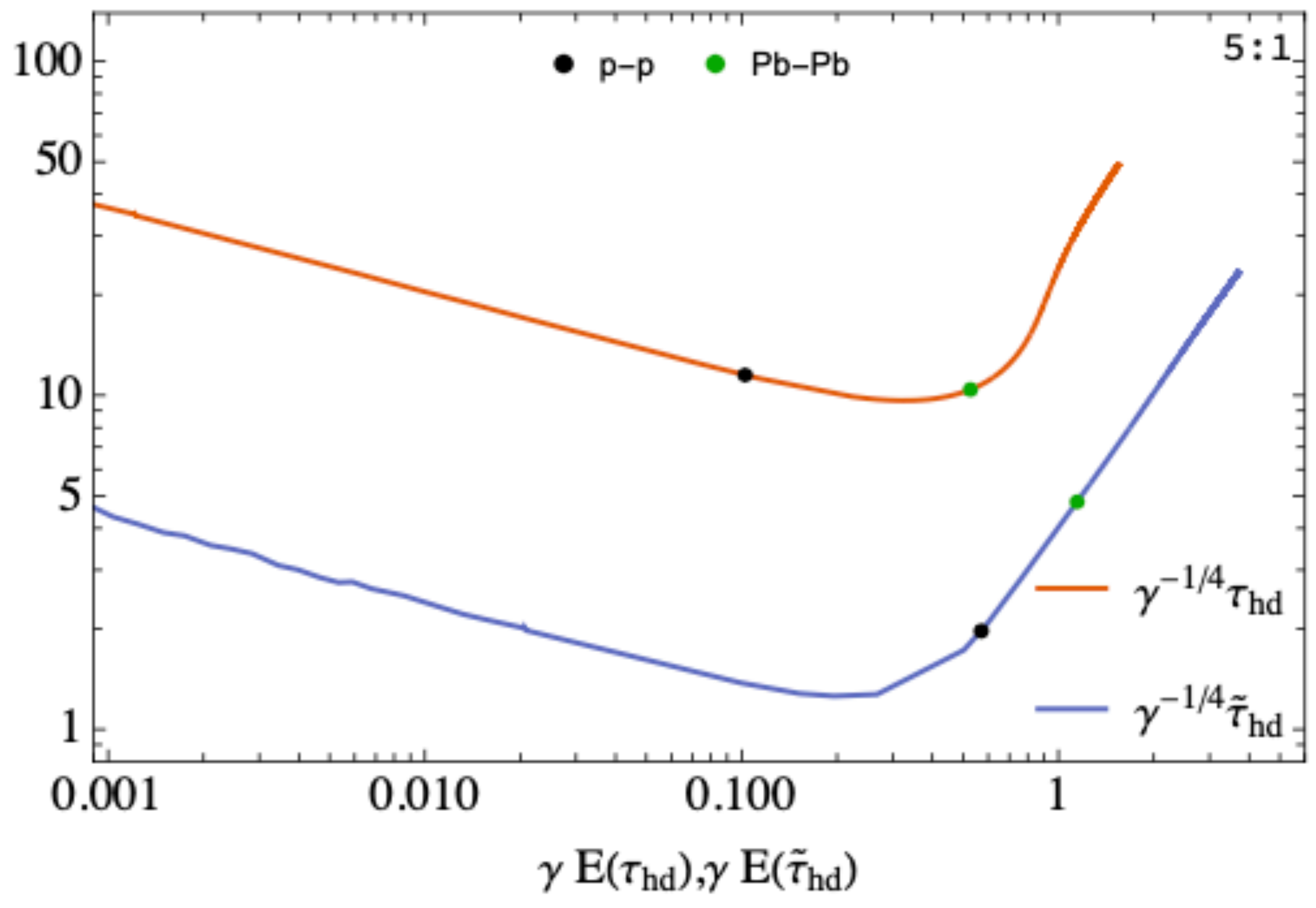
The IPsat model gives

$$Q^2_{s,A} \sim 0.4(x/0.001)^{-0.3} A^{1/3} Q^2_{s,p}.$$

The perturbative sector hydrodynamizes earlier in Pb-Pb collisions compared to p-p collisions.

This is because $\mathscr{E}(\tau_{hd})$ lies within or at the edge of the conformal window.

Furthermore, since hydrodynamization in conformal window, the hydrodynamization time is independent of initial conditions except for total energy density



The non-perturbative sector hydrodynamizes earlier in p-p collisions compared to Pb-Pb collisions.

This is because $\tilde{\mathscr{E}}(\tilde{\tau}_{hd})$ lies well outside the conformal window.

Summary-1

- The hybrid Bjorken flow attractor surface is a two-dimensional surface in four-dimensional phase space. The surface is ruled by curves labelled by g and k_2 (giving early time expansion) or α and β (giving hydro expansion).
- Bottom up scenario is universal with the perturbative sector dominating over the non-perturbative sector at $\tau \approx 0$ with an exponent determined by transport and relaxation data
- **At late time the full system can be described as a single fluid with equation of state and transport coefficients dependent on α and β**

Summary-2

- The hydrodynamization times (τ_{hd} and $\tilde{\tau}_{\mathrm{hd}}$) of both subsections have three regimes: conformal where τ_{hd} and $\tilde{\tau}_{\mathrm{hd}}$ decrease with the energy density at respective hydrodynamization times, intermediate where they reach minimal values and then increase.
- **The perturbative sector hydrodynamizes faster in large system collisions**
- **The non-perturbative sector hydrodynamizes faster in small system collisions**
- **The hydrodynamization time of non-perturbative sector is independent of the coupling and also initial conditions except for the total energy density**

Outlook

- It is necessary to understand the full kinetic theory plus holographic gravity hybrid setup. The dynamics has many non-trivial features which cannot be obtained by coupling two MIS systems.

- The superfluid story is also very rich where we need to also obtain the attractor quantum trajectory for the condensate.

Thanks for the invitation and for your attention!