Study of the initial state characteristics of heavy ion collision systems and their connections with the final state characteristics

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Emergent Topics in Relativistic Hydrodynamics, Chirality, Vorticity and Magntic field
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Outline

- Vorticity distribution at different stages
- Turbulence power spectrum analysis for velocity fields and temperature fluctuations.
- Study temperature fluctuations using Tsallis entropic index.

Why study vorticity?

- There are observational evidence of the polarization of Λ hyperons caused by vorticity.
- In off-central heavy-ion collisions, huge orbital angular momenta of order $10^3 - 10^5\hbar$ are generated. How such orbital angular momenta are distributed in the hot and dense matter is an interesting topic to be investigated. There is an inherent correlation between rotation and particle polarization.
- Vorticity contributes in the study of the fluid's viscous characteristics. We are interested in assessing how shear viscosity affects the vorticity patterns observed in the QGP.
- It would be interesting to see how the interplay between viscosity and angular momentum affects the nature of vorticity distribution across the collision energy range.
- Anomalous transport effects similar to CME can be generated in presence of vorticity.

- Our calculation is based on the hadronic transport approach referred to as AMPT (A Multi-Phase Transport) Model. We study the partonic as well as the hadronic matter with the string melting version of AMPT.

- We have generated 10^4 AMPT events for Au+Au collision for the centre of mass energy $20 GeV - 200 GeV$ and the particles are coarse-grained using a $3 + 1$-dimensional (3+1D) space-time grid.

- The average energy and velocities content in a cell at point (z, x) has been calculated by doing $v = \frac{\sum p_i}{\sum E_i}$ for each grid cell, where E_i and p_i are energy and momentum of the i-th particle.

- Then vorticity patterns are generated using the different definitions of vorticity.

Definitions

- The classical and relativistic vorticity can be defined as

$$\vec{\omega}(\mathbf{x}, t) = \vec{\nabla} \times \vec{v} \tag{1}$$

$$\omega_{\mu\nu} = \frac{1}{2}(\partial_\nu u_\mu - \partial_\mu u_\nu) \tag{2}$$

$u_\mu = \gamma(1, \mathbf{v}) \Longrightarrow$ fluid velocity four-vector
$\gamma = 1/\sqrt{1 - \mathbf{v}^2} \Longrightarrow$ Lorentz factor.

- For the reaction plane,

$$\omega_{xz} = \frac{1}{2}\gamma(\partial_z v_x - \partial_x v_z) + \frac{1}{2}(v_x \partial_z \gamma - v_z \partial_x \gamma) \tag{3}$$

- The thermal vorticity is defined by,

$$\omega^T_{\mu\nu} = \frac{1}{2}(\partial_\nu \beta_\mu - \partial_\mu \beta_\nu) \tag{4}$$

Here $\beta_\mu = \frac{u_\mu}{T}$.

F. Becattini et al. Annual Review of Nuclear and Particle Science, 70, 395-423 (2020).

Vorticity distribution

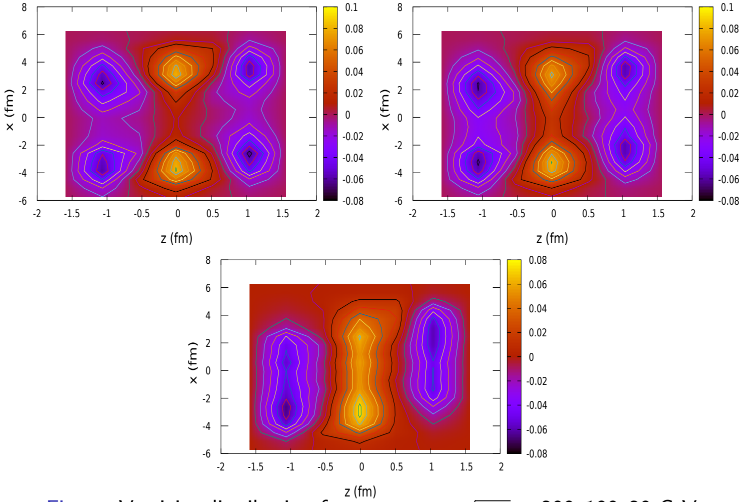


Figure: Vorticity distribution for partons at $\sqrt{s_{NN}} = 200, 100, 20$ GeV

Thermal vorticity distribution

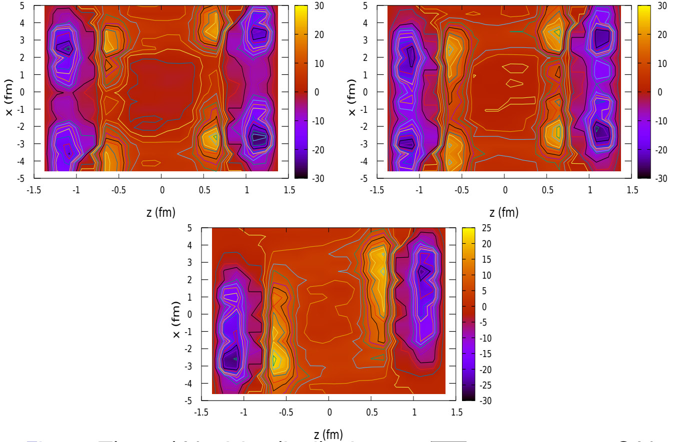


Figure: Thermal Vorticity distribution at $\sqrt{s_{NN}} = 200, 100, 20$ GeV

Vorticity distribution for hadrons

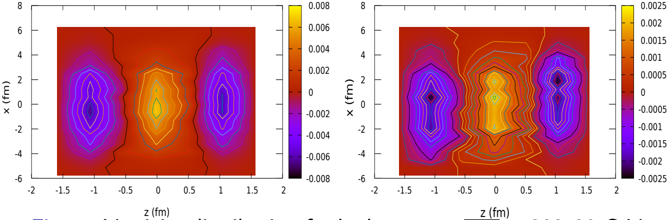


Figure: Vorticity distribution for hadrons at $\sqrt{s_{NN}} = 200, 20$ GeV

The global Λ polarization

$$P = 2\frac{< S^* > .J}{|J|} \tag{5}$$

$$\mathbf{S}^*(x, p) = \mathbf{S} - \frac{\mathbf{p}.\mathbf{S}}{E_p(m + E_p)}\mathbf{p} \tag{6}$$

The ensemble average of the spin vector for spin-1/2 fermions

$$S^\mu(x, p) = -\frac{1}{8m}(1 - n_F)\epsilon^{\mu\nu\rho\sigma} p_\nu \omega_{\rho\sigma}(x) \tag{7}$$

Average vorticity behavior

$$< \omega_{xz} > \; = \; \frac{\sum \epsilon_{ij} \omega_{xz}^{ij}}{\sum \epsilon_{ij}}$$

Hui Li et al. Phys. Rev. C 96, 054908 (2017)

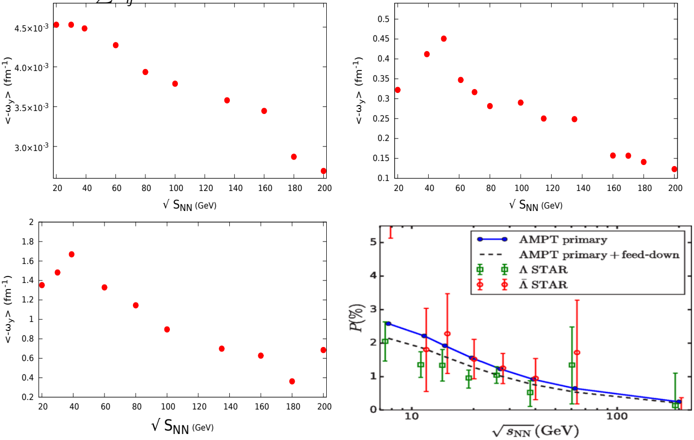


Figure: a)Non-relativistic and b)relativistic c) thermal $\langle \omega_{xz} \rangle \equiv \langle \omega_y \rangle$ at different $\sqrt{s_{NN}}$

η/s effect

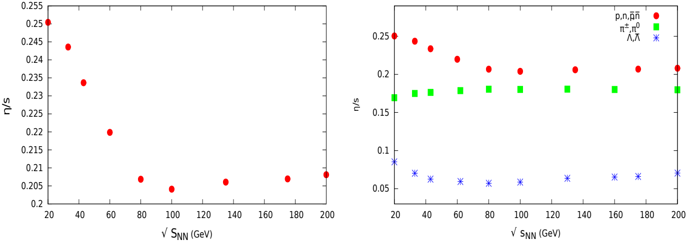


Figure: The specific shear viscosity at different $\sqrt{s_{NN}}$

EPJ Web of Conferences 259, 11001 (2022)

Phys. Rev. C 91, 064901 (2015)

Anisotropic turbulence in relativistic plasmas

- The Reynolds number is exceedingly high for QGP produced in HIC. As a result, many elements of viscous flow, especially whether or not turbulence emerges, remain unclear based on our simulations.

  Brett McInnes, Nucl. Phys. B 921, 39-58 (2017)

- In heavy ion collisions, we have anisotropy in the final stage particle distribution. This is shown in terms of the higher order flow harmonics in particle azimuthal distribution.

- These anisotropies are the results of initial stage nucleon distribution. This is given by higher harmonics of eccentricity, triangularity etc. This depends on the system under consideration e.g. (Au-Au, Pb-Pb, Pb-p) and the distribution of partons inside nucleons.

From large scales to small scales

- These are large scale anisotropies. These don't show us the initial energy density distribution and their dissipation in the QGP system. To observe the initial state anisotropy, we have to examine the small scale fluctuations.
- We proposed two ways of studying the small scale energy dissipation.
  - Velocity power spectra
  - Temperature power spectra
- We assume that the initial QGP system acts like a turbulent fluid and we obtain the turbulent spectra.
- By studying the turbulent spectra, we can study the small scale anisotropy and the energy dissipation in a turbulent fluid.

Turbulence Spectra

- Turbulent fluid means the velocity of the fluid elements have component other than the laminar component. $u(x) =< u > +u^{'}(x)$, here $< u >$ is the laminar component and $u^{'} = u- < u >$ is the turbulent component.
- The characteristic of a turbulent fluid is the presence of eddies. These are swirling patterns in the fluid.
- In a turbulent flow energy cascades from the largest length scale to smaller length scales.

- $\langle \vec{u} \rangle = \lim_{\Delta x \to 0} \int_V \frac{\vec{u} d^3 x}{V}$
- $u^{'}(x) = u(x) - < u >$ is the turbulent component. x represents the cell positions. The dimensions are

$dx = dy = dz = 0.3 fm$ and Lattice Size(LS) = 48

- The size of our system is LS$\times$ dx= 48$\times$0.3 = 14.4 fm $\equiv$ The diameter of Au nuclei.

The minimum k range $k_{min} = 0.52\ fm^{-1}$

- The smallest length scale or the Kolmogorov scale is given by,

$$\eta = \left(\frac{\nu^3}{\epsilon_d} \right)^{1/4} \tag{8}$$

ν is kinematic viscosity with $\nu_{QGP} \equiv 10^{-7} \frac{m^2}{s} \equiv 1.69 GeV^{-1}$

Kostya Trachenko et al. SciPost Phys. 10, 118 (2021)

- $\epsilon_d = 2\ GeV/fm^3 \rightarrow \eta = 1.24 fm$ and $k_{max} \approxeq 5$

- $Re_{QGP} \geqq 8.52$ and $k_{max} \approxeq 3$ Brett McInnes, Nucl. Phys. B 921, 39-58 (2017)

- In the simulations, we use the range of the energy spectrum from $k = 0.5$ to $k = 20$ to include all possible length scales of the $Au - Au$ collisions at RHIC energies.

- The energy spectrum is obtained by doing a Fourier transform of velocity correlation tensor R_{ij}.

'

$$E(k) = 2\pi k^2 \sum_{i,j} \phi_{ij}(k), \phi_{ij} = \frac{1}{(2\pi)^3} \int R_{ij} e^{-i\vec{k}.\vec{r}} dr \quad (9)$$

$$R_{ij}(r) \equiv < u_i^{'}(\vec{x}) u_j^{'}(\vec{x} + \vec{r}) > \quad (10)$$

Boost Effect

- Since the particles are colliding with relativistic velocities along the z-axis, we need to take care of the Lorentz boost effect when we calculate the velocity correlation in the longitudinal plane.

$$R_{ij} = \Lambda(d/2)\Lambda(-d/2) < u'_i(r - d/2), u'_j(r + d/2) > \tag{11}$$

  Here $\Lambda(\Delta x)$ is the boost which brings $u(x + \Delta x)$ to $u(x)$, $\Lambda(\Delta x)u(x + \Delta x) = u(x)$.

- The correlator is boosted to the local reference frame of the midpoint between the two points whose correlation we are interested in. We obtain the $\Lambda(d/2)$ matrices for an infinitesimal boost. If $\phi_A(x)$ is a hydrodynamic fluctuation field,

$$[\Lambda(\Delta x)\phi]_\mu = \phi_\mu - u_\mu(\Delta u.\phi) + \Delta u_\mu(u.\phi)$$

Xin An et al. Phys. Rev. C 100, 024910 (2019)

- In 1883, Kolmogorov hypothesized that the amount of energy in a turbulent flow carried by eddies of diameter d, gravitate towards $d^{5/3}$. But this is only valid within a specific range of length scales known as the inertial subrange. Thus in this range, Kolmogorov spectra will have a power-law nature where the energy is given by,

$$E(k) \approx k^{-\nu} \approx k^{-5/3} \tag{12}$$

V. E. Zakharov et al. Kolmogorov Spectra of Turbulence I 1992, ISBN : 978-3-642-50054-1

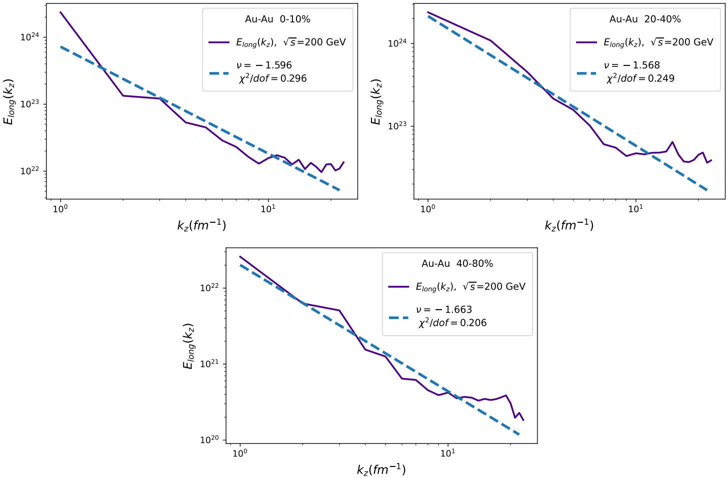
- This fits well for many turbulent flows on scales between the correlation length λ_t and the Kolmogorov length η .

- For isotropic spectra the power law exponent is -5/3.

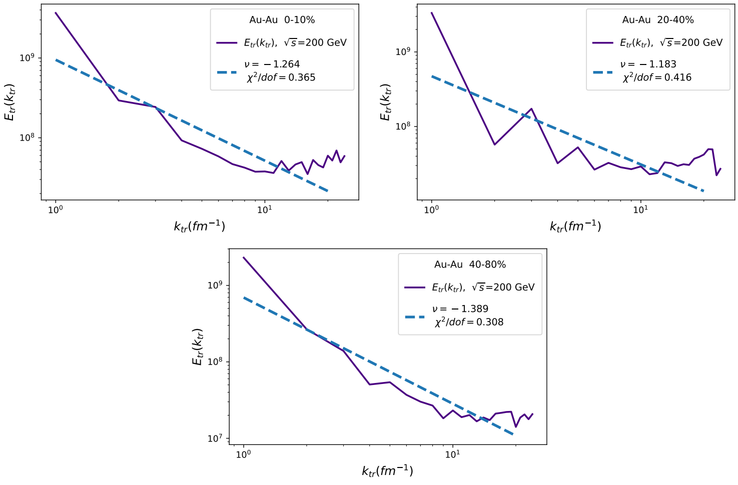
- This exponent can have various value and deviation from -5/3 will show the degree of anisotropy in energy dissipation.

Velocity Power Spectrum

Reaction Plane Spectra



Transverse plane spectra



Observations I

- The spectra on the reaction plane have $\nu \approx -1.64$ which is closer to the inertial range of Kolmogorov spectra. On this plane there is no anisotropy in velocity spectra.
- On the transverse plane the exponent ν is less than -5/3. It is closer to -4/3 and have values from -1.1 to -1.45 depending on the configuration.
- We have observed this behavior for all the RHIC energies starting from 7.7 GeV to 200 GeV with centralities 0-10%, 20-40% and 40-80%.
- So, on the transverse plane we observe the anisotropy. The large scale anisotropies are also seen on the transverse plane.
- We are out of inertial subrange of Kolmogorov spectra.

Temperature Power Spectrum

- We calculate the temperature power spectrum in a similar way as is done in the case of velocity power spectrum. Here temperature correlation $m = < T(\vec{x}) T(\vec{x} + \vec{r}) >$ is obtained first and then the Fourier transform is plotted against k.

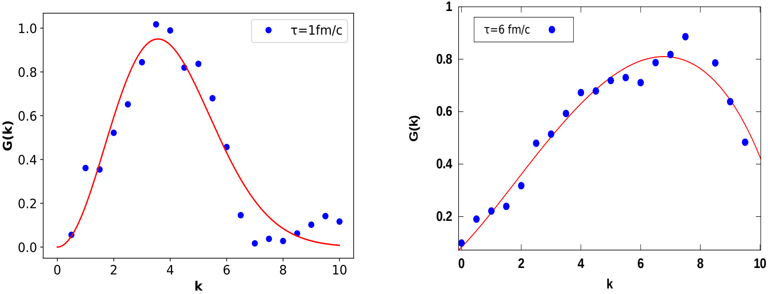


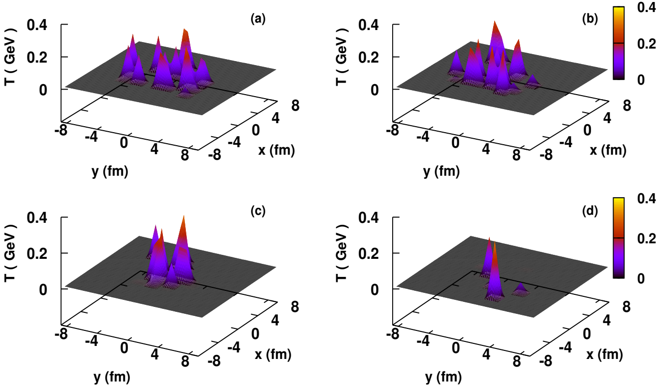
Figure: Temperature power spectrum of Au-Au central collision at a collision energy 200 GeV at $\tau = 1 fm/c$ and $\tau = 6 fm/c$. The unit of k is fm^{-1}

Observations II

- The minimum length scale of temperature spectrum in RHIC energies is 1.24 fm which is similar to the length scale of velocity turbulence.
- Although the initial spectrum is fitted with a Gaussian which is expected for a isotropic turbulence, there are points at the lower length scales which are far from the fitted line. This can be thought as a signature of anisotropy.
- At later times, the peak of the distribution shifted and it is no longer be fitted with a Gaussian distribution. This gives us the anisotropies in the system.

Temperature fluctuations and Tsallis statistics in relativistic heavy ion collisions

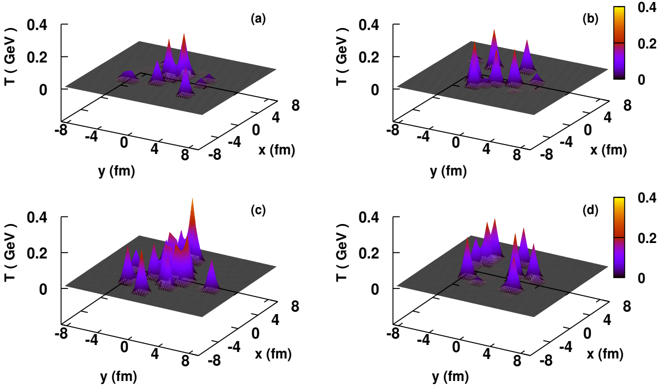
Temperature fluctuation



The fluctuations are higher in the initial stages but decrease with time.

Figure: Temperature fluctuations at times a) 1 fm/c b) 2 fm/c c) 3 fm/c d) 5fm/c at $\sqrt{s} = 100$ GeV

Temperature fluctuation



The general pattern of the fluctuations remain the same, only amplitude changes.

Figure: Temperature fluctuations a) $\sqrt{s} = 19.6$ GeV b) $\sqrt{s} = 62.4$ GeV c) $\sqrt{s} = 100$ GeV d) $\sqrt{s} = 200$ GeV at time $\tau = 1$ fm/c

Why do we need Tsallis Statistics?

- The standard distribution alone can not describe the whole transverse momentum spectra. Also for the p-p collision and the peripheral heavy ion collision.
- Standard distribution can give a good fit if one considers 2 or 3 emission sources and take their weighted contribution to match the overall distribution. Tsallis distribution, on the other hand, due to its generalized nature, have inbuilt power-law nature. So, it can be fitted for the soft transverse momentum spectra.

$$f(p_T) = A \sum_{i=1}^{n_0} k_{Si} \left[exp \left(\pm \frac{\sqrt{p_T^2 + m_0^2} coshy - \mu}{T} \right) + S \right]^{-1}$$

$$f(p_T) = \left[1 + \frac{q-1}{T} (\sqrt{p_T^2 + m_0^2} coshy - \mu) \right]^{-\frac{1}{q-1}}$$

q$\rightarrow$ asymmetry parameter, $q \rightarrow 1$ gives standard distribution.

H. Zheng et al., Adv. High Energy Phys. 2016, 9632126, 10 (2016)

- There are many experimental evidences that $q \neq 1$ yields a correct description of many complex physical phenomena, including
    - hydro-dynamic turbulence: Phys. Rev. 63E, 035303(R)(2001),J. Phys. 33A, L235 (2000)
    - scattering processes in particle physics: J. Miranda, Physica 286A, 156 (2000), Physica 286A, 164 (2000)
    - self-gravitating systems in astrophysics: Phys. Lett. 174A, 384 (1993), Astrophys. Lett. and Comm. 35, 449 (1998).

So, our aim is to study the temperature fluctuations in the initial stage and model these fluctuations using Tsallis statistics. After modeling we will get entropic index showing us the degree of anisotropy. We will study various system parameters in the initial stage to see the changes in the entropic index.

Modeling fluctuations to get non-extensive parameter

If a non-equilibrium system is formally described by a fluctuating β, then the generalized distribution functions of non-extensive Tsallis statistics are a consequence of integrating over all possible fluctuating β's provided that the β is χ^2 distributed.

$$(1+(q-1)\beta_0 H)^{-\frac{1}{q-1}} = \int_0^\infty e^{-\beta H} f(\beta) d\beta \qquad (13)$$

Since our system too has temperature fluctuations, we fit the probability distribution of β (i.e the temperature inverse) with a χ^2 distribution. The distribution that we use for the fit is given by,

$$f(\beta) = \frac{1}{\Gamma(\frac{1}{(q-1)})} \left(\frac{1}{(q-1)\beta_0} \right)^{\frac{1}{(q-1)}} \beta^{\frac{1}{q-1}-1} \exp\left(\frac{-\beta}{(q-1)\beta_0} \right) \qquad (14)$$

Euro. phys. Lett., 57 (3), pp. 329-333 (2002).

χ^2 fitting to the fluctuating β's

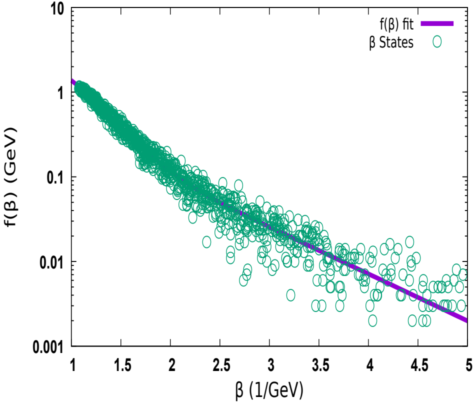


Figure: The plot of β for the temperature fluctuations. The value of (q-1) is obtained by fitting a χ^2 distribution to the plot.

- We obtain a good fit for our temperature fluctuations.
- In this fitting, we have taken q as one of the fitting parameters and q can be obtained from these fits.
- The constant β_0 is the average of the fluctuating β, $E(\beta) = \int_0^{\infty} \beta f(\beta) d\beta$
  q can be calculated from the relative variance, $q - 1 = \frac{E(\beta^2) - E(\beta)^2}{E(\beta)^2}$
- The q values we get in this way are higher compared to the experimentally observed q values which are calculated by fitting the transverse momentum spectra to the Tsallis distribution.
- We have taken into consideration the initial system which have has lot more fluctuations.

Results:

G. Wilk, Z. Włodarczyk, Phys. Rev. C 79, 054903 (2009)

$$T_{eff} = 0.22 - 1.25(q-1)$$

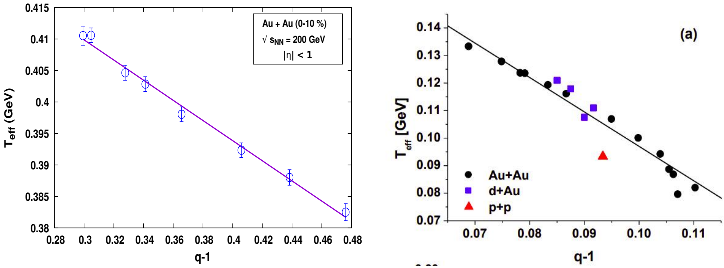


Figure: The plot shows the dependence of the effective temperature T_{eff} on the values of q at a collision energy of $\sqrt{s} = 200$ GeV

We find that there exists a linear relationship between the temperature and the entropic index which can be fitted with a straight line

q vs η plot

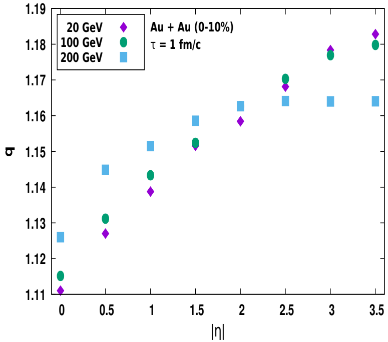


Figure: The plot shows the variation of q for different space time rapidity (η) values at different collision energies $(\sqrt{s})$

Remarks:

- Here we find that with increasing values of space time rapidity our q value increases plateauing out for higher collision energies.
- It has been shown in previous studies, the multiplicity distribution has to be of the negative binomial type and can be related to the entropic index.
- A higher multiplicity distribution will result in lower q values.

q vs $\sqrt{s}$ plot

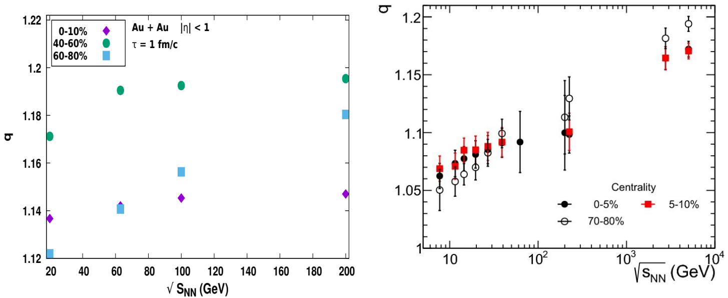


Figure: This plot shows the variation of q for different $\sqrt{s}$ values at different centralities.

Tapan K. Nayak et al. Eur. Phys. J. Plus 136, 702 (2021)

Remarks made from the q vs $\sqrt{s}$ plot

- We find that at lower collision energies the Tsallis entropic index is lower. However, the dependence is dependent on the centrality of the collisions.
- For peripheral collisions (60% - 80%), the q values are lower than the q value at central collision (0-10%) below 100 GeV. Above 100 GeV, the opposite is true, i.e the q value, for centralities (60% - 80%) is higher than the q value at centralities (0-10%).
- The total change in the q value for central collisions is also much smaller than the total change in the peripheral collisions.

This result has also been obtained from experimental data in a recent paper

Tapan K. Nayak et al. Eur. Phys. J. Plus 136, 702 (2021)

q vs τ plot

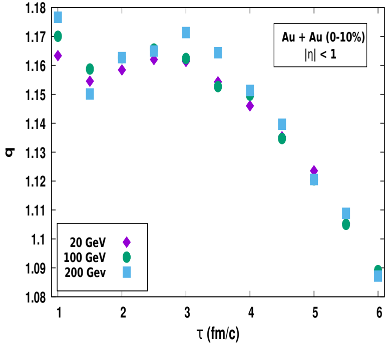


Figure: This plot shows the variation of (q) with proper time (τ) at different collision energies ($\sqrt{s}$)

Remarks:

- The nature of the graph does not change for different values of collision energy.
- The increase and subsequent decrease of the q value may be attributed to the increase and decrease in the energy density of the particles.
- We have also observed similar behavior while studying temperature fluctuations.

Summary and Conclusion:

- We studied vorticity distribution at different stages of HIC. We discuss various factors that affect the distribution. We also studied how the average vorticity changes with $\sqrt{s_{NN}}$.
- We have obtained the energy spectrum for the turbulent flow velocities on the transverse plane of the heavy-ion collision as well as on the longitudinal plane at different initial conditions. By studying the coefficients of the spectra, we find that the anisotropy is present in the transverse plane for velocity spectra. We find that there is an overall anisotropy after studying the temperature spectrum.
- The temperature fluctuations of an out of equilibrium system can be studied using the Tsallis statistics provided the inverse of the temperature β can be fitted with a χ^2 distribution. The β obtained from our simulations is fitted with a χ^2 distribution and then used to obtain the entropic index q. We have checked the dependency of the entropic index on various system parameters and compared it with the entropic index obtained from p_T spectra.

References and Acknowledgements

1. *Flow and vorticity with varying chemical potential in relativistic heavy ion collisions.*
   Abhisek Saha and Soma Sanyal
   Int. J. Mod. Phys. E Vol. 29, No. 01, 2050001 (2020)
2. *Anisotropic turbulence in relativistic plasmas.*
   Abhisek Saha and Soma Sanyal
   Eur. Phys. J. Plus 137, 1074 (2022)
3. *Temperature fluctuations and Tsallis statistics in relativistic heavy ion collisions*
   Abhisek Saha, Soma Sanyal
   Mod. Phys. Lett. A Vol. 36, No. 22, 2150152 (2021)

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Thank you all for listening!

Extra Slides

Extra Slides

The shear viscosity is given by,

$$\eta = \frac{5}{64\sqrt{8}r^2}\Sigma_i < |p| > \frac{n_i}{n} \tag{15}$$

n_i is the number density of the i th particle, r is the radius of the particles.

G.P. Kadam and H. Mishra, Nucl. Phys. A 934, 133-147 (2015)

Extra Slides

- The Kolmogorov length scale,
  $l_K = (\frac{\nu^3}{\epsilon_d})^{1/4}$
- The eddies of size l K rotate with a velocity,
  $u_k = (\nu \epsilon_d)^{1/4}$
- The power spectrum of the turbulence should have the form,
  $E(k,t) = u_K^2 l_K E(l_K k)$
- When the wavenumber k of a turbulent element lies in the range $\Lambda_t^{-1} < k < l_K$, then it is in the inertial subrange, in which negligible dissipation occurs, and the dominant energy process is the transfer of kinetic energy from large eddies to smaller eddies by inertial forces.
  $E(k) = \nu^{1/2}\epsilon_d^{1/2}\nu^{3/4}\epsilon_d^{-1/4}\alpha\nu^{3n/4}\epsilon_d^{n/4}k^n$
- Since viscous forces are negligible on this scale, the power spectrum $E(k)$ must also be independent of the value of the kinematic viscosity ν.
  $E(k) = \alpha\epsilon_d^{2/3}k^{-5/3}$