Dense matter equation of state in strong magnetic field model with density-dependent parametrization

Vivek Baruah Thapa

Indian Institute of Technology Jodhpur, Jodhpur 342037, India

In collaboration with Monika Sinha, Jia Jie Li, Armen Sedrakian

XXIV DAE-BRNS High Energy Physics Symposium
15 December 2020
Outline

1 Motivation
2 Neutron Stars
3 Neutron Star Interior
4 Model
5 Magnetic Field Profiles
6 Results
7 Conclusions
Motivation

The neutron stars (NSs) provide a unique astrophysical laboratory to test the properties of hadronic matter under extreme conditions, not attainable in any terrestrial facility now or in the future.

They can provide valuable information on crucial questions in hadronic physics like:

- Equation of State (EoS) of dense bulk matter
- Properties of neutron-rich nuclei above neutron drip density

The strong magnetic fields in the NS interior can significantly alter the quasi-particle spectrum of baryons, leading to the suppression of the superfluidity of protons and neutrons\(^1\)

We propose to study the EoS of $\Delta$-resonance admixed hypernuclear matter in strong magnetic fields within the density-dependent parameterization (DD-ME2) model

---

Neutron Stars: Magnetars

- NSs are stellar remnants of Supernovae explosions exhibiting matter densities from sub-saturation up to several times the nuclear saturation density
- Magnetars, a kind of pulsars possess surface magnetic fields of $10^{14}-10^{15}$ Gauss and the core magnetic field may reach $\geq 10^{18}$ Gauss
- Two types of magnetars: Anomalous X-ray Pulsars (AXPs), Soft Gamma Repeaters (SGRs)
- From recent NS mass-radius measurements, observed maximum mass of NS $> 2 \text{M}_\odot$ and radius in the range of $11.52 - 13.85 \text{ km}$

Figure: Artist's impression of magnetar\(^1\)

\(^1\)Image courtesy: The European Space Agency: Science & Exploration

Thapa et al. (2020) Equation of state of strongly magnetized matter with hyperons and $\Delta$-resonances
Neutron Star Interior

Motivation
Neutron Stars

Neutron Star Interior Model

Magnetic Field Profiles

Results

Conclusions


Thapa et al. (2020) Equation of state of strongly magnetized matter with hyperons and Δ-resonances

15 December 2020
We consider the Density-Dependent Relativistic Mean Field (DDRMF) model with matter composition as the full baryon octet $b$, the quartet of $\Delta$-resonances $d$ and leptons ($l = e^-, \mu^-$). Strong interactions between baryons are mediated by $\sigma, \omega, \rho, \sigma^*, \phi$-mesons.

In this work, DD-ME2 parametrization is incorporated.

The baryonic matter part of the Lagrangian density\(^1\),

\[
\mathcal{L}_B = \sum_b \bar{\psi}_b (i \gamma_\mu D^\mu - m_b + g_{\sigma b} \sigma + g_{\sigma^* b} \sigma^* - g_{\omega b} \gamma_\mu \omega^\mu - g_{\phi b} \gamma_\mu \phi^\mu \\
- g_{\rho b} \gamma_\mu \tau_b \cdot \rho^\mu) \psi_b + \sum_d (\psi_b \rightarrow \psi_d^\nu) + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) \\
+ \frac{1}{2} (\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}) - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\
- \frac{1}{4} \phi_{\mu \nu} \phi^{\mu \nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu - \frac{1}{4} \rho_{\mu \nu} \cdot \rho^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu
\]


Thapa et al. (2020) Equation of state of strongly magnetized matter with hyperons and $\Delta$-resonances 15 December 2020
Model for EoS (continued)

And, the leptonic matter part of the Lagrangian density,

\[ \mathcal{L}_l = \sum_l \bar{\psi}_l (i \gamma_\mu D^\mu - m_l) \psi_l \]  \hspace{1cm} (2)

where \( D^\mu = \partial^\mu + i e Q A^\mu \) is the covariant derivative, \( A^\mu \) is the electromagnetic vector potential, \( e Q \) is the charge of the particle.

- **Degeneracy factors:**
  - For spin-1/2 particles\(^1\) → First landau level: 1; Other levels: 2
  - For spin-3/2 particles\(^2\) → First landau level: 2; Second level: 3; Other levels: 4

- The maximum value of Landau level is defined by,

\[ \nu_{\text{max}} = \text{Int} \left( \frac{p_F c}{2 e |Q| B} \right) \] \hspace{1cm} (3)

---


**Equation of State: Pressure vs. Energy Density**

- The energy density of the matter is given by

\[
\varepsilon_m = \sum_b \varepsilon_b + \sum_d \varepsilon_d + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} + \frac{1}{2} m_\omega^2 \omega^2_0 + \frac{1}{2} m_{\phi}^2 \phi^2_0 + \frac{1}{2} m_{\sigma}^2 \rho^2_0 + \sum_l \varepsilon_l
\]  

(4)

- For uncharged particles:

\[
\varepsilon_u = \frac{2J_u + 1}{4\pi^2} \left[ p_{F_u} E_{F_u}^3 - \frac{m_u^*}{8} \left( p_{F_u} E_{F_u} + m_u^* \ln \left( \frac{p_{F_u} + E_{F_u}}{m_u^*} \right) \right) \right]
\]  

(5)

- For charged particles:

\[
\varepsilon_c = \frac{e|Q|B}{2\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} r_{\nu} \left[ p_{F_c} E_{F_c} + M^* \ln \left( \frac{p_{F_c} + E_{F_c}}{\sqrt{M^*}} \right) \right]
\]  

(6)

where \( r_{\nu} \) denotes degeneracy factor, \( M^* = m_c^* + 2\nu e|Q|B \)
The matter pressure can be evaluated from the thermodynamic (Gibbs-Duhem) relation as

$$p_m = \sum_b \mu_b n_b + \sum_d \mu_d n_d + \sum_l \mu_l n_l - \varepsilon_m,$$

(7)

Self-energy re-arrangement term is introduced in the chemical potential expression to maintain thermodynamic consistency\(^1\),\(^2\)

$$\Sigma^r = \sum_b \left[ \frac{\partial g_\omega}{\partial n} \omega_0 n_b - \frac{\partial g_{\sigma b}}{\partial n} \sigma n_b^s + \frac{\partial g_\rho}{\partial n} \rho_0 \tau_{b3} n_b - \frac{\partial g_{\sigma^* b}}{\partial n} \sigma^* n_b^s \right.$$

$$+ \frac{\partial g_\phi}{\partial n} \phi_0 n_b \left. \right] + \sum_d \left[ \frac{\partial g_\omega}{\partial n} \omega_0 n_d - \frac{\partial g_{\sigma d}}{\partial n} \sigma n_d^s + \frac{\partial g_\rho}{\partial n} \rho_0 \tau_{d3} n_d \right]$$

(8)


Magnetic Field Profiles

To model the magnetic field profile in the neutron star interior, two ad-hoc profiles are considered:

- **The exponential profile** is given by\(^1\), \((\beta = 0.01, \gamma = 3.2)\)

\[
B \left( \frac{n_b}{n_0} \right) = B_s + B_c \left\{ 1 - \exp \left[ -\beta \left( \frac{n_b}{n_0} \right)^\gamma \right] \right\}
\] (9)

- **The universal profile** is given as\(^2\),

\[
B(x) = B_c \left( 1 - 1.6x^2 - x^4 + 4.2x^6 - 2.4x^8 \right)
\] (10)

where, \(B_s\) and \(B_c\) denote the magnetic fields at surface and at center of the star respectively, \(x = r/r_{\text{mean}}\), \(r\) is the internal radius joining the center to the point of observation, \(r_{\text{mean}}\) is the mean radius of the star.

Here, \(B_s = 10^{15}\) G and \(B_c = 2.9 \times 10^{18}\) G.


Figure: Magnetic field profiles for the NYΔ composition as a function of baryon number density, $n$ (left panel) and internal radius $r$ (right panel)
Results ($B = 0$ case)

Figure: Left panel: Equation of State in the absence of magnetic field for three compositions. Right panel: $M$-$R$ relations corresponding to the EoSs
Results \((B \neq 0\) case\)

**Figure:** Equation of State in the presence of magnetic field for NY\(\Delta\) composition. Upper panel: Exponential profile, lower panel: Universal profile
Results (Particle fraction profile)

Figure: $\delta Y_i = n_i(B)/n_i(0)$ as a function of the baryon number density normalized to the nuclear saturation density $n_0$ with NY$\Delta$ composition. Upper panel: Exponential profile, lower panel: Universal profile
Results (Effective nucleon Dirac mass)

Figure: $X_{mn}^* = m_N^*(B)/m_N^*(0)$ as a function of baryon number density $n$ in units of $n_0$ with NYΔ composition
Conclusions

- The implementation of the magnetic field effects in a DDRMF model (DD-ME2 parameterization) is consistent with the recent astrophysical observations viz. GW190814 event\(^1\), MSP J0740+6620\(^2\) and NICER measurements on PSR J0030+0451\(^3,4\).

- The EoS is stiffened compared to \(B = 0\) case and oscillations in the pressure are associated with the occupation of the Landau levels.

- The oscillations of the particle fractions having their origin in the occupation of the Landau levels as well and are substantially different for the two profiles at low density regime.

- Dirac nucleon effective masses exhibit similar oscillations implying that the physical properties viz. specific heat, thermal conductivity, etc. may show this behavior too.

---


V. B. Thapa and M. Sinha acknowledge the funding support from Science and Engineering Research Board, Department of Science and Technology, Government of India through Project No. EMR/2016/006577.

A. Sedrakian acknowledges the support through the Deutsche Forschungsgemeinschaft (Grant No. SE 1836/5-1) and European COST Actions “PHAROS” (CA16214)
Thank You!