Formation of Marginally Trapped Surfaces in Gravitational Collapse

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Outline of the talk

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Gravitational Collapse

- The quest of finding the final outcome of a gravitational collapse is central in gravitational theory and relativistic astrophysics.
- Pioneer work:
  1. Oppenheimer-Snyder-Dutt (OSD/Homogeneous dust) model.
  2. Lemaitre, Tolman and Bondi (LTB/Inhomogeneous dust) model.
- Event horizon (EH) forms as collapse progresses, before the singularity formation and thus hiding singularity from far away observers.
- Black hole (BH) regions are usually defined using Event Horizons.
- For general/complex collapsing matter cloud it is difficult to determine the BH region using the EH formulation.
- We require an alternative formulation to determine the BH region.
Trapped Surfaces

- Trapped surface is an useful formalism for analytical and numerical study of black holes region.
  1. Trapped surfaces are precursors of spacetime singularities in gravitational collapse. *A spacetime singularity must occur in the future of a trapped surfaces for physical matter fields [Penrose, (1969)].*
  2. Trapped surfaces encapsulate the essential feature of black hole: outgoing light rays cannot escape to observer at infinity.
  3. Trapped surfaces are the quasi local description of black hole region.

- A trapped region $S$ is trapped in the black hole region and marginally trapped on the boundary.
**Trapped/Untrapped Surfaces**

**Figure:** All $S$ are trapped in the BH region and marginally trapped on the boundary.

$\delta A$ increases along $l^{a}$ and $n^{a}$. $S$ is untrapped. 

$\Theta_{(w)} > 0$, $\Theta_{(m)} > 0$.

$\delta A$ decreases along $l^{a}$ as well as $n^{a}$. So, $S$ is a trapped surface. 

$\Theta_{(w)} < 0$, $\Theta_{(m)} < 0$. 
Marginally Trapped Surfaces (MTT)

Figure: All $S$ are trapped in the BH region and marginally trapped on the boundary.

Marginalia trapped tube (MTT) foliated by spheres, $S$ with $\Theta(\omega) = 0, \Theta_{cn} < 0$

$t^\mu = (\nu^\mu - c n^\mu)$ is the vector field tangent to MTT.

$IH = \text{Isolated Horizon (c=0)}$

$DH = \text{Dynamical Horizon (c>0)}$
Spherical Symmetric Gravitational Collapse

- We consider a general spherically symmetric distribution of collapsing cloud

\[
ds^2 = -e^{2\Phi(r,t)} dt^2 + e^{2\psi(r,t)} dr^2 + R(r,t)^2 d\theta^2 + R(r,t)^2 \sin^2 \theta d\phi^2,
\]

(1)

- The energy-momentum tensor for general anisotropic fluid as

\[
T_{\mu\nu} = (p_\theta + \rho)u_\mu u_\nu + p_\theta g_{\mu\nu} + (p_r - p_\theta)X_\mu X_\nu - 2\eta \sigma_{\mu\nu} - \zeta \Theta P_{\mu\nu}.
\]

(2)

where

- \(\eta \geq 0; \zeta \geq 0\) are the coefficients of shear and bulk viscosity.
- \(u^\mu\) & \(X^\mu\) are unit time-like and space-like vectors.
- \(\sigma_{\mu\nu}, \Theta, P_{\mu\nu}\) are shear, expansion and projection tensor fluids.
- \(\rho, p_r \& p_\theta\) are the energy density and radial and tangential pressures.
Einstein’s Equations

\[
\rho = \frac{F'}{R^2 R'}; \quad p_r = -\frac{\dot{F}}{R^2 \dot{R}} + \frac{4}{3} \eta \sigma + \zeta \Theta,
\]

(3)

\[
\Phi' = \frac{2 (\Theta - \sigma)'}{3 \sigma} \frac{p_\theta - p_r + 2 \eta \sigma}{\rho + p_r - \frac{4}{3} \eta \sigma - \zeta \Theta} - \frac{p_r - \frac{4}{3} \eta \sigma' - \zeta \Theta'}{\rho + p_r - \frac{4}{3} \eta \sigma - \zeta \Theta},
\]

(4)

\[
2 \dot{R}' = \frac{R' \dot{G}}{G} + \frac{\dot{R} H'}{H} \quad \Leftrightarrow \quad \frac{\dot{G}}{G} = -\frac{2 \dot{R} \Phi'}{R'},
\]

(5)

\[
F = R(1 - G + H).
\]

(6)

where,

- \( H = e^{-2\Phi(r,t)} \dot{R}^2; \quad G = e^{-2\psi(r,t)} R'^2. \)
- The dynamics of the marginally trapped surfaces depends upon the sign of the expansion parameter \( C, \) at \( R = F, \) defined by

\[
C = \frac{L_l \theta_{(l)}}{L_n \theta_{(l)}} = \frac{\rho + p_\theta - (4/3) \eta \sigma - \zeta \Theta + (p_\theta - p_r)}{(4\pi/A) - (1/2) [\rho - \{ p_\theta - (4/3) \eta \sigma - \zeta \Theta + (p_\theta - p_r) \}]}
\]
Dynamics of MTT for Gaussian Density Distribution
Dynamics of MTT for Two Shells falling consecutively
An important aspect of study of the MTTs or trapped surfaces involve identifying boundary of a black hole region.

In this study our main focus was to obtain the trapped regions and locate the marginally trapped surfaces for some general class of energy- momentum tensors.

For the purpose of generality, we have included the homogeneous as well as inhomogeneous dust models.

While the dust models have been studied earlier [3, 4], the detail study of the formation and time- development of the EH and the MTTs for a generic class of energy- momentum tensors, through analytical as well as numerical means have not been carried out in the literature.
Summary and Discussion

- We have obtained the signature of the MTTs, using $C$, during each of the collapse scenarios and a general conclusion may be reached:
  1. The MTTs in the OSD models are timelike.
  2. The MTTs in the LTB models are spacelike, and are dynamical horizons and reach the isolated horizon phase in equilibrium.
  3. These parameter ranges, of the coefficients arising in the energy-momentum tensor, have been utilised to numerically study the evolution of the MTTs in these cases.
  4. We observe that, MTT formation may be delayed or accelerated, compared to the dust models, by suitable choices in the fluid parameters.
  5. We believe that the results obtained in this paper may help in forming a general outlook about the time development of Marginally Trapped Surfaces during gravitational collapse in GR as well as in other alternate gravity theories.
Thank You.