Theory of Fast Flavor Conversion of Supernova neutrinos

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CONTENT OF THE TALK:

● Equations of motion and a bit of background in the field

● Complicacy and Challenges in the field

● Our Progress

● Theory of SN neutrino oscillations:
   1) Multipole diffusion
   2) Transverse relaxation

● Main results

● Phenomenological Consequences and Conclusion
SN Explosion & Neutrinos:

Onion Like Structure

Implosion

Bounce & Shock

Neutrinos emitted: Shock stalls.

Successful Explosion

Manibrata Sen, Northwestern University, Evanston (N3AS Network)

Talk at SNNu ECT*, May 16, 2019
Neutrino Oscillations

Electron antineutrinos / neutrinos decouple earlier / later

forward scatter with each other and undergo collective oscillations

forward scatter off electrons and undergo MSW conversions

R < 10 km
Trapping
No Oscillation (?)

R ~ 10 km
Decoupling
Fast Collective conversion 1/t ~ μ

R ~ 100 km
Free-streaming
Slow Collective conversion 1/t ~ (μω)^1/2
Swaps at μ ~ ω

R ~ 1000 km
Free-streaming
MSW conversion
Resonance at λ ~ ω

Interstellar space
Free-streaming
Kinematic decoherence

Inside Earth
Free-streaming
Regeneration

Basudeb Dasgupta (TIFR Mumbai)

Talk at Neutrino 2018, Heidelberg, June 9, 2018
Fast Flavor Conversions: A review

- Conversion occurs very close to the SN core (few km’s) with rate $\propto$ Neutrino number density
- $10^5$ times faster compared to neutrino oscillation in vacuum/ordinary matter
- Requires a “zero” Crossing in neutrino angular distribution and independent of energy (and mass hierarchy)

Chakraborty, Hansen, Izaguirre, Raffelt (2016)  
Ray Sawyer (2005, 2015)
EQUATION OF MOTION:

• Neutrino density matrix: \( \hat{\rho}[r, E, \vec{p}, t] \equiv \hat{\rho}_{E,\vec{p}} \) \[\rho = \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle & \langle \nu_e | \nu_\tau \rangle \\ \\ \\
\langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle & \langle \nu_\mu | \nu_\tau \rangle \\ \\ \\
\langle \nu_\tau | \nu_e \rangle & \langle \nu_\tau | \nu_\mu \rangle & \langle \nu_\tau | \nu_\tau \rangle \end{pmatrix} \]

\( \langle \nu_i | \nu_i \rangle \rightarrow \) Total flavor content

\( \langle \nu_i | \nu_j \rangle \rightarrow \) Amount of flavor conversion

• Equations governing flavor evolution:

\[ \left( \partial_t + \vec{v} \cdot \vec{\nabla} \right) \hat{\rho}_{E,\vec{p}} = \hat{H}_{E,\vec{p}} \hat{\rho}_{E,\vec{p}} - \hat{\rho}_{E,\vec{p}} \hat{H}_{E,\vec{p}} \]

\( H_{E}^{\text{vac}} = \frac{\Delta m^2}{2E} \)

\( H_{E}^{\text{mat}} = \sqrt{2G_F} n_e \)

\( H_{\vec{p}}^{\text{self}} = \int d^3 \vec{q} / (2\pi)^3 \left( 1 - \vec{v}_q \cdot \vec{v}_{\vec{p}} \right) \left( \hat{\rho}_{E',\vec{q}} - \hat{\rho}_{E',\vec{q}} \right) (\hat{\rho}_{E,\vec{p}}) \)

- Vacuum oscillation
- Dependence on energy

- MSW potential
- Dependence on electron number density

- Neutrino-Neutrino self interaction potential
- Depends on neutrino number density
- Responsible for collective effects (e.g.: fast conversion)
EQUATION OF MOTION (FFC):

We stick to two-flavor framework:

\[ \hat{\rho}_{E,\bar{v}} = \begin{pmatrix} \langle \nu_e | \nu_e \rangle_{E,\bar{v}} & \langle \nu_e | \nu_x \rangle_{E,\bar{v}} \\ \langle \nu_x | \nu_e \rangle_{E,\bar{v}} & \langle \nu_x | \nu_x \rangle_{E,\bar{v}} \end{pmatrix} = \frac{Tr(\hat{\rho}_{E,\bar{v}})}{2} \mathbb{I}_{2 \times 2} + \frac{g_{E,\bar{v}}}{2} \vec{S}_{E,\bar{v}} \cdot \sigma \]

\[ \hat{H}_{E,\bar{v}} = \frac{Tr(\hat{H}_{E,\bar{v}})}{2} \mathbb{I}_{2 \times 2} + \frac{1}{2} \vec{H}_{E,\bar{v}} \cdot \sigma \]

(\partial_t + \bar{v} \cdot \vec{\nabla}) \hat{\rho}_{E,\bar{v}} = \hat{H}_{E,\bar{v}} \hat{\rho}_{E,\bar{v}} - \hat{\rho}_{E,\bar{v}} \hat{H}_{E,\bar{v}} \rightarrow (\partial_t + \bar{v} \cdot \vec{\nabla}) S_{\omega,\bar{v}} = (H_{\omega}^{\text{vac}} + H_{\omega}^{\text{mat}} + H_{\omega}^{\text{self}}) \times S_{\omega,\bar{v}} \]

\[ H_{\omega}^{\text{mat}} = \sqrt{2} G_F (n_{e^-} - n_{e^+}) (0, 0, 1) \quad H_{\omega}^{\text{vac}} = \omega \left( \sin 2\vartheta, 0, \cos 2\vartheta \right) \]

\[ H_{\omega}^{\text{self}} = \int d^3 \vec{p}_{\omega',\bar{v}'} / (2\pi)^3 g_{\omega',\bar{v}'} \left( 1 - \bar{v} \cdot \bar{v}' \right) S_{\omega',\bar{v}'} \]
**EQUATION OF MOTION (FFC):**

We stick to two-flavor framework:

\[
\hat{\rho}_{E,\bar{v}} = \begin{pmatrix} \langle \nu_e | \nu_e \rangle_{E,\bar{v}} & \langle \nu_e | \nu_x \rangle_{E,\bar{v}} \\ \langle \nu_x | \nu_e \rangle_{E,\bar{v}} & \langle \nu_x | \nu_x \rangle_{E,\bar{v}} \end{pmatrix} = \frac{Tr(\hat{\rho}_{E,\bar{v}})}{2} \mathbb{I}_{2\times2} + \frac{g_{E,\bar{v}}}{2} \vec{S}_{E,\bar{v}} \cdot \sigma
\]

\[
\hat{H}_{E,\bar{v}} = \frac{Tr(\hat{H}_{E,\bar{v}})}{2} \mathbb{I}_{2\times2} + \frac{1}{2} \vec{H}_{E,\bar{v}} \cdot \sigma
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\[
(\partial_t + \nabla \cdot \vec{v}) \hat{\rho}_{E,\bar{v}} = \hat{H}_{E,\bar{v}} \hat{\rho}_{E,\bar{v}} - \hat{\rho}_{E,\bar{v}} \hat{H}_{E,\bar{v}}
\]

**Zero crossing**: FFC

\[
(\partial_t + \nabla \cdot \vec{v}) S_{\omega,\bar{v}} = (H_{\omega}^{\text{vac}} + H_{\bar{v}}^{\text{mat}} + H_{\bar{v}}^{\text{self}}) \times S_{\omega,\bar{v}}
\]

\[
H_{\text{mat}} = \sqrt{2} G_F (n_{e^-} - n_{e^+}) (0,0,1) \quad H_{\omega}^{\text{vac}} = \omega (\sin 2\theta, 0, \cos 2\theta)
\]

\[
H_{\bar{v}}^{\text{self}} = \int d^3 \vec{p}' / (2\pi)^3 g_{\omega',\bar{v}'} (1 - \bar{v} \cdot \bar{v}') S_{\omega',\bar{v}'}
\]

\[
S_{\bar{v}} \parallel \text{: Total Flavor content} \quad S_{\bar{v}} \perp \text{: Flavor conversion}
\]

**g_{\bar{p}}**: Neutrino angular distribution
Difficulties and Challenges:

<table>
<thead>
<tr>
<th>What are the challenges?</th>
<th>What did we do?</th>
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<tbody>
<tr>
<td>Huge Phase space dimensionality 3 sp. + 3 mom. + 1 time = 7 dim</td>
<td>Partially resolved 2 sp. + 2 mom. + 1 time = 5 dim.</td>
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| Large set of coupled nonlinear P.D.E’s | Developed:
  a) Analytical techniques
  b) Numerical code |
| Lack of numerical techniques to give accurate and precise result in the nonlinear regime | Developed our own code that gives accurate and precise answer even in the nonlinear regime |
| Lack of analytical development / theory beyond the linear regime (Linear stability) which can predict the final outcome | Developed our own theory that can predict how, when and to what extent fast conversion can happen. |
OUR NUMERICAL RECIPE:

1. Discretize in space for each velocity
   - Method: Fast Fourier Transform
   - Solver: scipy.fftpack.diff in python
   - B.C: Periodic in space

2. A set of coupled nonlinear O.D.E
   - Method: Backward differentiation
   - Solver: Zvode solver in python

3. Solve O.D.E’s in time
RESULTS : Irreversibility

Model :
1Time + 1 Sp. + 1 Mom.

Initial Cond :
\[ S_{v}^{\parallel} \text{ini} = +1 \]
\[ |S_{v}^{\perp} \text{ini}| = 10^{-6} \delta (z) \]

- Irreversible and steady state behaviour in time
- length shrinks

\[ G_v = 1, v > 0; A - 1, v < 0 \]

Bhattacharyya, Dasgupta (2020)
RESULTS : Multipole Diffusion

• In terms of multipole moments, \( M_n = \int_{-1}^{+1} dv G_v L_n S_v \) and considering \( n \) as continuum we get:

\[
\partial_t M_n - M_0 \times M_n = \partial_z \left( M_n + \delta_n M_n/(2n + 1) + \delta_n^2 M_n/2 \right) - M_1 \times \left( M_n + \delta_n M_n/(2n + 1) + \delta_n^2 M_n/2 \right)
\]

• Further coarse graining over \( z \), and using \( 2n + 1 \approx 2n \)

\[
\partial_t \langle M_n \rangle = \frac{\langle M_1 \rangle}{2} \left( \delta_n^2 \langle M_n \rangle + \frac{1}{n} \partial_n \langle M_n \rangle \right)
\]

• The above equation remains same under \( n \to an, \ t \to a^2 t \) \( \longrightarrow \langle M_n(t) \rangle = f \left( \frac{n^2}{t} \right) = f(\xi) \)

\[
2 \frac{d^2}{d\xi^2} f(\xi) + \left( 1/\langle M_1 \rangle + 2/\xi \right) \frac{d}{d\xi} f(\xi) = 0
\]

\[
\langle M_n(t) \rangle = c_1 \text{Ei} \left[ -n^2 / (2\langle M_1 \rangle t) \right] + c_2
\]
Power flow in multipole space from low to high $n$ values and coarse-graining causes irreversibility in time and also shrinking in the length for high $n$ multipole moments.
RESULTS: Transverse Relaxation

- Modes for which $|H^\perp_v| \approx |H^\parallel_v|$, $S_v$ crosses the transverse plane and gets depolarized.
- Amount depends on lepton asymmetry and choice of $v$.

Bhattacharyya, Dasgupta (2020)
RESULTS: Flavor Depolarization

Depolarization factor:
\[ f_v^D = \frac{1}{2} \left( 1 - \frac{\langle S_v \rangle_{\text{fin}}}{\langle S_v \rangle_{\text{ini}}} \right) \]

\[ f_v^D = 0.5 \quad \text{Complete depolarization} \]
\[ f_v^D = 0 \quad \text{No depolarization} \]

Multipole expansion upto linear order:
\[ G_v S_{v}^{\|} f_{\text{fin}} = \frac{M_0^{\text{fin}}}{2} + \frac{3vM_1^{\text{fin}}}{2} + O(v^2) \]

Lepton number conservation:
\[ \langle M_0^{\text{ini}} \rangle = \langle M_0^{\text{fin}} \rangle = A \quad \langle M_1^{\text{fin}} \rangle = \frac{A}{2} \]

\[ f_v^D \approx 0.5, \text{if } v < 0 \]
\[ f_v^D \approx \frac{1}{2} - \frac{A}{4} - \frac{3A}{8} v, \text{if } v > 0 \]
CONCLUSION

- We have presented an **analytical theory** of fast neutrino flavor conversions in the **nonlinear regime**.

- We showed fast conversions can bring different neutrino flavors **close to each other** (Flavor Depolarization) and **irreversibility** in the system.

- **$T2$ relaxation** and **multipole diffusion** governs such behaviour.

- We gave a strategy and a formula for computing the **extent of flavor depolarization**
Phenomenological Consequences:

- Flavor depolarization can cause significant increase in neutrino heating rate and change the explosion scenario.

- Including MSW conversions, propagation and earth effects, our formula will allow one to determine the final neutrino signal from a SN explosion.

\[
F_{\nu_e, \nu_\mu}^{\text{fin}}[\vec{p}] = (1 - f_p^D)F_{\nu_e, \nu_\mu}^{\text{ini}}[\vec{p}] + f_p^D F_{\nu_\mu, \nu_e}^{\text{ini}}[\vec{p}]
\]

- This signal can be the first ever direct probe of testing the neutrino-neutrino self-interaction.

- The final output of fast conversions can have implications even in the nucleosynthesis of elements, astrophysics of binary neutron star mergers, diffuse SN background, and can be detected in future experiments.