Structure of magnetic field quantization in viscosity expression for relativistic fluid

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Outline

1. Introduction & Motivation
2. Spectral Function of the EMT
3. Viscous Coefficients from the Spectral Function
4. Results
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Introduction & Motivation

The study of the nuclear matter under extreme conditions of temperature and/or density has been a subject of intense investigation over the past few decades. Where does such a state might exist?

- The microsecond old universe after big-bang.
- The core of a neutron star.

Can we create such a state in the laboratory?

- YES: In the Heavy ion collision (HIC) experiments.

However, in a non-central or asymmetric HIC, extremely high magnetic fields are created ($eB \sim 15 \text{ m}^2\pi$ or $B \sim 5 \times 10^{15}$ Tesla).

Thus, in a HIC, we create hot and dense 'strongly' interacting magnetized matter. Many exotic effects/phenomenon can take place:

- Chiral Magnetic Effect (CME)
- Magnetic Catalysis (MC) and Inverse Magnetic Catalysis (IMC).
- Superconductivity of the Vacuum.

In this work, we aim to study the transport properties (mainly the calculation of the viscous coefficients) of a hot and magnetized Bosonic and Fermionic systems using the Kubo Formalism.
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Spectral Function of the EMT at $B = 0$

The key microscopic quantity to calculate the viscous coefficients is the in-medium spectral function $\rho_{\mu\nu\alpha\beta}(q)$, given by

$$\rho_{\mu\nu\alpha\beta}(q) = \text{Im} i \int d^4 x e^{iq \cdot x} \langle T_{\mu\nu}(x) T_{\alpha\beta}(0) \rangle_R.$$  

$T_{\mu\nu}(x)$: Local Energy Momentum Tensor (EMT)  
$\langle \cdot \cdot \cdot \rangle_R$: Ensemble average of the retarded two point correlation function.

Free Lagrangian (densities):
- $L_{\text{Scalar}} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi$,  
- $L_{\text{Dirac}} = i \frac{1}{2} (\psi \gamma_\mu \partial^\mu \psi - \partial^\mu \psi \gamma_\mu \psi) - m \bar{\psi} \psi$.

Symmetric EMTs can be constructed out of the above Lagrangians as

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Spectral Function of the EMT at $B = 0$

Calculation of the spectral function at finite temperature is done employing the Real Time Formalism of Finite Temperature Field Theory. In the static limit:

$$S_{\mu\nu\alpha\beta} = \frac{\partial \rho_{\mu\nu\alpha\beta}}{\partial q_0} \bigg| \begin{array}{c} \vec{q} = \vec{0}, q_0 \to 0 \end{array} \right.$$ 

where,

$$\omega_k = \sqrt{\vec{k}^2 + m^2},$$

$$f_a(x) = \left[ e^{x/T} - a \right]^{-1}$$

This interaction is introduced by considering finite values of $\Gamma$ for a dissipative system. This $\Gamma$ can be identified as the thermal width or collision rate of the constituent particles.
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$$\times \left[ N^{\mu\nu\alpha\beta}(k, k) \bigg|_{k^0 = \omega_k} + N^{\mu\nu\alpha\beta}(k, k) \bigg|_{k^0 = -\omega_k} \right]$$

- where, $\omega_k = \sqrt{k^2 + m^2}$, $f_a(x) = [e^{x/T} - a]^{-1}$ and

$$N^{\mu\nu\alpha\beta}_{\text{Scalar}}(k, k) = 4k^\mu k^\nu k^\alpha k^\beta - 2(k^2 - m^2) g^{\mu\nu} k^\alpha k^\beta + g^{\alpha\beta} k^\mu k^\nu + (k^2 - m^2)^2 g^{\mu\nu} g^{\alpha\beta},$$

$$N^{\mu\nu\alpha\beta}_{\text{Dirac}}(k, k) = -8k^\mu k^\nu k^\alpha k^\beta + (k^2 - m^2) \left\{ g^{\mu\alpha} k^\nu k^\beta + g^{\nu\alpha} k^\mu k^\beta + g^{\mu\beta} k^\nu k^\alpha + g^{\nu\beta} k^\mu k^\alpha + 4g^{\mu\nu} k^\alpha k^\beta + 4g^{\alpha\beta} k^\mu k^\nu \right\} - 4(k^2 - m^2)^2 g^{\mu\nu} g^{\alpha\beta}.$$
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Spectral Function of the EMT at $B \neq 0$

In presence of external electromagnetic field, described by the four-potential $A^{\mu}_{\text{ext}}(x)$, the Lagrangians:

- **Scalar Lagrangian**:
  $$L_{\text{Scalar}} = \bar{\phi} D^{\mu} \phi - m^2 \phi$$

- **Dirac Lagrangian**:
  $$L_{\text{Dirac}} = i \frac{2}{4} \left( \bar{\psi} \gamma^{\mu} D_{\mu} \psi - D^{\mu} \bar{\psi} \gamma^{\mu} \psi \right) - m \bar{\psi} \psi$$

$D^{\mu} = \partial^{\mu} + ie A^{\mu}_{\text{ext}}(x)$ and $D^{\ast \mu} = \partial^{\mu} - ie A^{\mu}_{\text{ext}}(x)$ are the covariant derivatives.

Symmetric EMTs are:

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  $$T^{\mu \nu}_{\text{Scalar}} = \bar{\phi} D^{\ast \mu} \phi D^{\nu} \phi - \frac{1}{2} g^{\mu \nu} L_{\text{Scalar}} + (\mu \leftrightarrow \nu)$$

- **Dirac EMT**:
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\mathcal{L}_{\text{Scalar}} = D^*\phi^\dagger D_\mu \phi - m^2 \phi^\dagger \phi , \\
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Calculation of the spectral function at finite magnetic field is done employing the Schwinger Proper-time Formalism. In the static limit:

\[ S_{\mu\nu\alpha\beta} = \lim_{\Gamma \to 0} \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{2} T \int d^3k (2\pi)^3 \frac{1}{2} \left( \omega_{kl} - \omega_{kn} \right)^2 + \Gamma^2 \left\{ a f_a \left( \omega_{kl} \right) + a f_a \left( \omega_{kn} \right) + 2 f_a \left( \omega_{kl} \right) f_a \left( \omega_{kn} \right) \right\} \]

\[ N_{\mu\nu\alpha\beta} \ln \left( \frac{k}{k_0} \right) \mid \left| k_0 = \omega_{kl} + N_{\mu\nu\alpha\beta} \ln \left( \frac{k}{k_0} \right) \mid \right| k_0 = -\omega_{kl} \]

where, \( \omega_{kl} = \sqrt{k^2 z + (2l + 1 - 2s) eB + m^2} \) and \( N_{\mu\nu\alpha\beta} \); Scalar \((k,k) = 4A \ln \left( \frac{k_2}{k_\perp} \right) \{
\begin{align*}
4k_\mu k_\nu k_\alpha k_\beta &- 2(k^2 - m^2)(g_\mu\nu k_\alpha k_\beta + g_\alpha\beta k_\mu k_\nu) + (k^2 - m^2)^2 g_\mu\nu g_\alpha\beta \\
&+ g_\mu\nu g_\alpha\beta k^2_{\perp} (k_\alpha - k_\parallel) - g_\alpha\beta k_\nu k_\mu_{\perp} (k_\alpha - k_\parallel) + 2 g_\mu\nu g_\alpha\beta k^2_{\perp}
\end{align*}\}
\]

\( N_{\mu\nu\alpha\beta} \); Dirac \((k,k) = -16B \ln \left( \frac{k_2}{k_\perp} \right) \{
\begin{align*}
k_\nu k_\beta (2k_\mu k_\perp k_\alpha - g_\mu\nu k_\perp (k_\alpha - k_\parallel)) &- g_\mu\nu k_\beta k^2_{\perp} (k_\alpha - k_\parallel) - g_\alpha\beta k_\nu k_\mu_{\perp} (k_\alpha - k_\parallel) + 2 g_\mu\nu g_\alpha\beta k^2_{\perp} \\
&+ (k_2 - m^2) g_\mu\nu g_\alpha\beta k^2_{\perp}
\end{align*}\}
\]

\(-2C \ln \left( \frac{k_2}{k_\perp} \right) \{
\begin{align*}
k_\nu k_\beta (2k_\mu_{\perp} k_\alpha - g_\mu\nu k_\perp (k_\alpha - k_\parallel)) &- g_\mu\nu k_\beta k^2_{\perp} (k_\alpha - k_\parallel) - g_\alpha\beta k_\nu k_\mu_{\perp} (k_\alpha - k_\parallel) + 2 g_\mu\nu g_\alpha\beta k^2_{\perp} \\
&+ (k_2 - m^2) g_\mu\nu g_\alpha\beta k^2_{\perp}
\end{align*}\}
\]
Spectral Function of the EMT at $B \neq 0$

- Calculation of the spectral function at finite magnetic field is done employing the Schwinger Proper-time Formalism.
Spectral Function of the EMT at $B \neq 0$

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- In the static limit:

$$S_{\mu \nu \alpha \beta} = \lim_{\Gamma \to 0} \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{2T} \int \frac{d^3k}{(2\pi)^3} \frac{1}{4\omega_{kl}\omega_{kn}} \frac{\Gamma}{(\omega_{kl} - \omega_{kn})^2 + \Gamma^2} \left\{ a f_a(\omega_{kl}) + a f_a(\omega_{kn}) ight\}$$

$$+ 2 f_a(\omega_{kl}) f_a(\omega_{kn}) \left[ N_{ln}^{\mu \nu \alpha \beta}(k, k) \bigg|_{k^0 = \omega_{kl}} + N_{ln}^{\mu \nu \alpha \beta}(k, k) \bigg|_{k^0 = -\omega_{kl}} \right]$$
Spectral Function of the EMT at $B \neq 0$

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$$+ 2 f_a(\omega_{kl}) f_a(\omega_{kn}) \} \left[ N^{\mu\nu\alpha\beta}_{ln}(k,k) \bigg|_{k^0=\omega_{kl}} + N^{\mu\nu\alpha\beta}_{ln}(k,k) \bigg|_{k^0=-\omega_{kl}} \right]$$

- where, $\omega_{kl} = \sqrt{k^2_z + (2l + 1 - 2s)eB + m^2}$ and

$$N^{\mu\nu\alpha\beta}_{ln;\text{Scalar}}(k,k) = 4 A_{ln}(k^2_\perp) \left\{ 4 k^\mu k^\nu k^\alpha k^\beta - 2(k^2 - m^2)(g^{\mu\nu} k^\alpha k^\beta + g^{\alpha\beta} k^\mu k^\nu) $$

$$+ (k^2 - m^2)^2 g^{\mu\nu} g^{\alpha\beta} \right\}$$

$$N^{\mu\nu\alpha\beta}_{ln;\text{Dirac}}(k,k) = -16 B_{ln}(k^2_\perp) \left[ k^\nu k^\beta (2 k^\mu_\perp k^\alpha_\perp - k^2_\perp g^{\mu\alpha}) - g^{\mu\nu} k^\alpha k^\beta_\perp (k^\perp_\alpha - k^\alpha_\perp) - g^{\alpha\beta} k^\nu k^\beta k^\perp_\alpha (k^\mu_\perp - k^\mu_\perp) $$

$$+ g^{\mu\nu} g^{\alpha\beta} k^\mu_\perp k^\perp_\alpha (k^2_\perp - k^2_\perp + m^2) \right] - 2 C_{ln}(k^2_\perp) \left[ k^\nu k^\beta \{ 2 k^\mu_\perp k^\alpha_\perp - (k^2 - m^2) g^{\mu\alpha} \} - (k^2 - m^2) g^{\mu\nu} k^\alpha k^\beta_\perp $$

$$- (k^2 - m^2) g^{\alpha\beta} k^\nu k^\perp_\mu + g^{\mu\nu} g^{\alpha\beta} (k^2_\perp - m^2)^2 \right] - 2 D_{ln}(k^2_\perp)(k^2_\perp - m^2) \left[ k^\nu k^\beta g^{\mu\alpha}_\perp - g^{\mu\nu} k^\beta k^\perp_\alpha $$

$$- g^{\alpha\beta} k^\nu k^\mu_\perp + g^{\mu\nu} g^{\alpha\beta} k^2_\perp \right] - 4 E_{ln}(k^2_\perp) \left[ k^\nu k^\beta (k^\mu_\perp k^\alpha_\perp + k^\mu_\perp k^\alpha_\perp) - g^{\mu\nu} k^\beta \{ (k^2_\perp - m^2) k^\alpha_\perp + k^2_\perp k^\alpha_\perp \} $$

$$- g^{\alpha\beta} k^\nu \{ (k^2_\perp - m^2) k^\mu_\perp + k^2_\perp k^\mu_\perp \} + 2 g^{\mu\nu} g^{\alpha\beta} k^2_\perp (k^2_\perp - m^2) \right] + (\mu \leftrightarrow \nu) + (\alpha \leftrightarrow \beta) + (\mu \leftrightarrow \nu, \alpha \leftrightarrow \beta)$
Outline

1. Introduction & Motivation

2. Spectral Function of the EMT

3. Viscous Coefficients from the Spectral Function

4. Results
Viscosities at $B = 0$

Owing to the Kubo relation the viscous coefficients (shear and bulk) can be calculated from the spectral functions of the EMT.

The shear viscosity ($\eta$) and the bulk viscosity ($\zeta$) are obtained from

$$\nu = P(\nu)_{\mu \nu \alpha \beta} S_{\mu \nu \alpha \beta};$$

where,

$$P(\eta)_{\mu \nu \alpha \beta} = \frac{1}{10} (\Delta \sigma_{\mu} \Delta \rho_{\nu} - \frac{1}{3} \Delta \sigma_{\rho} \Delta \mu_{\nu}) (\Delta \sigma_{\alpha} \Delta \rho_{\beta} - \frac{1}{3} \Delta \sigma_{\rho} \Delta \alpha_{\beta}),$$

$$P(\zeta)_{\mu \nu \alpha \beta} = \left( \frac{1}{3} \Delta \rho_{\mu} \rho_{\nu} + \theta u_{\mu} u_{\nu} \right) \left( \frac{1}{3} \Delta \rho_{\alpha} \rho_{\beta} + \theta u_{\alpha} u_{\beta} \right),$$

in which $\Delta_{\mu \nu} = (g_{\mu \nu} - u_{\mu} u_{\nu})$, $\theta = \left( \frac{\partial P}{\partial \epsilon} \right)$, $P$ is the pressure and $\epsilon$ is the energy density of the system being considered.
Owing to the Kubo relation the viscous coefficients (shear and bulk) can be calculated from the spectral functions of the EMT.
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$$\mathcal{P}^{(\zeta)}_{\mu\nu\alpha\beta} = \left( \frac{1}{3} \Delta_{\mu\nu} + \theta u_{\mu} u_{\nu} \right) \left( \frac{1}{3} \Delta_{\alpha\beta} + \theta u_{\alpha} u_{\beta} \right),$$
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$$P_{\mu\nu\alpha\beta}^{(\zeta)} = \left( \frac{1}{3} \Delta_{\mu\nu} + \theta u_\mu u_\nu \right) \left( \frac{1}{3} \Delta_{\alpha\beta} + \theta u_\alpha u_\beta \right),$$

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Viscosities at $B = 0$

Simplifications yields the final well known expressions:

\[ \eta_{\text{Scalar}} = 2 \frac{1}{15} T \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^4}{\omega^2 k \Gamma f(\omega k)} \left\{ 1 + f(\omega k) \right\}, \]

\[ \eta_{\text{Dirac}} = 4 \frac{1}{15} T \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^4}{\omega^2 k \Gamma \tilde{f}(\omega k)} \left\{ 1 - \tilde{f}(\omega k) \right\}, \]

\[ \zeta_{\text{Scalar}} = 2 \frac{2}{9} T \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega^2 k \Gamma} \left\{ m^2 + (3\theta - 1) \omega^2 k \right\}^2 f(\omega k) \left\{ 1 + f(\omega k) \right\}, \]

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\]

\[
\eta_{\text{Dirac}} = \frac{4}{15T} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^4}{\omega_k^2 \Gamma} \tilde{f}(\omega_k) \left\{ 1 - \tilde{f}(\omega_k) \right\},
\]

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\zeta_{\text{Dirac}} = \frac{4}{9T} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k^2 \Gamma} \left\{ m^2 + (3\theta - 1)\omega_k^2 \right\}^2 \tilde{f}(\omega_k) \left\{ 1 - \tilde{f}(\omega_k) \right\}.
\]
Viscosities at $B \neq 0$

At $B \neq 0$, one can get five shear viscosity coefficients $\eta_n (n = 0, 1, 2, 3, 4)$ and two bulk viscosity coefficients $\zeta_\perp, \zeta_\parallel$. The shear and the bulk viscous coefficients are obtained from

$$\nu = -\xi(\nu) \eta_0 + P(\nu) \mu_{\alpha\beta} S_{\mu\nu\alpha\beta};$$

where,

$$\xi(\nu) = \begin{cases} 
4/3 & \text{if } \nu = \eta_1 \\
1 & \text{if } \nu = \eta_2 \\
0 & \text{otherwise} 
\end{cases}$$

and $P(\nu)_{\mu\nu\alpha\beta}$ are given by

$$P(\eta_0)_{\mu\nu\alpha\beta} = \frac{1}{4} (\xi_{\sigma\mu} \xi_{\rho\nu} - \frac{1}{2} \xi_{\sigma\rho} \xi_{\mu\nu}) (\xi_{\sigma\alpha} \xi_{\rho\beta} - \frac{1}{2} \xi_{\sigma\rho} \xi_{\alpha\beta})$$,

$$P(\eta_1)_{\mu\nu\alpha\beta} = 2 (b_{\mu} b_{\nu} - \theta u_{\mu} u_{\nu}) (\frac{1}{2} \xi_{\alpha\beta} + (\theta + \phi) u_{\alpha} u_{\beta})$$,

$$P(\eta_2)_{\mu\nu\alpha\beta} = -\frac{1}{2} \xi_{\sigma\mu} b_{\nu} \xi_{\sigma\alpha} b_{\beta}$$,

$$P(\eta_3)_{\mu\nu\alpha\beta} = -\frac{1}{8} (\xi_{\sigma\mu} \xi_{\rho\nu} - \frac{1}{2} \xi_{\sigma\rho} \xi_{\mu\nu}) b_{\lambda} \sigma (\xi_{\lambda\alpha} \xi_{\rho\beta} - \frac{1}{2} \xi_{\lambda\rho} \xi_{\alpha\beta})$$,

$$P(\eta_4)_{\mu\nu\alpha\beta} = \frac{1}{2} b_{\rho} \sigma \xi_{\rho\mu} b_{\nu} \xi_{\sigma\alpha} b_{\beta}$$,

$$P(\zeta_\perp)_{\mu\nu\alpha\beta} = \frac{1}{3} (\Delta_{\mu\nu} + (3 \theta + 2 \phi) u_{\mu} u_{\nu}) (\frac{1}{2} \xi_{\alpha\beta} + (\theta + \phi) u_{\alpha} u_{\beta})$$,

$$P(\zeta_\parallel)_{\mu\nu\alpha\beta} = -\frac{1}{3} (\Delta_{\mu\nu} + (\theta + 2 \phi) u_{\mu} u_{\nu}) (b_{\alpha} b_{\beta} - \theta u_{\alpha} u_{\beta})$$.
Viscosities at $B \neq 0$

- At $B \neq 0$, one can get five shear viscosity coefficients $\eta_n$ ($n = 0, 1, 2, 3, 4$) and two bulk viscosity coefficients $\zeta_{\perp, \parallel}$.
Viscosities at $B \neq 0$

- At $B \neq 0$, one can get five shear viscosity coefficients $\eta_n$ \(n = 0, 1, 2, 3, 4\) and two bulk viscosity coefficients $\zeta_{\perp, \parallel}$.
- The shear and the bulk viscous coefficients are obtained from

\[
u = -\xi^{(v)} \eta_0 + P^{(v)}_{\mu\nu\alpha\beta} S^{\mu\nu\alpha\beta}; \quad v \in \{\eta_0, \eta_1, \eta_2, \eta_3, \eta_4, \zeta_{\perp}, \zeta_{\parallel}\}
\]
Viscosities at $B \neq 0$

- At $B \neq 0$, one can get five shear viscosity coefficients $\eta_n \ (n = 0, 1, 2, 3, 4)$ and two bulk viscosity coefficients $\zeta_\perp, \zeta_\parallel$.
- The shear and the bulk viscous coefficients are obtained from

$$
\nu = -\xi^{(v)} \eta_0 + \mathcal{P}^{(v)}_{\mu\nu\alpha\beta} \mathcal{S}^{\mu\nu\alpha\beta} \ ; \ \nu \in \{\eta_0, \eta_1, \eta_2, \eta_3, \eta_4, \zeta_\perp, \zeta_\parallel\}
$$

where, $\xi^{(v)} = \begin{cases} 
4/3 \text{ if } \nu = \eta_1 \\
1 \text{ if } \nu = \eta_2 \\
0 \text{ otherwise }
\end{cases}$ and $\mathcal{P}^{(v)}_{\mu\nu\alpha\beta}$ are given by

$$
\begin{align*}
\mathcal{P}^{(\eta_0)}_{\mu\nu\alpha\beta} &= \frac{1}{4} \left( \Xi^{\sigma\rho} \Xi_{\nu}^{\sigma\rho} - \frac{1}{2} \Xi^{\sigma\rho} \Xi_{\mu\nu}^{\sigma\rho} \right) \left( \Xi^{\sigma\alpha} \Xi_{\rho\beta}^{\sigma\rho} - \frac{1}{2} \Xi^{\sigma\rho} \Xi_{\alpha\beta}^{\sigma\rho} \right), \\
\mathcal{P}^{(\eta_1)}_{\mu\nu\alpha\beta} &= 2 \left( b_{\mu} b_{\nu} - \theta \ u_{\mu} u_{\nu} \right) \left( \frac{1}{2} \Xi_{\alpha\beta}^{\sigma\rho} + (\theta + \phi) \ u_{\alpha} u_{\beta} \right), \\
\mathcal{P}^{(\eta_2)}_{\mu\nu\alpha\beta} &= -\frac{1}{2} \Xi^{\sigma\rho} b_{\nu} \Xi^{\gamma\alpha} b_{\alpha}, \\
\mathcal{P}^{(\eta_3)}_{\mu\nu\alpha\beta} &= -\frac{1}{8} \left( \Xi^{\sigma\rho} \Xi_{\nu}^{\sigma\rho} - \frac{1}{2} \Xi^{\sigma\rho} \Xi_{\mu\nu}^{\sigma\rho} \right) b_{\lambda} \left( \Xi^{\lambda\alpha} \Xi_{\rho\beta}^{\lambda\rho} - \frac{1}{2} \Xi^{\lambda\rho} \Xi_{\alpha\beta}^{\lambda\rho} \right), \\
\mathcal{P}^{(\eta_4)}_{\mu\nu\alpha\beta} &= \frac{1}{2} b_{\rho\sigma} \Xi^{\rho\beta} \Xi^{\sigma\alpha} b_{\alpha}, \\
\mathcal{P}^{(\zeta_\perp)}_{\mu\nu\alpha\beta} &= \frac{1}{3} \left( \Delta_{\mu\nu} + (3\theta + 2\phi) \ u_{\mu} u_{\nu} \right) \left( \frac{1}{2} \Xi_{\alpha\beta}^{\sigma\rho} + (\theta + \phi) \ u_{\alpha} u_{\beta} \right), \\
\mathcal{P}^{(\zeta_\parallel)}_{\mu\nu\alpha\beta} &= -\frac{1}{3} \left( \Delta_{\mu\nu} + (\theta + 2\phi) \ u_{\mu} u_{\nu} \right) \left( b_{\alpha} b_{\beta} - \theta \ u_{\alpha} u_{\beta} \right).
\end{align*}
$$
Viscosities at $B \neq 0$

The viscous coefficients $\nu \in \{\eta_0, \eta_1, \eta_2, \eta_3, \eta_4, \zeta_\perp, \zeta_\parallel\}$ at $B \neq 0$ are:

$$
\nu = \xi(\nu) \eta_0 + \infty \sum_{l=0}^{\infty} \sum_{n=0}^{1} T \int_{-\infty}^{\infty} dk z(2\pi)^{1/4} \omega_{kl} \omega_{kn} \Gamma(\omega_{kl} - \omega_{kn})^2 + \Gamma^2 \times \{ a_f a(\omega_{kl}) + a_f a(\omega_{kn}) + 2 f a(\omega_{kl}) f a(\omega_{kn}) \}
\tilde{N}(\nu) \ln(k z)
$$
Viscosities at $B \neq 0$

The viscous coefficients $\nu \in \{\eta_0, \eta_1, \eta_2, \eta_3, \eta_4, \zeta_\perp, \zeta_\parallel\}$ at $B \neq 0$ are:

$$\nu = \xi^{(\nu)} \eta_0 + \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{T} \int_{-\infty}^{\infty} \frac{dk_z}{(2\pi)^{1/4}} \frac{1}{4\omega_{kl}\omega_{kn}} \frac{\Gamma}{(\omega_{kl} - \omega_{kn})^2 + \Gamma^2}$$

$$\times \{ a f_a(\omega_{kl}) + a f_a(\omega_{kn}) + 2 f_a(\omega_{kl}) f_a(\omega_{kn}) \} \tilde{N}^{(\nu)}_{ln}(k_z)$$
Viscosities at $B \neq 0$

\[ \tilde{\mathcal{N}}_{\ln;\text{Scalar}}^{(\eta_0)}(\vec{k}) = 2A_{\ln}^{(4)}, \]

\[ \tilde{\mathcal{N}}_{\ln;\text{Scalar}}^{(\eta_1)}(\vec{k}) = 8\left[ A_{\ln}^{(0)} \{ (1 - \theta)\omega_{kl}^2 + k_z^2 \} - (1 - \theta - \phi)\omega_{kl}^2 + (1 + \theta + \phi)(k_z^2 + m^2) \right] + A_{\ln}^{(2)} \{ - (1 + \theta - 2\theta^2 - 2\theta\phi)\omega_{kl}^2(1 + \theta + 2\theta^2 + 2\theta\phi)k_z^2 + (1 + \theta + 1)(1 + 2\theta + 2\phi)m^2 \} - A_{\ln}^{(4)} (1 + \theta)(6) \]

\[ \tilde{\mathcal{N}}_{\ln;\text{Scalar}}^{(\eta_2)}(\vec{k}) = -8A_{\ln}^{(2)} k_z^2, \]

\[ \tilde{\mathcal{N}}_{\ln;\text{Scalar}}^{(\eta_3)}(\vec{k}) = 0, \]

\[ \tilde{\mathcal{N}}_{\ln;\text{Scalar}}^{(\eta_4)}(\vec{k}) = 0, \]

\[ \text{Looks Very Nasty .....} \]

\[ \text{.... DOES NOT EVEN FIT IN THE SLIDE} \]

\[ \tilde{\mathcal{N}}_{\ln;\text{Scalar}}^{(\zeta_{\perp})}(\vec{k}) = \frac{1}{3} \left[ 4A_{\ln}^{(0)} \{ (1 - \theta - \phi)\omega_{kl}^2 - (1 + \theta + \phi)(k_z^2 + m^2) \} \{ (3 - 3\theta - 2\phi)\omega_{kl}^2 - (1 + 3\theta + 2\phi)k_z^2 - (3 + 3\theta + 2\phi) \} + A_{\ln}^{(2)} \{ 4(1 + 10\theta + 4\phi - 6\theta^2 - 10\theta\phi - 4\phi^2)\omega_{kl}^2 - 4(1 + 2\theta + 2\phi)(1 + 3\theta + 2\phi)k_z^2 - 4(1 + 7\theta + 6\phi + 6\theta^2 + 10\theta\phi + 4\phi^2)m^2 \} + 4A_{\ln}^{(4)} (\theta + \phi)(1 + 3\theta + 2\phi) \right], \]

\[ \tilde{\mathcal{N}}_{\ln;\text{Scalar}}^{(\zeta_{\parallel})}(\vec{k}) = \frac{4}{3} \left[ A_{\ln}^{(0)} \{ (1 - \theta)(\omega_{kl}^2 + k_z^2) - (1 + \theta)m^2 \} \{ (3 - 3\theta - 2\phi)\omega_{kl}^2 - (1 + 3\theta + 2\phi)k_z^2 - (3 + 3\theta + 2\phi) \} + 2A_{\ln}^{(2)} \{ (2 + \theta - 3\theta^2 - 2\theta\phi)\omega_{kl}^2 - \theta(1 + 3\theta + 2\phi)k_z^2 - (1 + \theta)(2 + 3\theta + 2\phi)m^2 \} + A_{\ln}^{(4)} (1 + \theta)(1 + 3\theta + 2\phi) \right]. \]
Viscosities at $B \neq 0$

No Approximation on the strength of the magnetic field. Large Number of Landau Levels ($\sim 10^4$) are summed up so that the results are valid for low as well as high magnetic field strength.

Numerical consistency check: Numerical limit $B \to 0$ of the viscous coefficients at $B \neq 0 \Rightarrow$ for sufficiently small values of $B$, $\eta_0 \to \eta$, $\eta_1 \to 0$, $\eta_2 \to 0$, $\zeta_{\perp} \to \zeta$ and $\zeta_{\parallel} \to \zeta$.

$\eta$ and $\zeta$ are respectively the shear and bulk viscosities at $B = 0$. Hence, for sufficiently small values of $B$, a large number of Landau levels contribute to $\nu$ in (the Landau levels become infinitesimally close to each other reaching the continuum), which in turn numerically reproduce the exact continuum results of $B = 0$. 
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Viscosities at $B \neq 0$

- **No Approximation** on the strength of the magnetic field.
- Large Number of Landau Levels ($\sim 10000$) are summed up so that the results are valid for low as well as high magnetic field strength.
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- $\eta$ and $\zeta$ are respectively the shear and bulk viscosities at $B = 0$.
- Hence, for sufficiently small values of $B$, a large number of Landau levels contribute to $\nu$ in (the Landau levels become infinitesimally close to each other reaching the continuum), which in turn **numerically** reproduce the exact continuum results of $B = 0$. 


Outline

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Shear Viscosities

Figure: (Color Online) The variation of scaled shear viscosities as a function of $T$ for system of massless charged Dirac Fermions (spin-$\frac{1}{2}$) with relaxation time $\tau_c = 1/\Gamma = 1$ fm.
Figure: (Color Online) The variation of scaled bulk viscosities as a function of $T$ for system of massless charged Dirac Fermions (spin-$\frac{1}{2}$) with relaxation time $\tau_c = 1/\Gamma = 1$ fm.
Collaborators

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Thank You