Thermal Chaos in Extended Phase space of AdS black holes in Einstein Gauss-Bonnet Gravity with quadratic non-linear electrodynamics

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Abstract

We study the inception of chaos in Einstein Gauss-Bonnet gravity with quadratic Maxwell invariant term in AdS space by using Melnikov function under both, temporal and spatial perturbations. In temporal case, when a small time periodic perturbation at subcritical temperature, is applied upon the black hole where it is quenched in spherical region of phase space, chaos starts. It is controlled by the saturation limit of tuning parameter. By analyzing the simple zeros of appropriate Melnikov function exact analytic limit for was obtained. It depends on topological parameter k, Gauss-Bonnet coupling γ and nonlinear correction parameter. For spatial case small perturbation, periodic in space, was applied in subcritical region and it shows that chaos is almost present for all possible topologies.

Motivation: The Melnikov method consists of systematic search of certain structures in phase space, the Smale horseshoe. When a Smale horseshoe exist there are techniques to identify invariant subsets where dynamics is chaotic. Smale horseshoe occur when a homoclinic orbit split under perturbation, in such a way that the two daughter orbits cross transversally. Melnikov method based on evaluation of Melnikov integral along unperturbed homoclinic orbit. If Melnikov integral shows isolated zeros, homoclinic orbits will split and transversal cuts will occur under perturbation. There for their invariant Smale horseshoe in phase space. Specifically analytic nature of this procedure provides us a useful tool to diagnose chaos.

Objective: Determine appropriate Melnikov function of the black hole system when it is quenched in spherical region of its Thermodynamic phase space and analyze it's simple zeros.

Homoclinic and Heteroclinic Orbit: Now homoclinic and heteroclinic orbits looks like for a system in phase space, as following

\[ dQ(t) = \left. \frac{dQ}{dt} \right|_{t\rightarrow\infty} - dQ(t) \]

Now we perturb the Hamiltonian with a function periodic in time. There still will be fixed point \( \alpha \) at same place but stable and unstable points will not join anymore. Suppose after time \( t \) stable and unstable manifold will get separated by a distance \( \delta(t) \)

Melnikov Method for perturbed Hamiltonian system

Suppose perturbed Hamiltonian can be written as \( H_0 + \epsilon f \), \( H_0 \) should have a homoclinic orbit surrounded by a class of periodic orbits as mentioned below - (Figure 1). Period of the orbits should vary continuously with \( \epsilon \).

Hamiltonian can be derived as following by Taylor expansion of P-v eqns about \( \gamma \) and ignoring the coefficient of \( \gamma \)

\[ H = \frac{1}{2}(P^2 + V(q)) \]

Solution of homoclinic orbit is already calculated in [2,3,4]. We substitute that in \( P \) and \( V \). For the heteroclinic orbit in following

\[ P = P_0 + \epsilon \Delta P \]

\[ V = V_0 + \epsilon \Delta V \]

Writing zeros in following form we get \( \Delta P \) and \( \Delta V \)

\[ \Delta P = 0 \]

\[ \Delta V = 0 \]

Required Thermodynamics: Equation of state can be expressed as:

\[ T = \frac{\beta \gamma}{\alpha \gamma} \]

Regions of P-v Diagram

For a subcritical temp, \( P-v \) phase space has three domains

1. \( \Delta \alpha \) corresponds to small black hole region \( \gamma < 0 \)

2. \( \Delta \beta \) corresponds to mixed or spinodal region \( \gamma = 0 \)

3. \( \Delta \theta \) corresponds to large black hole region \( \gamma > 0 \)

Later on we will use \( P_0 \) denote a subcritical temperature and corresponding volume by \( v_0 \) by \( \epsilon^2 + \epsilon^4 \)

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Temporal Fluctuation: We impat a fluctuation at spinodal region(\( T_0 \),\( v_0 \)) of phase space. The corresponding Hamiltonian can be considered as

\[ H = \frac{1}{2}(P^2 + V(q)) \]

Where \( f \) and \( g \) comes from equation of motion of the system.

Equation of motion can be calculated as

\[ \dot{q} = \frac{\partial H}{\partial P} \]

Final equation can be written as

\[ \dot{q} = \frac{\partial H}{\partial P} = \frac{f(q) - \epsilon g(q)}{\alpha} \]

Writing eqn in following form we get \( \Delta P \) and \( \Delta V \)

\[ \Delta P = 0 \]

\[ \Delta V = 0 \]

Conclusion: Appropriate Melnikov function calculated.

1) Analyzing the simple zeros Critical limit of \( \gamma \) obtained. For \( \gamma = 0 \), \( \Delta \alpha, \Delta \beta \) has simple zero for \( \gamma = 0 \) \( (2m+1) \pi/2 \) for any \( m \), for \( \gamma > 0 \), \( \Delta \alpha, \Delta \beta \) has simple zero for \( \gamma = 0 \) \( (2m+1) \pi/2 \) for any \( m \), comes from predicted zeros of homoclinic orbit of spatial dependency. So \( \Delta \alpha, \Delta \beta \) always have simple zeros which indicates chaos.

References: