QCD critical point, universality and small quark mass

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QCD phase diagram and locating the critical point

\[ QGP \Rightarrow \text{hadrons} \]

Crossover at \( \mu = 0 \)

How does \( m_q \) affect these \#s?

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\[ \langle \delta N^n \rangle_c = V T^{n-1} \frac{\partial^n P}{\partial \mu^n} \]

\[ \langle \delta N^2 \rangle \sim \#_2 \xi^2, \langle \delta N^3 \rangle \sim \#_3 \xi^{4.5} \]

\[ \langle \delta N^4 \rangle_c \sim \#_4 \xi^7 \]

\#s depend on microscopic details

Stephanov, 08

2015 LRPNS

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QCD EoS near CP at small \( m_q \)
QCD Equation of State near the critical point

- Taylor expansions about $\mu = 0$ diverge near CP *Hot QCD, 20* and Datta, Gavai, Gupta, 18
- The non-analytic part of the EoS is universal around all critical points with same restored symmetry in spatial dimension $d$ : $\phi^4$ theory in $d = 3$

\[ Z_{QCD}(\mu, T) = Z_{\text{Ising}}(r, h) + \text{subleading singular terms} \]

Static universality class of QCD: 3D Ising model

Fisher, 1998
Equation of State near QCD CP from universality

\[ P_{QCD}(\mu, T) = -G(r, h) + \text{less singular terms} \]

Non-universal mapping between QCD and Ising variables [Parotto et.al.(2018)]

- Connecting to observables
- Chiral limit
- Phenomenological consequences
Relating $\alpha_1$ and $\alpha_2$ to observables

\[
\tan \alpha_1 = \lim_{r \to 0^+} \frac{P_{\mu\mu}}{P_{\mu T}} \\
\tan \alpha_2 = \lim_{h=0 \atop r \to 0^+} \frac{P_{\mu\mu\mu}}{P_{T T T} P_{\mu\mu}}
\]

$\alpha_1 \approx 13^\circ$, $\alpha_2 \approx 1^\circ$

Contours of $\chi_4$ obtained using RMM EoS, Halasz et. al, 1998

Contours of $\chi_4$ obtained using $\phi^4$ theory mapped to RMM
Effective Landau Ginsburg potential near a critical point which is close to a tri-critical point

\[
\Omega(\phi, \mu, T) = -m_q \phi + \frac{a(\mu, T)}{2} \phi^2 + \frac{b(\mu, T)}{4} \phi^4 + \frac{c}{6} \phi^6 \ldots
\]

\[\tan \alpha_1 - \tan \alpha_2 = \frac{20}{7a_T^2} \frac{\partial(a, b)}{\partial(\mu, T)} \phi_c^2 \sim m_q^{2/5} > 0\]
Ginsburg region and beyond mean-field

Mean field theory breaks down when \( \times \sim \times \)

Mean-field theory has a significant range of validity around the critical point and breaks down in a parametrically small region around it.

We studied the affect of fluctuations within the framework of dimensional regularization. Fluctuations donot modify the scaling of slope difference with quark mass.
Sign of skewness along the cross-over line depends on the sign of $\alpha_2$

$$\alpha_1 = 13^\circ, \alpha_2 = 1^\circ$$

- If the value of skewness along the freeze-out curve is negative, it is indicative that the slope of $h$ axis is negative.
Phenomenological consequences

\[ \alpha_1 - \alpha_2 \sim m_q^{2/5} > 0 \]

- Enhanced baryon cumulants relative to the often favored assumption,

\[ h \perp \mu, r \perp T, \frac{\partial^2 P}{\partial \mu^2} \sim \frac{\partial^2 G}{\partial r^2} \sim \xi^{0.17} \]

\[ \alpha_1 - \alpha_2 \sim m_q^{2/5}, \frac{\partial^2 P}{\partial \mu^2} \sim \frac{\partial^2 G}{\partial h^2} \sim \xi^2 \]

- Recently, Mroczek et al., 2020, showed that the dip in the kurtosis may become prominent when \( 0 < \alpha_2 < \alpha_1 \)

M. Stephanov slides, QM 2012
Back up 1: $r$ axis is steeper than $h$ axis

$$\tan \alpha_1 - \tan \alpha_2 = \frac{20}{7a^2_a \partial(\mu, T)} \phi_c^2 \sim m_q^{2/5}$$

$\alpha_1 > \alpha_2$