Studying explicit $U(1)_A$ symmetry breaking in hot and magnetised two flavour non-local NJL model constrained using lattice results

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Introduction

- With the help of the NJL model one can study the chiral properties of QCD. [Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345(1961); 124, 246(1961)]

- In the NJL model the chiral condensate which breaks the chiral symmetry spontaneously, increases with magnetic field(eB) for all temperature(T), termed as magnetic catalysis(MC). Whereas lattice QCD simulation obtained decrease in chiral condensate around the crossover T as eB increases which is termed as inverse magnetic catalysis(IMC). [G. S. Bali et al., Phys. Rev. D 86, 071502]

- One can consider non-local interaction in NJL model which is more realistic as it captures some aspects of the asymptotic freedom of QCD through the non-local form factor. [V. P. Pagura et al., Phys. Rev. D 95, 034013]
Motivation

- Standard NJL model assume the strength of the axial $U(1)$ symmetry breaking ’t Hooft determinant term to be equal to that of the axial $U(1)$ symmetric term, even in the non-local one.

- Our main goals:
  
  (a) To constrain the strength of the $U(1)_A$ symmetry breaking interactions using lattice QCD results.
  
  (b) Study the effect of $T$ and $eB$ on it.
  
  (c) Look at topological susceptibility to get a better understanding on the axial symmetry restoration as a function of $T$. 
NJL Model with Different $U(1)_A$ Breaking Strength


\[ \mathcal{L}_{NJL} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 \]

\[ \mathcal{L}_0 = \bar{\psi} \left( i \not{\partial} - m \right) \psi \]

\[ \mathcal{L}_1 = G_1 \left\{ (\bar{\psi} \psi)^2 + (\bar{\psi} \not{\tau} \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 + (\bar{\psi} i \gamma_5 \not{\tau} \psi)^2 \right\} \]

\[ \mathcal{L}_2 = G_2 \left\{ (\bar{\psi} \psi)^2 - (\bar{\psi} \not{\tau} \psi)^2 - (\bar{\psi} i \gamma_5 \psi)^2 + (\bar{\psi} i \gamma_5 \not{\tau} \psi)^2 \right\} \]

- $\mathcal{L}_2$ explicitly breaks $U(1)_A$.

- Symmetry only allows the $\langle \bar{\psi} \psi \rangle$ condensate, which depends on $(G_1 + G_2)$.

- With $\mu_1$ or magnetic field as the $SU(2)$ symmetry is broken one can have $\langle \bar{\psi} \tau_3 \psi \rangle$ which has a $(G_1 - G_2)$ dependence.

- $G_1$ and $G_2$ can be parametrized as $G_1 = (1 - c)G_0/2$ and $G_2 = cG_0/2$

- $c = 1/2$ corresponds to the standard NJL model.
The NJL Lagrangian with non-local interaction

\[ \mathcal{L} = \bar{\psi} \left( i\partial^\mu - m \right) \psi + \mathcal{L}_1 + \mathcal{L}_2. \]

\( \mathcal{L}_1 \) respects \( U(1)_A \) but \( \mathcal{L}_2 \) does not,

\[ \mathcal{L}_1 = G_1 \{ j_a(x) j_a(x) + j_b(x) j_b(x) \} \]

\[ \mathcal{L}_2 = G_2 \{ j_a(x) j_a(x) - j_b(x) j_b(x) \} \]

with the above definition of \( j_{a/b}(x) \) where \( \Gamma_a = (\mathbb{I}, i\gamma_5 \vec{\tau}) \) and \( \Gamma_b = (i\gamma_5, \vec{\tau}) \).

\( j_{a/b}(x) \) are the non-local currents, given by

\[ j_{a/b}(x) = \int d^4 z \ G(z) \bar{\psi} \left( x + \frac{z}{2} \right) \Gamma_{a/b} \psi(x - \frac{z}{2}), \]

\[ \Gamma_a = (\mathbb{I}, i\gamma_5 \vec{\tau}) \text{ and } \Gamma_b = (i\gamma_5, \vec{\tau}) \]

\( G(z) \) is the non-locality form factor.

Symmetries

\[ SU(2)_V \times SU(2)_A \times U(1)_V \]
With the help of Hubbard-Stratonovich transformation one can bosonize the above Lagrangian.

With non-zero mean field $\langle \bar{\psi} \psi \rangle$, the free energy becomes

$$\frac{\Omega_{MF}}{V^{(4)}} = \frac{\sigma^2}{2G_0} - 2N_c \sum_f \int \frac{d^4p}{(2\pi)^4} \log \left[ p^2 + M_f^2(p) \right]$$

with $M(p) = m + g(p^2)\sigma$

With Lorentz symmetry the Fourier transformation $G(z)$ can only depends on $p^2$, which is written as $g(p^2)$.

We have consider $g(p^2)$ to be Gaussian in nature

$$g(p^2) = \exp[-p^2/\Lambda^2]$$

The model parameters are fixed to get physical pion mass and decay constant at zero $T$. 
Condensate and $F_\pi$ taken from LQCD calculation

\[ \langle \bar{\psi} \psi \rangle^{1/3} = 261(13)(1) \text{ MeV} \text{ and } F_\pi = 90(8)(2) \text{ MeV}. \]

With the physical pion mass ($M_\pi = 135 \text{ MeV}$). [B. B. Brandt, A. Juttner, and H. Wittig, JHEP 11, 034 (2013)]

For a given value of $\langle \bar{\psi} \psi \rangle^{1/3}$, $F_\pi$ increases as $\Lambda$ decreases and/or $G_0$ increases. This is other way around for condensate.

$\Lambda$ defines how fast the effective coupling decreases with momentum. This information will be helpful in understanding our results.

Another parameter set with $\langle \bar{\psi} \psi \rangle^{1/3} = 238.9(4) \text{ MeV}$ and $F_\pi = 87.3(5.6) \text{ MeV}$. [H. Fukaya et al.(JLQCD), Phys. Rev. D 77, 074503 (2008)]
For the non-local interaction the currents should transform as

\[ j_{a/b}(x) \rightarrow \int d^4z \mathcal{G}(z) \bar{\psi}(x + \frac{z}{2}) W^\dagger(x + z/2, x) \Gamma_a W(x, x - z/2) \psi(x - \frac{z}{2}) \]

\[ W(s, t) = P \exp \left[ -iQ \int_s^t dr_\mu A_\mu(r) \right] \]

The bosonized action

\[ S_{bos} = - \ln \det(D) + \int d^4x \left[ \frac{\sigma^2(x)}{2G_0} + \frac{\Delta\sigma^2(x)}{2(1-2c)G_0} \right] \]

In Lorentz gauge the inverse of fermionic propagator is given by

\[ \mathcal{D}_{MF}(x, x') = \delta^{(4)}(x - x')( -i \slashed{\partial} - QBx_1 \gamma_2 + m) + \mathcal{G}(x - x') \times (\sigma + \tau_3 \Delta \sigma) \exp \left[ iQB(x_2 - x'_2)(x_1 + x'_1) \right]. \]

Using Ritus eigenfunction one obtain the Fourier transform of the above.
Fitting of $c$ using LQCD data

- As mentioned earlier the average condensate does not depend on $c$. 

- Condensate difference depends on $c$. $c$ is fitted to 0.058 to match the LQCD results with $\chi^2/\text{DoF} = 1.5$. [MSA, Chowdhury Aminul Islam, Rishi Sharma, arXiv:2009.13563]

- For JLQCD $c$ is fitted to 0.16 with $\chi^2/\text{DoF} = 0.12$
Average condensate at finite temperature

- **CC parameter of Brandt13**

- **LH parameter of Brandt13**
Crossover Temperature ($T_{CO}$) as a function of $eB$

- Transition temperature does not depend on $c$ as the condensate is almost independent of $c$.

![Graph showing $T_{CO}$ as a function of $eB$.](image)

**Figure:** Phase diagram in $T_{CO} - eB$ plane. Brandt13(left) JLQCD(right)

- Low condensate value and high $F_\pi$ produce a stronger IMC effect around the crossover temperature.
- In terms of the parameters, smaller $\Lambda$ and higher $G_0$ produces better IMC around the crossover temperature.
Condensate difference at finite temperature

- **CC parameter of Brandt13**

- **LH parameter of Brandt13**

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Topological Susceptibility

- Topological susceptibility ($\chi_t$) can be formally related to the mass of the axion field. [S. Weinberg, Phys. Rev. Lett. 40, 223 (1978)]
- The topological term $\frac{\theta g^2}{32\pi^2} F \tilde{F}$ breaks the CP symmetry of strong interaction.
- As the dynamical axion is considered to be a possible solution of strong CP problem, $\theta$ can be related to the axion fields, $\theta = a/f_a$.
- With a chiral rotation of the quark fields by an angle $a/f_a$ one obtain
  \[
  \mathcal{L}_2 = 2 G_2 \left\{ e^{i \frac{a}{f_a}} \det \bar{\psi}(1 + \gamma_5) \psi + e^{-i \frac{a}{f_a}} \det \bar{\psi}(1 - \gamma_5) \psi \right\}
  \]
- $\Omega(T, eB, a)$ being the free energy with these modification, the topological susceptibility is
  \[
  \frac{d^2 \Omega(T, eB, a)}{da^2} \bigg|_{a=0} = \frac{\chi_t}{f_a^2}.
  \]
Topological susceptibility

Topological susceptibility in presence of $eB$

- Topological susceptibility for Brandt13(left) and JLQCD(right) as a function of scaled $T$ with fitted $c$ for different value of $eB$.
• $c$ does not effect the average condensate. Has effect on condensate difference in presence of a magnetic field.

• The effect of magnetic field on axial symmetry at zero $T$ can be studied through the effect of the same on $c$.

• For our parameter set $\chi^2/\text{DoF}$ is reasonably good without any eB dependence.

• For finite $T$ we need an order parameter associated with axial symmetry breaking. Topological susceptibility can be treated as one.


• With more precise lattice QCD data on $\chi_t$, we might be able to obtain the $T$ dependence of $c$.

• The finite chemical potential region can be another interesting direction to explore.
Thank You
The free energy is divergent, and is regularized as follows:

\[ \Omega_{\text{MF}}^{(\text{reg})} = \Omega_{\text{MF}} - \Omega_{\text{free}} + \Omega_{\text{free}}^{(\text{reg})} \]

The gap equations are free from these divergence due to the non-locality form factor.

The condensates are given by the derivative of \( \Omega_{\text{MF}}^{(\text{free})} \) with respect to bare quark masses. So the regularization follows.

\[ \langle \bar{\psi}\psi \rangle^{\text{(reg)}} = \langle \bar{\psi}\psi \rangle - \langle \bar{\psi}\psi \rangle_{\text{free}} + \langle \bar{\psi}\psi \rangle_{\text{free}}^{(\text{reg})} \]

Finite temperature extension is straightforward following Matsubara formalism.

Magnetic field makes the analytic calculation complicated but the procedure is the same.
Topological Susceptibility as a function of $c$ (Backup)

Figure: $\chi_t$ as a function of the explicit $U(1)_A$ symmetric breaking parameter $c$ at different temperatures for the same set of parameter.