Measurements of Neutron Stars and the Dense Matter EOS

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Main Topics

- Neutron Stars and How They Depend on the Equation of State
- Maximum Mass and Causality Constraints
- Nuclear Physics and Unitary Gas Constraints
- Measuring Neutron Star Properties From Radio, X-ray and Gravitational Wave Observations
- Estimating Neutron Star Properties from Neutron Star Mergers and NICER
A NEUTRON STAR: SURFACE and INTERIOR

CORE:
Homogeneous Matter

CRUST:
Nuclei
Neutron Superfluid

ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE

Polar cap
Cone of open magnetic field lines

Neutron Superfluid
Neutron Vortex
Nuclei in a lattice
Magnetic Flux Tube

Measurements of Neutron Stars and the Dense Matter EOS
Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

\[
\frac{dp}{dr} = -\frac{G (mc^2 + 4\pi pr^3)(\varepsilon + p)}{c^4 r(r - 2Gm/c^2)} \\
\frac{dm}{dr} = 4\pi \frac{\varepsilon}{c^2} r^2
\]

Equation of State

Observations
Mass-Radius Diagram and Theoretical Constraints

GR:
\[ R > \frac{2GM}{c^2} \]

\[ P < \infty : \quad R > \left( \frac{9}{4} \right) \frac{GM}{c^2} \]

causality:
\[ R \gtrsim 2.9\frac{GM}{c^2} \]

— normal NS
— SQS

\[ R_\infty = \frac{R}{\sqrt{1 - 2\frac{GM}{Rc^2}}} \]
The Radius – Pressure Correlation


$R P^{-1/4} (\text{km fm}^{3/4} \text{MeV}^{-1/4})$


$9.52 \pm 0.49$

$7.06 \pm 0.24$

$5.68 \pm 0.14$
The symmetry energy is the difference between the energies of pure neutron matter \((x = 0)\) and symmetric \((x = 1/2)\) nuclear matter:

\[
S(n) = E(n, x = 0) - E(n, x = 1/2)
\]

Usually approximated as an expansion around the saturation density \((n_s)\) and isospin symmetry \((x = 1/2)\):

\[
E(n, x) = E(n, 1/2) + (1 - 2x)^2 S_2(n) + \ldots
\]

\[
S_2(n) = S_V + \frac{L}{3} \frac{n - n_s}{n_s} + \ldots
\]

\[
S_V \approx 31 \text{ MeV}, \quad L \approx 50 \text{ MeV}
\]

Extrapolated to pure neutron matter:

\[
E(n_s, 0) \approx S_V + E(n_s, 1/2) \equiv S_V - B, \quad p(n_s, 0) = L n_s / 3
\]

Neutron star matter (beta equilibrium) is nearly neutron matter:

\[
\frac{\partial(E + E_e)}{\partial x} = 0, \quad p(n_s, x_\beta) \approx \frac{L n_s}{3} \left[ 1 - \left( \frac{4S_V}{\hbar c} \right)^3 \frac{4 - 3S_V / L}{3\pi^2 n_s} \right]
\]
The Conjecture: Neutron matter energy is larger than that of the unitary gas \( E_{UG} = \xi_0 (3/5) E_F \), or

\[
E_{UG} \simeq 12.6 \left( \frac{n}{n_s} \right)^{2/3} \text{ MeV}
\]

The unitary gas consists of fermions interacting via a pairwise short-range s-wave interaction with infinite scattering length and zero range. Cold atom experiments show a universal behavior with the Bertsch parameter \( \xi_0 \simeq 0.37 \).

\[
S_v \geq 28.6 \text{ MeV}; \quad L \geq 25.3 \text{ MeV}; \quad p_0(n_s) \geq 1.35 \text{ MeV fm}^{-3}; \quad R_{1.4} \geq 9.7 \text{ km}
\]
Recently developed chiral effective field theory allows a systematic expansion of nuclear forces at low energies based on the symmetries of quantum chromodynamics. It exploits the gap between the pion mass (the pseudo-Goldstone boson of chiral symmetry-breaking) and the energy scale of short-range nuclear interactions established from experimental phase shifts. It provides the only known consistent framework for estimating energy uncertainties.
Surface and volume symmetry energies of nuclei are highly correlated, $L \propto S_V$.

Neutron skin thicknesses depend primarily on $L$:
$$\Delta r_{np}^{208}(\text{fm}) = 0.00147L + 0.101 \pm 0.022.$$
$R_{1.4} = (9.52 \pm 0.49) \left( \frac{p_s}{\text{MeV fm}^{-3}} \right)^{1/4} \text{ km}$

$p_s \approx n_s L/3$

$30 \text{ MeV} \lesssim L \lesssim 70 \text{ MeV}.$

Causality and $M_{\text{max}} \gtrsim 2.0 M_\odot$: $R_{1.4} \gtrsim 8.2 \text{ km}$

Imposing the unitary gas conjecture: $R_{1.4} \gtrsim 9.7 \text{ km}$

Theoretical neutron matter studies:

$10.3 \text{ km} \lesssim R_{1.4} \lesssim 13.5 \text{ km}.$
Pulsar timing can accurately (> 0.0001\(M_\odot\)) measure masses. Most are between 1.2\(M_\odot\) and 1.5\(M_\odot\); lowest is 1.174 ± 0.004\(M_\odot\), highest are 2.14\(^{+0.10}_{-0.09}\)\(M_\odot\) and 2.01 ± 0.04\(M_\odot\). Higher estimates have large uncertainties.

Thermal and bursting observations of X-rays yield radii, but uncertain to a few km.

- Quiescent sources in globular clusters
- Thermonuclear explosions on accreting neutron stars in binaries
- Pulse profile modeling of hot spots on rapidly rotating neutron stars (NICER experiment)

Gravitational waves from merging neutron stars measure masses and tidal deformabilities. GW170817 suggests \(R = 10.5 \pm 1.5\) km
LIGO-Virgo (LVC) detected a signal consistent with a BNS merger, followed 1.7 s later by a weak sGRB.

10100 orbits observed over 317 s.

\[ M = 1.186 \pm 0.001 \, M_\odot \]

\[ M_{T,\text{min}} = 2^{6/5} M = 2.725 M_\odot \]

\[ E_{GW} > 0.025 M_\odot c^2 \]

\[ D_L = 40^{+8}_{-14} \, \text{Mpc} \]

\[ 75 < \tilde{\Lambda} < 560 \, (90\%) \]

\[ M_{\text{ejecta}} \sim 0.06 \pm 0.02 \, M_\odot \]

Blue ejecta: \sim 0.01 M_\odot

Red ejecta: \sim 0.05 M_\odot

Possible r-process production

Ejecta + GRB: \[ M_{\text{max}} \lesssim 2.22 M_\odot \]
Properties of Known Double Neutron Star Binaries

- Both component masses are accurately measured (11)
  — Only the total binary mass is accurately measured (6)

**Binaries with \( \tau_{GW} > t_{\text{universe}} \) (7)**

- \( q = \frac{M_2}{M_1} \) is the binary mass ratio for a system
- \( \chi = \frac{cJ}{(GM^2)} \) is the dimensionless spin parameter for individual pulsars

\[ \mathcal{M} = \left( \frac{M_1 M_2}{M_1 + M_2} \right)^{3/5} \] is the chirp mass
There are 13 parameters in third PN order \((v/c)^6\) models which include finite-size effects. LVC17 used a 13-parameter model; De et al. (2018) used a 9-10 parameter model.

- Sky location (2) EM data
- Distance (1) EM data
- Inclination (1)
- Coalescence time (1)
- Coalescence phase (1)
- Polarization (1)
- Component masses (2)
- Spin parameters (2)
- Tidal deformabilities (2) correlated with masses

Extrinsic

Intrinsic
Tidal Deformability

The tidal deformability $\lambda$ is the ratio of the induced dipole moment $Q_{ij}$ to the external tidal field $E_{ij}$, $Q_{ij} \equiv -\lambda E_{ij}$.

We use the dimensionless quantity $\Lambda = \frac{\lambda c^{10}}{G^4 M^5} \equiv \frac{2}{3} k_2 \left( \frac{Rc^2}{GM} \right)^5$

$k_2$ is the dimensionless Love number.

For a neutron star binary the mass-weighted $\tilde{\Lambda}$ is the relevant parameter:

$$\tilde{\Lambda} = \frac{16}{13} \frac{(1 + 12q)\Lambda_1 + (12 + q)q^4\Lambda_2}{(1 + q)^5}, \quad q = \frac{M_2}{M_1} \leq 1$$
The Effect of Tides

Tides accelerate the inspiral and produce a phase shift compared to the case of two point masses.

\[ \delta \Phi_t = -\frac{117}{256} \frac{(1 + q)^4}{q^2} \left( \frac{\pi f_{GW} GM}{c^3} \right)^{5/3} \tilde{\Lambda} + \cdots. \]

credit: Jocelyn Read
\( \Lambda \) is Highly Correlated With \( M \) and \( R \)

- \( \Lambda = a \beta^{-6} \)
  \( \beta = GM/Rc^2 \)
- \( a = 0.0086 \pm 0.0011 \)
  for
  \( M = (1.35 \pm 0.25) M_\odot \)
- If \( R_1 \sim R_2 \sim R_{1.4} \)
  it follows that
  \( \Lambda_2 \sim q^{-6} \Lambda_1 \).

\[ \Lambda = a \beta^{-6} \]

\[ \beta = GM/Rc^2 \]

\[ a = 0.0086 \pm 0.0011 \]

for

\[ M = (1.35 \pm 0.25) M_\odot \]

Zhao & Lattimer (2018)
Binary Deformability and the Radius

\[ \tilde{\Lambda} = \frac{16}{13} \left( 1 + 12q \right) \Lambda_1 + q^4 \left( 12 + q \right) \Lambda_2 \]

\[ \sim \frac{16a}{13} \left( \frac{R_{1.4}c^2}{G \mathcal{M}} \right)^6 q^{8/5} (12 - 11q + 12q^2) \]

\[ (1 + q)^{26/5} \]

- For \( \tilde{\Lambda} \): \( a' = 0.0035 \pm 0.0006 \)
- For \( \tilde{\Lambda} \): \( a' = 0.00375 \pm 0.00025 \)
- \( R_{1.4} = \) \[ (11.5 \pm 0.3) \frac{\mathcal{M}}{M_\odot} \left( \frac{\tilde{\Lambda}}{800} \right)^{1/6} \] km
- For GW170817: \( R_{1.4} = (13.4 \pm 0.1) \left( \frac{\tilde{\Lambda}}{800} \right)^{1/6} \) km

Zhao & Lattimer (2018)

GW170817: \( M_{\text{max}} > 2.01 M_\odot \)
68.3%, 90%, 95.4% and 99.7% Confidence Bounds

Zhao and Lattimer (2019)

Uniform $\Lambda_s$

Zhao and Lattimer (2019)
Pulsar observations imply non-rotating $M_{\text{max}} \gtrsim 2M_\odot$.

Remnant differential rotation uniformizes within $\sim 0.1$ s.

Inspiralling mass $M_T = M q^{-3/5} (1 + q)^{6/5}$ is $2.73M_\odot$ ($q = 1$) to $2.78M_\odot$ ($q = 0.7$), smaller than $M_{\text{max,d}}$.

Maximally uniformly rotating stars have $M_{\text{max,u}} = \xi M_{\text{max}}$ with $1.17 \lesssim \xi \lesssim 1.21$. *Hypermassive* stars, with $M_T > M_{\text{max,u}}$, promptly collapse to a BH.

*Supermassive* stars, with $M_{\text{max}} \leq M_T \leq M_{\text{max,u}}$, are metastable but have much longer lifetimes. Such a remnant pumps too much energy into the ejecta to be consistent with observations.

Taking into account gravitational binding energy, the condition $M_T > M_{\text{max,u}}$ implies $M_{\text{max}} \leq 2.22M_\odot$. 
Neutron Star Interior Composition ExploreR (NICER)

Science Measurements

Reveal stellar structure through lightcurve modeling, long-term timing, and pulsation searches

Lightcurve modeling constrains the compactness \( (M/R) \) and viewing geometry of a non-accreting millisecond pulsar through the depth of modulation and harmonic content of emission from rotating hot-spots, thanks to gravitational light-bending...
... while phase-resolved spectroscopy promises a direct constraint of radius $R$. 

PSR J0437–4715

$M = 1.67 ~ M_\odot$

$R = 12 ~ km, 13.25 ~ km$

$T_{\text{exp}} = 1 ~ M\text{sec}$
LVC O3 Detections

36 BBH mergers, plus 3 mergers potentially containing a neutron star:

- GW190425 (156 ± 41 Mpc, FAR = 4.5 \cdot 10^{-13}, \mathcal{M} = 1.44 \pm 0.02 M_\odot)
  Either a BNS with \( m_1 = 1.85_{-0.19}^{+0.27} M_\odot \) and \( m_2 = 1.47_{-0.08}^{+0.16} M_\odot \), or
  a BHNS with \( m_{BH} = 2.19_{-0.17}^{+0.21} M_\odot \) and \( m_{NS} = 1.26_{-0.08}^{+0.10} M_\odot \).

- GW190426 (377 ± 100 Mpc, FAR = 1.9 \cdot 10^{-8}, \mathcal{M} = 2.41_{-0.18}^{+0.08} M_\odot)
  A likely BHNS with \( m_{BH} = 5.7_{-2.3}^{+4.0} M_\odot \) and \( m_{NS} = 1.5_{-0.5}^{+0.8} M_\odot \).

- GW190814 (267 ± 52 Mpc, FAR=2.0 \cdot 10^{-33}, \mathcal{M} = 6.09 \pm 0.06 M_\odot, q = 0.112 \pm 0.009)
  Either a BHNS or a BBH with \( m_1 = 23.2_{-1.0}^{+1.1} M_\odot \) and
  \( m_2 = 2.59_{-0.09}^{+0.08} M_\odot \).
GW170817 provided $R$ and EOS information compatible with expectations from nuclear theory, experiment and other astrophysical observations, considering existing systematic uncertainties.

GW170817 also hints that $M_{\text{max}}$ is not far above the $2M_\odot$ minimum provided by pulsar timing, supported by possible identification of low-mass black holes with $M < 3M_\odot$.

NICER provides masses and radii from pulse-profile models of rapidly rotating X-ray pulsars. It will measure radii of both typical and also $2 - 2.1M_\odot$ stars.

Future GW measurements of BNS will be additive since sources should be similar.

There is some tension between PREX neutron skin measurements and other nuclear experiments and astrophysical observations concerning $L$. 