LHC bounds on $R_{D(*)}$ motivated Leptoquark models

$S_1$ & $U_1$ models

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December 16, 2020
Outline

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- Single Leptoquark (LQ) models
  - $S_1$ Model
  - $U_1$ Model
- Production Modes
- Recasting, Analysis
- Exclusion Limits
- Conclusions
$R_{D(*)}$ Anomalies

$R_{D(*)} = \frac{Br(B \rightarrow D(*)\tau\nu)}{Br(B \rightarrow D(*)\ell\nu)}$

- SM Expectation:
  $R_D = 0.299 \pm 0.003$
  $R_D^* = 0.258 \pm 0.005$

- Updated values (Experimental average):
  $R_D = 0.340 \pm 0.027 \pm 0.013$
  $R_D^* = 0.295 \pm 0.011 \pm 0.008$, combined excess of $\sim 3.08\sigma$

LQ solution: Processes Involved

Figure: $B \to D^{(*)}\tau\nu$ decay in SM

Figure: $B \to D^{(*)}\tau\nu$ decay in (a) $S_1$ LQ model, (b) $U_1$ LQ model
Related Searches

Pair Production searches

- For $\text{Br}(LQ \rightarrow t\tau) = 1$
  
  Scalar LQ: excluded masses below 900 GeV

- For $\text{Br}(LQ \rightarrow b\nu) = 1$
  
  Scalar LQ: excluded masses below 1100 GeV
  Vector LQ: excluded masses below 1475 GeV

- For $\text{Br}(LQ \rightarrow t\nu) = 1$
  
  Scalar LQ: excluded masses below 1020 GeV
  Vector LQ: excluded masses below 1460 GeV

- For vector $LQ \rightarrow t\nu, b\tau$ with $\text{Br}(t\nu) = \text{Br}(b\tau) = 50\%$, LQs with mass below 1115 GeV are excluded.
Objective

Obtain **complimentary exclusion limits** from LHC data that are independent of the other flavour bounds.

How?
By recasting LHC dilepton (i.e. $\tau\tau$, $\tau\nu$) search results.
$S_1$ Model

The possible interaction terms that would affect the $R_D(\ast)$ observables can be expressed as follows:

$$
\mathcal{L} \supset \left[ \lambda^L_{3\alpha} \bar{Q}^c_3 (i\tau_2) L_\alpha + \lambda^L_{23} \bar{Q}^c_2 (i\tau_2) L_3 + \lambda^R_{23} \bar{c}^c \tau_R \right] S_1^\dagger + h.c.
$$

where

- $Q_\alpha (L_\alpha)$ denotes the $\alpha$-th generation quark (lepton) doublet
- $\lambda^H_{ab}$ denotes the coupling of $S_1$ with a charge-conjugate quark from generation $a$ and a lepton of chirality $H$ from generation $b$
Minimal Scenarios

\[ \mathcal{L} \supset \left[ \lambda_{3\alpha}^L \bar{Q}_3^c (i\tau_2) L_\alpha + \lambda_{23}^L \bar{Q}_2^c (i\tau_2) L_3 + \lambda_{23}^R \bar{c}^c \tau_R \right] S_1^\dagger + h.c. \]

**Scenario 1**: \( S_1 \) is aligned to the up-type quark basis

\[ \mathcal{L} \supset \lambda_{23}^L \left[ \bar{c}^c \tau - (V_{cb} \bar{b}^c + V_{cs} \bar{s}^c + V_{cd} \bar{d}^c) \nu \right] S_1^\dagger + h.c. \]

**Scenario 2**: \( S_1 \) is aligned to the down-type quark basis

\[ \mathcal{L} \supset \lambda_{33}^L \left[ (V_{cb} \bar{c}^c + V_{tb} \bar{t}^c + V_{ub} \bar{u}^c) \tau - \bar{b}^c \nu \right] S_1^\dagger + h.c. \]

**Interesting for the LHC**: A large \( \lambda_{23}^L \) opens up the possibility of producing \( S_1 \) through \( s \)- and/or \( c \)-quark initiated processes (\( s \)- or \( c \)-PDF \( \gg \) \( b \)-PDF).
Direct Production processes: (Scenario 1: $\lambda_{23}^L$ is non-zero.)

- Pair Production

\[
p p \rightarrow \begin{cases} 
    S_1 S_1 & \rightarrow \ c\tau \ c\tau \ \equiv \ \tau\tau + 2j \\
    S_1 S_1 & \rightarrow \ c\tau \ s\nu \ \equiv \ \tau + 2j + \not{E}_T \\
    S_1 S_1 & \rightarrow \ s\nu \ s\nu \ \equiv \ \not{2j} + \not{E}_T
\end{cases}
\]
Production Modes, Decays

Direct Production processes: (Scenario 1: $\lambda_{23}^L$ is non-zero)

- Pair Production

  \[
  pp \rightarrow \begin{cases} 
  S_1 S_1 \rightarrow c\tau \ c\tau & \equiv \tau\tau + 2j \\
  S_1 S_1 \rightarrow c\tau \ s\nu & \equiv \tau + 2j + E_T \\
  S_1 S_1 \rightarrow s\nu \ s\nu & \equiv 2j + E_T
  \end{cases}
  \]

- Single Production

  - $\tau\tau +$ jets:

    \[
    pp \rightarrow \begin{cases} 
    S_1 \tau \rightarrow \tau j \ \tau \\
    S_1 \tau j \rightarrow \tau j \ \tau j \\
    S_1 \tau jj \rightarrow \tau j \ \tau jj
    \end{cases}
    \]

  - $\tau + E_T +$ jets:

    \[
    pp \rightarrow \begin{cases} 
    (S_1 \tau + S_1 \nu) \rightarrow \nu j \ \tau + \tau j \ \nu \\
    (S_1 \tau j + S_1 \nu j) \rightarrow \nu j \ \tau j + j \ \nu j \\
    (S_1 \tau jj + S_1 \nu jj) \rightarrow \nu j \ \tau jj + j \ \nu jj
    \end{cases}
    \]

Weak limits

[1808.04169]
Production Modes, Decays

Indirect processes: (Scenario 1: $\lambda^{L}_{23}$ is non-zero.)
- $t$-channel exchange of $S_1$
- Interferes with SM (destructively)

$\lambda$ dependence of all productions modes:

$$\sigma^{S_1}_{total} = \sigma_{p} + \sigma^{incl}_{s} + \sigma_{t} - \sigma_{\times}$$

$$\approx \lambda^{0} \quad \lambda^{2} \quad \lambda^{4} \quad \lambda^{2}$$
**ATLAS $\tau\tau$ search. 36 fb$^{-1}$, 13 TeV**

Latest $pp \rightarrow Z' \rightarrow \tau\tau$ data is used. [1709.07242]

Event Selection Criteria:

- **$\tau\tau$ search: $\tau_{had}\tau_{had}$ channel**
  1. Two $\tau_{had}$'s are tagged, no electrons or muons
  2. Two $\tau_{had}$'s have $p_T(\tau_{had}) > 65$ GeV, they are oppositely charged, in the azimuthal plane by $|\Delta\phi(p_T^{\tau_1}, p_T^{\tau_2})| < 2.7$ rad.

- **$\tau\tau$ search: $\tau_{lep}\tau_{had}$ channel**
  1. Any event has one $\tau_{had}$, one $\ell = e, \mu$
  2. $p_T(\tau_{had}) > 25$ GeV, $|\eta(\tau_{had})| < 2.3$ (excluding $1.37 < |\eta| < 1.52$)
  3. if $\ell = e$, $|\eta| < 2.4$ (excluding $1.37 < |\eta| < 1.52$), if $\ell = \mu$, $|\eta| < 1.52$
  4. $p_T(\ell) > 30$ GeV, $|\Delta\phi(p_T^{\tau_1}, p_T^{\tau_2})| < 2.4$ rad.
  5. $m_T(p_T^\ell, \not{E}_T) > 40$ GeV, where

$$m_T(p_T^A, p_T^B) = \left[2p_T^Ap_T^B\left\{1 - \cos\Delta\phi(p_T^A, p_T^B)\right\}\right]^{1/2}$$

Total transverse mass, $m_T^{tot}$ is binned which is defined

$$m_T^{tot}(\tau_1, \tau_2, \not{E}_T) = \left[m_T^2(p_T^{\tau_1}, p_T^{\tau_2}) + m_T^2(p_T^{\tau_1}, \not{E}_T) + m_T^2(p_T^{\tau_2}, \not{E}_T)\right]^{1/2}$$
**ATLAS $\tau\nu$ search, $36 \text{ fb}^{-1}, 13 \text{ TeV}$**

Latest $pp \rightarrow W' \rightarrow \tau\nu$ data is used. [1801.06992]

**Event Selection Criteria:**

- **$\tau\nu$ search:**
  1. At least one $\tau_{had}$ with $p_T(\tau_{had}) > 25$ GeV, $|\eta(\tau_{had})| < 2.4$
  2. $\slashed{E}_T > 150$ GeV with $0.7 < p_T(\tau_{had})/\slashed{E}_T < 1.3$
  3. $\Delta \phi(p_T^{\tau_1}, \slashed{E}_T) < 2.4$
  4. Events are rejected if they contain any $\ell = e, \mu$ with $p_T(\ell) > 20$ GeV, $|\eta(\ell)| < 2.47$ (excluding barrel-endcap region)

- For Scenario 1, CMS $2j + \slashed{E}_T$ limits are also considered - assuming the jets originate from $s$ quarks (pair production)

- For Scenario 2, CMS searches for the pair production of third generation LQ with $tt\tau\tau$ and $bb + \slashed{E}_T$ are used for recasting.
Process Contributions

$S_1$ model

<table>
<thead>
<tr>
<th>$M_{S_1}$ (TeV)</th>
<th>Pair (NLO) ($\lambda \approx 0$)</th>
<th>Indirect ($\lambda^2$, $\lambda = 1$)</th>
<th>BSM ($\lambda^4$, $\lambda = 1$)</th>
<th>Inclusive single ($\lambda^2$, $\lambda = 1$)</th>
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<tbody>
<tr>
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<td>$\sigma_p$</td>
<td>$\varepsilon_p$</td>
<td>$N_p$</td>
<td>$-\sigma_X$</td>
</tr>
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<td>3.4</td>
<td>1.63</td>
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<td>1.5</td>
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<td>2.0</td>
<td>0.003</td>
<td>3.1</td>
<td>0.00</td>
<td>-15.30</td>
</tr>
</tbody>
</table>

Interference contribution ($\sigma_X$) $\gg$ other modes
Data Recast, Exclusion Limits

- Chi-square test is performed, with the test statistic:

\[
\chi^2 = \sum_i \left[ \frac{N^i_T - N^i_D}{\Delta N^i} \right]^2
\]

Events are combined as follows:

\[
N^i_T = N^i_{S_1} + N^i_{BG} = \left[ N_p + N_{incl}^s + N_t - N_x \right]^i + N^i_{BG}
\]

using total uncertainty,

\[
\Delta N^i = \sqrt{(\Delta N^i_{Stat})^2 + (\Delta N^i_{Syst})^2}
\]

where, \(\Delta N^i_{stat} = \sqrt{N^i_D}\) and we assume \(\Delta N^i_{sys} = \delta^i \times N^i_D\)
LHC Exclusion Limits

\( \lambda_{23}^L \) is non-zero

Figure: The 1\( \sigma \) and 2\( \sigma \) CL exclusion limits on \( \lambda = \lambda_{23}^L \) in Scenario-I as functions of \( M_{S_1} \) using the ATLAS (a) \( \tau\tau \) and (b) \( \tau\nu \) resonance search data. The coloured regions are excluded. We keep \( \lambda \leq 3.5 \) to ensure \( \lambda^2 / 4\pi < 1 \).
LHC + flavour

\( \lambda_{23}^L \) is non-zero

\( \lambda_{33}^L \) is non-zero

(a) Scenario-I

(b) Scenario-II

Figure: The 95% CL (2\( \sigma \)) exclusion limits from the LHC in the \( M_{S_1} - \lambda \) plane for the three scenarios in the minimal model with \( S_1 \) and the preferred regions by the \( R_{D(*)} \) anomalies with (a) \( \lambda = \lambda_{23}^L, \lambda_{33}^L = 0 \) (Scenario-I), (b) \( \lambda_{23}^L = 0, \lambda = \lambda_{33}^L \) (Scenario-II)
LHC + flavour: Combined Scenario

\[ \lambda_{33}^L = 0.5 \]

\[ \lambda_{33}^L = 1 \]

**Figure:** The 95% CL (2\(\sigma\)) exclusion limits from the LHC in the \(M_{S_1} - \lambda\) plane for the three scenarios in the minimal model with \(S_1\) and the preferred regions by the \(R_{D(*)}\) anomalies with (a) \(\lambda = \lambda_{23}^L, \lambda_{33}^L = 0.5\) (Scenario-III) and (b) \(\lambda = \lambda_{23}^L, \lambda_{33}^L = 1\) (Scenario-III).
The interaction between $U_1$ and the SM quarks and leptons can be expressed as:

$$
\mathcal{L} \supset \lambda^L_{23} \bar{Q}^2 \gamma_\mu U^\mu_1 P_L \ell^3 + \lambda^L_{33} \bar{Q}^3 \gamma_\mu U^\mu_1 P_L \ell^3 + \lambda^R_{33} \bar{b} \gamma_\mu U^\mu_1 \tau_R + \text{H.c.}
$$

where

- $Q^i$ and $\ell^j$ - SM left-handed quark and lepton doublets
- $i, j = \{1, 2, 3\}$ stand for the generation indices
Production Modes, Decays

For example, considering $\lambda_{23}^L$ is non-zero (Scenario RD1A)

- Pair Production

\[
pp \rightarrow \begin{cases} 
U_1 U_1 \rightarrow s\tau s\tau \equiv \tau\tau + 2j \\
U_1 U_1 \rightarrow s\tau c\nu \equiv \tau + E_T + 2j \\
U_1 U_1 \rightarrow c\nu c\nu \equiv E_T + 2j
\end{cases}
\]

- Single Production

\[
pp \rightarrow \begin{cases} 
U_1 \tau + U_1 \tau j \rightarrow (s\tau)\tau + (s\tau)\tau j \equiv \tau\tau + n_j \\
U_1 \nu + U_1 \nu j \rightarrow (c\nu)\nu + (c\nu)\nu j \equiv E_T + n_j \\
U_1 \tau + U_1 \tau j \rightarrow (c\nu)\tau + (c\nu)\tau j \equiv \tau + E_T + n_j \\
U_1 \nu + U_1 \nu j \rightarrow (s\tau)\nu + (s\tau)\nu j \equiv \tau + E_T + n_j
\end{cases}
\]

- Contributions from the exclusive modes are considered in the $\tau\tau$ channels
LHC Exclusion + $R_D(*)$ plots - RD1A, RD1B

Figure: The 95% CL (2σ) exclusion limits from the LHC in the $M_{U1} - \lambda$ plane for the one coupling scenarios in the minimal model with $U_1$ and the preferred regions by the $R_D(*)$ anomalies with (a) $\lambda = \lambda^L_{23}, \lambda^L_{33} = 0$ (Scenario-I), (b) $\lambda^L_{23} = 0, \lambda = \lambda^L_{33}$ (Scenario-II). $R_D(*)$ is ruled out at 2σ for both scenarios.
LHC Exclusion + $R_D(\ast)$ plots - RD2A

$\lambda_{23}^L, \lambda_{33}^L$ is non-zero

Figure: The 95% CL (2$\sigma$) allowed regions from the LHC in the $\lambda_{23}^L - \lambda_{33}^L$ plane for the two coupling scenarios, (a) $M_{U_1} = 1500$ GeV, (b) $M_{U_1} = 2000$ GeV
LHC Exclusion + $R_D^{(*)}$ plots - RD2B

$\lambda_{23}^L, \lambda_{33}^R$ is non-zero

**Figure:** The 95% CL (2$\sigma$) allowed regions from the LHC in the $\lambda_{23}^L$ - $\lambda_{33}^R$ plane, for the two coupling scenarios, (a) $M_{U_1} = 1500$ GeV, (b) $M_{U_1} = 2000$ GeV
LHC puts competitive and stringent (model dependant) bounds on $S_1$, $U_1$ LQs.

Contributions from interference ($\sigma_\times$) dominates over other production modes.

For $U_1$, multiple couplings are needed to account for $R_{D(\ast)}$ anomalies.
Thank You for Your Attention!
σ vs mass plot

The graph shows the cross section \( \sigma \) as a function of mass \( M_{S_1} \), with different lines representing different scenarios.

- \( S_1S_1 \) (red solid line)
- \( S_1\tau \) (orange dashed line)
- \( S_1\tau j \) (green dotted line)
- \( S_1\nu \) (blue dot-dashed line)
- \( S_1\nu j \) (purple dash-dot line)
- \( \tau\tau \) (brown dotted line)
- \( \tau\nu \) (brown dash-dot line)

The x-axis represents the mass \( M_{S_1} \) in TeV, ranging from 0.5 to 3.0 TeV. The y-axis shows the cross section \( \sigma \) in fb (femtobarns), with a logarithmic scale ranging from \( 10^{-2} \) to \( 10^3 \) fb.
Pair Production Diagrams

Figure: pair production diagrams
\( r_{D^(*)} \) calculations

For \( S_1 \) and single coupling scenarios of \( U_1 \),

\[
\begin{align*}
  r_{D^*} &= \frac{R_{D^*}^i}{R_{SM}^i} = \left| 1 + C_V^i \right|^2 \\
  C_V^i &= \frac{1}{2 \sqrt{2} G_F V_{cb}} \frac{V_{cb} (\lambda^i)^2}{2 M_{S_1/U_1}^2} = \frac{(\lambda^i)^2}{4 \sqrt{2} G_F M_{S_1/U_1}^2}
\end{align*}
\]

where \( i = \{1, 2\} \) indicates scenarios.
2-coupling scenarios

In 2-coupling scenarios of $U_1$,

$$r_D \equiv \frac{R_D}{R_{SM}^D} \approx \left| 1 + C_{V_L}^{U_1} \right|^2 + 1.02 \left| C_{S_L}^{U_1} \right|^2 + 1.49 \Re \left[ (1 + C_{V_L}^{U_1})C_{S_L}^{U_1*} \right],$$

$$r_{D^*} \equiv \frac{R_{D^*}}{R_{SM}^D} \approx \left| 1 + C_{V_L}^{U_1} \right|^2 + 0.04 \left| C_{S_L}^{U_1} \right|^2 - 0.11 \Re \left[ (1 + C_{V_L}^{U_1})C_{S_L}^{U_1*} \right]$$

where,

$$C_{V_L}^{U_1} = \frac{1}{2\sqrt{2} G_F V_{cb}} \frac{\lambda_{23}^{L*} \lambda_{33}^{L}}{M_{U_1}^2},$$

$$C_{S_L}^{U_1} = -\frac{1}{2\sqrt{2} G_F V_{cb}} \frac{2\lambda_{23}^{L*} \lambda_{33}^{R}}{M_{U_1}^2}.$$
$U_1$ model: One coupling Scenarios

- Scenario RD1A: $\lambda^L_{23} = 1, \lambda^L_{33} = \lambda^R_{33} = 0$

  \[ \mathcal{L} \supset \lambda^L_{23} [\bar{c}_L \gamma_{\mu} \nu_L + (V_{cd} \bar{d}_L + V_{cs} \bar{s}_L + V_{cb} \bar{b}_L) \gamma_{\mu} \tau_L)] U_1^{\mu} \] (1)

- Scenario RD1B: $\lambda^L_{33} = 1, \lambda^R_{33} = \lambda^L_{23} = 0$

  \[ \mathcal{L} \supset \lambda^L_{33} [(V^*_{ub} \bar{u}_L + V^*_{cb} \bar{c}_L + V^*_{tb} \bar{t}_L) \gamma_{\mu} \nu_L) + \bar{b}_L \gamma_{\mu} \tau_L] U_1^{\mu} \] (2)
**U₁ model: Two coupling Scenarios**

- **Scenario RD2A:** $\lambda_{23}^L = \lambda_{33}^L = 1$

\[
\mathcal{L} \supset [\lambda_{23}^L (\bar{c}_L \gamma_{\mu} \nu_L + \bar{s}_L \gamma_{\mu} \tau_L) + \lambda_{33}^L (\bar{t}_L \gamma_{\mu} \nu_L + \bar{b}_L \gamma_{\mu} \tau_L)]U_1^\mu
\]

\[
= [\lambda_{23}^L (V_{us} \bar{u}_L \gamma_{\mu} \nu_L + V_{cs} \bar{c}_L \gamma_{\mu} \nu_L + V_{ts} \bar{t}_L \gamma_{\mu} \nu_L + \bar{s}_L \gamma_{\mu} \tau_L)]U_1^\mu
\]

\[
+ \lambda_{33}^L (V_{ub} \bar{u}_L \gamma_{\mu} \nu_L + V_{cb} \bar{c}_L \gamma_{\mu} \nu_L + V_{cb} \bar{t}_L \gamma_{\mu} \nu_L + \bar{b}_L \gamma_{\mu} \tau_L)]U_1^\mu
\]

- **Scenario RD2B:** $\lambda_{23}^L = \lambda_{33}^L = 1$

\[
\mathcal{L} \supset [\lambda_{23}^L (\bar{c}_L \gamma_{\mu} \nu_L + \bar{s}_L \gamma_{\mu} \tau_L) + \lambda_{33}^R \bar{b}_R \gamma_{\mu} \tau_R]U_1^\mu
\]

\[
= [\lambda_{23}^L (V_{us} \bar{u}_L \gamma_{\mu} \nu_L + V_{cs} \bar{c}_L \gamma_{\mu} \nu_L + V_{ts} \bar{t}_L \gamma_{\mu} \nu_L + \bar{s}_L \gamma_{\mu} \tau_L)]U_1^\mu
\]

\[
+ \lambda_{33}^R \bar{b}_R \gamma_{\mu} \tau_R]U_1^\mu
\]
Cross-section Parametrization: Pair Production

Total cross section:

\[
\sigma^p (M_{U_1}, \lambda) = \sigma^{p_0} (M_{U_1}) + \sum_{i}^{n} \lambda_i^2 \sigma^{p_2}_i (M_{U_1}) + \sum_{i \geq j}^{n} \lambda_i^2 \lambda_j^2 \sigma^{p_4}_{ij} (M_{U_1})
\]

No. of surviving events:

\[
\mathcal{N}^p = \sigma^p \circ \epsilon^p (M_{U_1}, \lambda) \times B^2 (M_{U_1}, \lambda)
\]

\[
= \left\{ \sigma^{p_0} \times \epsilon^{p_0} + \sum_{i}^{n} \lambda_i^2 \sigma^{p_2}_i \times \epsilon^{p_2}_i + \sum_{i \geq j}^{n} \lambda_i^2 \lambda_j^2 \sigma^{p_4}_{ij} \times \epsilon^{p_4}_{ij} \right\} \\
\times B^2 (M_{U_1}, \lambda) \times L
\]
Cross-section Parametrization: Single Production

Total cross section:

\[
\sigma^S(M, \lambda_i) = \sum_{i}^{n} \lambda_i^2 \sigma^s_i(M_{U_1}) + \sum_{i \geq j \geq k}^{n} \lambda_i^2 \lambda_j^2 \lambda_k^2 \sigma^s_{ijk}(M_{U_1})
\]

No. of surviving events:

\[
\mathcal{N}^S = \sigma^S \circ \epsilon^S (M_{U_1}, \lambda) \times \mathcal{B}(M_{U_1}, \lambda) \times L
\]

\[
= \left\{ \sum_{i}^{n} \lambda_i^2 \sigma^s_i(M_{U_1}) \times \epsilon^s_i(M_{U_1}) + \sum_{i \geq j \geq k}^{n} \lambda_i^2 \lambda_j^2 \lambda_k^2 \sigma^s_{ijk}(M_{U_1}) \times \epsilon^s_{ijk}(M_{U_1}) \right\}
\]

\[
\times \mathcal{B}(M_{U_1}, \lambda_i) \times L
\]
Cross-section Parametrization: Non-resonant Production

Total cross section:

\[
\sigma^{nr}(M_{U_1}, \lambda) = \sum_{i}^{n} \lambda_i^2 \sigma_i^{nr2}(M_{U_1}) + \sum_{i \geq j}^{n} \lambda_i^2 \lambda_j^2 \sigma_{ij}^{nr4}(M_{U_1})
\]

No. of surviving events:

\[
\mathcal{N}^{nr} = \sigma^{nr} \circ \epsilon^{nr}(M_{U_1}, \lambda) \times L
\]

\[
= \left\{ \sum_{i}^{n} \lambda_i^2 \sigma_i^{nr2}(M_{U_1}) \times \epsilon_i^{nr2}(M_{U_1}) + \sum_{i \geq j}^{n} \lambda_i^2 \lambda_j^2 \sigma_{ij}^{nr4}(M_{U_1}) \times \epsilon_{ij}^{nr4}(M_{U_1}) \right\} \times L
\]