Simulation studies of $R_2(\Delta \eta, \Delta \phi)$ and $P_2(\Delta \eta, \Delta \phi)$ correlation functions in p–p collisions with the PYTHIA and HERWIG models

Baidyanath Sahoo$^1$
Basanta Kumar Nandi$^1$, Prabhat Pujahari$^2$
Sumit Basu$^3$ and Claude A. Pruneau$^4$

$^1$IIT Bombay, $^2$IIT Madras, $^3$Lund University and $^4$WAYNE STATE

B. Sahoo, B.K. Nandi, P. Pujahari, S. Basu and C. Pruneau
Outline

- Motivation
- Definition of Observables: $R_2$ and $P_2$
- Analysis Details
- Results
- Summary
Motivation

Why do we study Two-particle Correlations?

These explore the underlying physics phenomena of particle production in collisions of both protons and heavy ions by measuring the distributions in $\Delta \eta \Delta \varphi$ space.

Goal

How do two-particle correlation functions behave in these different $p_T$ regions in small systems?

- Low $p_T$: 0.2 - 2.0 GeV/c (Underlying Event)
- Mid $p_T$: 2.0 - 5.0 GeV/c (Quark Coalescences)
- High $p_T$: 5.0 - 30.0 GeV/c (Jets)
Definition of Observables: $R_2$ and $P_2$

- **Single-Particle Density:** $\rho_1(x) = \frac{1}{N} \frac{dN}{dx}$
- **Two-Particle Density:** $\rho_2(x_1, x_2) = \frac{1}{N} \frac{d^2N}{dx_1 dx_2}$

1st Observable:

Two-particle differential number Correlation $^1$:

$$R_2(\Delta \eta, \Delta \phi) = \frac{\rho_2(\Delta \eta, \Delta \phi)}{\rho_1(\eta_1, \phi_1) \times \rho_1(\eta_2, \phi_2)} - 1$$

✓ Sensitive to particle production mechanisms

2nd Observable:

Two-particle differential transverse momentum Correlation $^1$:

$$P_2(\Delta \eta, \Delta \phi) = \frac{\left\langle \Delta p_{T,1} \times \Delta p_{T,2} \right\rangle(\Delta \eta, \Delta \phi)}{\left\langle p_T \right\rangle^2}$$

✓ Sensitive to transverse momentum fluctuations

where

- $\left\langle \Delta p_{T,1} \times \Delta p_{T,2} \right\rangle(\Delta \eta, \Delta \phi) = \int \rho_2(x_1, x_2) \Delta p_{T,1} \Delta p_{T,2} dp_{T,1} dp_{T,2} \rho_2(\Delta \eta, \Delta \phi)$
- $\Delta p_{T,i} = p_{T,i} - \left\langle p_T \right\rangle$

**Why did we use $R_2$ & $P_2$?**

1. Dimensionless quantity
2. Robust observable:
   - Independent of detection efficiency for $\eta$ & $\phi$, but dependent on $p_T$ efficiency$^1$.

$^1$ M. Sharma and C. A. Pruneau, PRC 79, 024905 (2009)
Definition of Observables: CI and CD

4 different charge combinations O:

\[ O^{(+,-)}, O^{(-,+)}, O^{(+,+)}, O^{(-,-)} \]

where \( O \equiv \{R_2, P_2\} \)

1. US: Unlike-sign pairs
\[ O^{US} = \frac{1}{2}(O^{(+,-)} + O^{(-,+)}) \]
- Coulomb Int., Jet, Resonance, flow (in Heavy Ion) etc

2. LS: Like-sign pairs
\[ O^{LS} = \frac{1}{2}(O^{(+,+)} + O^{(-,-)}) \]
- B-E corr., Coulomb Int., Jet, Resonance, flow (in Heavy Ion) etc

3. CI: Charge Independent
\[ O^{CI} = \frac{1}{2}(O^{US} + O^{LS}) \]
- Measure the average correlation strength between all charge particles

4. CD: Charge Dependent
\[ O^{CD} = \frac{1}{2}(O^{US} - O^{LS}) \]
- Keep effects related to balancing pairs

✓ Balance function is proportional to \( R_2^{CD} \) i.e.

\[ B(\Delta \eta, \Delta \phi) \equiv \frac{dN}{d\eta} R_2^{CD}(\Delta \eta, \Delta \phi) \]

Analysis Details

Models:
1. HERWIG: Cluster model of hadronization
2. PYTHIA6 Perugia-0: String model of hadronization

- $pp \sqrt{s} = 2.76$ TeV
- # of events = 200M
- Particles Selected: $h^{\pm}$

Kinematical Cuts:

I) $p_T$ ranges:
1. $0.2 < p_T \leq 2.0$ GeV/$c$
2. $2.0 < p_T \leq 5.0$ GeV/$c$
3. $5.0 < p_T \leq 30.0$ GeV/$c$

II) $\eta$ range: $|\eta| \leq 1.0$

III) $\phi$ range: $0 < \phi \leq 2\pi$
Results: $R^\text{CI}_2(\Delta \eta, \Delta \phi)$ and $P^\text{CI}_2(\Delta \eta, \Delta \phi)$

**HERWIG, pp $\sqrt{s} = 2.76$ TeV**

(a) $0.2 < p_T \leq 2.0$ GeV/c

(b) $2.0 < p_T \leq 5.0$ GeV/c

(c) $5.0 < p_T \leq 30.0$ GeV/c

Increasing $p_T$

$P^\text{CI}_2(\Delta \eta, \Delta \phi) = \frac{\langle \Delta p_{T,1} \times \Delta p_{T,2} \rangle(\Delta \eta, \Delta \phi)}{\langle p_T \rangle^2}$

Narrowing of the near-side peak with increasing $p_T$ for CI in HERWIG due to angular ordering.
Results: Projection on $\Delta \eta$

$R_{2}^{\text{CI}}(\Delta \eta)$

- (a) $0.2 < p_{T} \leq 2.0 \text{ GeV}/c$
- $p_{T}$ distribution for $|\Delta \phi| \leq \pi/2$
- $h^{z}$
- $R_{2}^{\text{CI}}(\Delta \eta)$ for $N_{pp} = 2.76 \text{ TeV}$

$P_{2}^{\text{CI}}(\Delta \eta)$

- (a) $0.2 < p_{T} \leq 2.0 \text{ GeV}/c$
- $p_{T}$ distribution for $|\Delta \phi| \leq \pi/2$
- $h^{z}$
- $P_{2}^{\text{CI}}(\Delta \eta)$ for $N_{pp} = 2.76 \text{ TeV}$

- $P_{2}^{\text{CI}}(\Delta \eta)$ is narrower than $R_{2}^{\text{CI}}(\Delta \eta)$ due to angular ordering which implies $P_{2}$ is more precise observable to probe the internal structure of jet.

- The shift observed in HERWIG events likely results from larger event-by-event multiplicity fluctuations in $P_{2}^{\text{CI}}(\Delta \eta)$. 

Jet
What will happen for narrower $p_T$ bins in CI?

- $P_2^{CI}(\Delta \eta)$ is narrower than $R_2^{CI}(\Delta \eta)$ due to angular ordering which implies $P_2$ is more precise observable to probe the internal structure of jet.
- The shift observed in HERWIG events likely results from larger event-by-event multiplicity fluctuations in $P_2^{CI}(\Delta \eta)$. 
Smooth fall of widths with increasing $p_T$.

$P_2$ is narrower than $R_2$, although for some $p_T$ bins, it is broader.
Narrowing of the near-side peak with increasing \( p_T \) for CD in HERWIG due to angular ordering.

\( R^\text{CD}_2(\Delta \eta, \Delta \phi) \) features an isolated peak centered at \((\Delta \eta, \Delta \phi) = (0, 0)\) resulting from the fact that correlated charged particle production occurs almost exclusively within the confines of a single jet.

Back-to-back gluon jets should yield no contributions to the away-side of \( P^\text{CD}_2 \) correlation functions but quark-quark jet pairs should have a finite CD correlation as quark jets are charge correlated.
$P_{2}^{CD}(\Delta \eta)$ is narrower than $R_{2}^{CD}(\Delta \eta)$ due to angular ordering which implies $P_{2}$ is more precise observable to probe the internal structure of jet.
Results: Projection on $\Delta \eta$

$P_{2CD}^{\Delta \eta}$ is narrower than $R_{2CD}^{\Delta \eta}$ due to angular ordering which implies $P_2$ is more precise observable to probe the internal structure of jet.
Results: Width for narrower $p_T$ bins in CD

- Smooth fall of widths with increasing $p_T$ in PYTHIA.
- Irregular $p_T$ dependence of widths in HERWIG.
- $P_2$ is narrower than $R_2$, although for some $p_T$ bins, it is broader.
Result: Multiplicity wise study for CI

\( \sigma_{\Delta \eta} \)

(a) \(0.2 < p_T \leq 2.0\) GeV/c

- PYTHIA6 Perugia-0
- HERWIG

(b) \(2.0 < p_T \leq 5.0\) GeV/c

- \(R_2^{C}(\Delta \eta)\)
- \(P_2^{C}(\Delta \eta)\)

(c) \(5.0 < p_T \leq 30.0\) GeV/c

\(\times 10^{-3}\)

(a) \(2.0 < p_T \leq 5.0\) GeV/c

HERWIG

pp \(\sqrt{s} = 2.76\) TeV, \(|\Delta \phi| \leq \pi/2\)

- \(N_{\text{total}} > 0\)
- \(N_{\text{total}} > 25\)
- \(N_{\text{total}} > 50\)

\(\sum_{\Delta \eta}^{\Delta \eta} \eta\)

\(h^+\)

\(\Delta \eta\)

- Quark Jet
- Gluon Jet

\(\checkmark\) High-multiplicity events favour gluon jets.

\(\checkmark\) Exhibit a discontinuity near or above \(N_{\text{total}} = 30\).
We study the predictions of the PYTHIA and HERWIG models relative to their dependence on the particle momenta, the particle species, and we focus, in particular, on the differences between $R_2$ and $P_2$ correlation functions.

$P_2$ is narrower than $R_2$ due to angular ordering, although in some $p_T$ bins, $P_{CD}^2$ is broader than $R_{CD}^2$.

$R_2$ correlation function receives positive definite contributions from all particle pairs of a jet and is thus not sensitive to the ordering of the particles of the pair but only the overall width of the jet.

High-multiplicity events favour gluon jets.

$P_{CI}^2$ is a more precise observable to probe the internal structure of jet than $R_{CI}^2$.

Analysis is on going with ALICE data with $pp@13$ TeV.
Thank you for your attention
Definition of CI

- 4 different charge combinations: (+ -), (- +), (+ +), (- -)

Unlike-Sign (US) pairs

\[ O^{US} = \frac{1}{2}(O^{(+,-)} + O^{(-,+)}) \]

Like-Sign (LS) pairs

\[ O^{LS} = \frac{1}{2}(O^{(+,+)} + O^{(-,-)}) \]

Charge Independent (CI)

\[ O^{CI} = \frac{1}{2}(O^{US} + O^{LS}) \]

\[ O \equiv (R_2, P_2) \]

- Measure the average correlation strength between all charge particles.


\[ \rightarrow \text{Coulomb Int., Jet, Resonance, flow(In Heavy Ion) etc} \]

\[ \rightarrow \text{Coulomb Int., Jet, B-E corr., flow(In Heavy Ion) etc} \]
Definition of CD

- 4 different charge combinations: (+ -), (- +), (+ +), (- -)

Unlike-Sign (US) pairs

\[ O^{US} = \frac{1}{2} (O^{(+,-)} + O^{(-,+)}). \]

Like-Sign (LS) pairs

\[ O^{LS} = \frac{1}{2} (O^{(+,+)} + O^{(-,-)}). \]

Charge Dependent (CD)

\[ O^{CD} = \frac{1}{2} (O^{US} - O^{LS}). \]

Coulomb Int., Jet, Resonance, flow (In Heavy Ion) etc

Keep effects related to balancing pairs


Angular Ordering: probe the internal structure of jets
$P^\text{CD}(\Delta \eta, \Delta \varphi)$ for $K^\pm$

- **Delta Eta Cut:**
  - Range: -2.0 to 2.0
  - # of Bins = 79
  - Bin Width = 0.05

- **Delta Phi Cut:**
  - Range: -0.5 to 1.5
  - # of Bins = 72
  - Bin Width = 0.087

Increasing $p_T$

$\phi^- > k^+ k^- - \text{Decay On}$

$\phi^- > k^+ k^- - \text{Decay Off}$

$$0.2 \leq p_T \leq 2$$

$$2 \leq p_T \leq 5$$

$$5 \leq p_T \leq 30$$
\( \langle p_T \rangle \) (GeV/c)

(a) \( 0.2 < p_T \leq 2.0 \) GeV/c

(b) \( 2.0 < p_T \leq 5.0 \) GeV/c

(c) \( 5.0 < p_T \leq 30.0 \) GeV/c

PPHERWIG

\( \langle p_T \rangle \) (GeV/c)
pp $\sqrt{s} = 2.76$ TeV

- $h^+$ PYTHIA6 Perugia-0, pp
- $h^-$ PYTHIA6 Perugia-0, pp
- $h^+$ HERWIG, pp
- $h^-$ HERWIG, pp
Result (VIII): $R_{2}^{CI}(\Delta \eta)$ & $P_{2}^{CI}(\Delta \eta)$ for identified species

- $R_{2}^{CI}(\Delta \eta)$
  - $pp \sqrt{s} = 2.76$ TeV
  - $0.2 < p_{T} \leq 2.0$ GeV/c
  - $|\Delta \phi| \leq \pi/2$

- $P_{2}^{CI}(\Delta \eta)$
  - $pp \sqrt{s} = 2.76$ TeV
  - $0.2 < p_{T} \leq 2.0$ GeV/c
  - $|\Delta \phi| \leq \pi/2$

Resonance plays a vital role for kaons!
Result (IX) : \( R_{2}^{CD}(\Delta \eta) \) & \( P_{2}^{CD}(\Delta \eta) \) for identified species

- (a) \( h^{\pm} \)
- (b) \( \pi^{\pm} \)
- (c) \( K^{\pm} \)
- (d) \( p\bar{p} \)

PP, \( \sqrt{s} = 2.76 \) TeV, \( 0.2 < p_T \leq 2.0 \) GeV/c, \( |\Delta \phi| \leq \pi/2 \)

Resonance plays a vital role for kaons!
pp $\sqrt{s} = 2.76$ TeV, $|\Delta \eta| \leq \pi/2$

(a) $0.2 < p_T \leq 2.0$ GeV/c

PYTHIA6 Perugia-0

- $\triangle$ $R_{2}^{CD}(\Delta \eta)$
- $\bigcirc$ $R_2^{CD}(\Delta \eta)$
- $\blacktriangle$ $P_{2}^{CD}(\Delta \eta)$
- $\bullet$ $P_2^{CD}(\Delta \eta)$

HERWIG

(b) $2.0 < p_T \leq 5.0$ GeV/c

(c) $5.0 < p_T \leq 30.0$ GeV/c

$\sigma_{\Delta \eta}$

$N_{ch}$

$\sigma_{\Delta \phi}$ (rad)

$N_{ch}$