Gravitational Wave Signatures from First-order Phase transitions in an Extended Inert Doublet Dark Matter Model


Avik Paul (Saha Inst.), Debasish Majumdar (Saha Inst.), Biswajit Banerjee (Saha Inst.)

Avik Paul
Saha Institute of Nuclear Physics, HBNI, Kolkata

XXIV DAE-BRNS HIGH ENERGY PHYSICS SYMPOSIUM 2020

14th December 2020
Introduction

- For GW production, the interesting situation is when there is a first-order phase transition. If the transition is a smooth crossover no significant GW production.
- The order of electroweak phase transition depends in particular on the value of Higgs mass. Unfortunately for the observed value of Higgs mass $m_H \sim 125.09$ GeV the phase transition due to electroweak symmetry breaking is not a first-order phase transition but is a smooth crossover at a temperature of around $T = 150.5 \pm 1.5$ GeV.
- Therefore, it is worthwhile to explore the extented Standard Model for which a strong first-order phase transition is possible at the electroweak scale.
- With this motivation, the Standard Model is extended by an inert Higgs and a singlet scalar so that both a particle dark matter candidate and the GW production from first-order phase transition at the electroweak scale may be realised.
The Model

- SM of particle physics is extended by the addition of an extra Higgs doublet and a real singlet scalar.
- $\Phi_I$ is $Z_2$ odd, the SM and the other added scalar singlet is $Z_2$ even.
- The potential of the scalar sector of the model can be expressed as

$$V = m_1^2 \Phi_H^\dagger \Phi_H + m_2^2 \Phi_I^\dagger \Phi_I + \frac{1}{2} m_s S^2 + \lambda_1 \left( \Phi_H^\dagger \Phi_H \right)^2 + \lambda_2 \left( \Phi_I^\dagger \Phi_I \right)^2$$

$$+ \lambda_3 \left( \Phi_H^\dagger \Phi_H \right) \left( \Phi_I^\dagger \Phi_I \right) + \lambda_4 \left( \Phi_I^\dagger \Phi_H \right) \left( \Phi_I^\dagger \Phi_I \right)$$

$$+ \frac{\lambda_5}{2} \left[ \left( \Phi_I^\dagger \Phi_H \right)^2 + \left( \Phi_H^\dagger \Phi_I \right)^2 \right] + \rho_1 \left( \Phi_H^\dagger \Phi_H \right) S$$

$$+ \rho'_1 \left( \Phi_I^\dagger \Phi_I \right) S + \rho_2 \left( \Phi_H^\dagger \Phi_H \right) S^2 + \rho'_2 \left( \Phi_I^\dagger \Phi_I \right) S^2 + \frac{\rho_3}{3} S^3 + \frac{\rho_4}{4} S^4.$$  

(1)
The Model

• After spontaneous symmetry breaking (SSB) we have,

\[ \phi_H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} (v + h) \end{pmatrix}, \quad \phi_I = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (H_0 + iA_0) \end{pmatrix}, \quad S = v_s + s, \] (2)

• From the minimisation conditions

\[ \frac{\partial V}{\partial h} \bigg|_{h=s=H^+=H_0=A_0=0} = \frac{\partial V}{\partial s} \bigg|_{h=s=H^+=H_0=A_0=0} = 0, \] (3)

one obtains

\[ m_1^2 + \lambda_1 v^2 + \rho_1 v_s + \rho_2 v_s^2 = 0, \]
\[ m_s^2 + \rho_3 v_s + \rho_4 v_s^2 + \frac{\rho_1 v^2}{2v_s} + \rho_2 v^2 = 0. \] (4)
\[ \mu_h^2 = m_1^2 + 3\lambda_1 v^2 + \rho_1 v_s + \rho_2 v_s^2 = 2\lambda_1 v^2, \]  
\[ \mu_s^2 = m_s^2 + \rho_2 v^2 + 2\rho_3 v_s + 3\rho_4 v_s^2 = \rho_3 v_s + 2\rho_4 v_s^2 - \frac{\rho_1 v^2}{2v_s}, \]  
\[ m_{H^\pm}^2 = m_2^2 + \frac{\lambda_3 v^2}{2} + \rho'_1 v_s + \rho'_2 v_s^2, \]  
\[ m_{H_0}^2 = m_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) \frac{v^2}{2} + \rho'_1 v_s + \rho'_2 v_s^2, \]  
\[ m_{A_0}^2 = m_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) \frac{v^2}{2} + \rho'_1 v_s + \rho'_2 v_s^2, \]  
\[ \mu_{h,s}^2 = \rho_1 v + 2\rho_2 v_s v. \]
• The mass matrix $\mathcal{M}$ is therefore given as

$$
\mathcal{M} = \begin{pmatrix}
\mu_h^2 & \mu_{hs}^2 \\
\mu_{hs}^2 & \mu_s^2
\end{pmatrix}
$$

(11)

• The mass eigenstates $h, s$ in the diagonal basis

$$
\begin{pmatrix}
h_1 \\
h_2
\end{pmatrix} = U
\begin{pmatrix}
h \\
s
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
h \\
s
\end{pmatrix},
$$

(12)

which implies

$$
h_1 = h \cos \theta - s \sin \theta, \quad h_2 = h \sin \theta + s \cos \theta,
$$

(13)

where

$$
tan \theta = \frac{y}{1 + \sqrt{1 + y^2}}, \quad \text{where} \quad y = \frac{2\mu_{hs}^2}{\mu_h^2 - \mu_s^2}.
$$

(14)

• One can obtain the following expressions

$$
n_{h_1,h_2}^2 = \frac{\left(\mu_h^2 + \mu_s^2\right)}{2} \pm \frac{\left(\mu_h^2 - \mu_s^2\right)}{2} \sqrt{1 + y^2},
$$

(15)
The Model

\begin{align}
\lambda_1 &= \frac{m^2_{h_2} \sin^2 \theta + m^2_{h_1} \cos^2 \theta}{2v^2}, \\
\rho_4 &= \frac{1}{2v_s^2} \left( m^2_{h_2} \cos^2 \theta + m^2_{h_1} \sin^2 \theta + \frac{v^2 \rho_1}{2v_s} - v_s \rho_3 \right), \\
\rho_2 &= \frac{m^2_{h_1} - m^2_{h_2}}{4vv_s} \sin 2\theta - \frac{\rho_1}{2v_s}, \\
m_1^2 &= -\lambda_1 v^2 - \rho_1 v_s - \rho_2 v_s^2, \\
m_2^2 &= m^2_{H_0} - \frac{v^2}{2} \left( \lambda_3 + \lambda_4 + \lambda_5 \right) - \rho'_1 v_s - \rho'_2 v_s^2, \\
m_s^2 &= -\rho_3 v_s - \rho_4 v_s^2 - \frac{\rho_1 v^2}{2v_s} - \rho_2 v^2.
\end{align}
Constraints

- **Theoretical Constraints**
- **Vacuum Stability**

\[ \lambda_1, \lambda_2, \rho_4 > 0, \lambda_3 + 2\sqrt{\lambda_1 \lambda_2} > 0, \quad (22) \]

\[ \lambda_3 + \lambda_4 - |\lambda_5| + 2\sqrt{\lambda_1 \lambda_2} > 0, \rho_2 + \sqrt{\lambda_1 \rho_4} > 0, \quad (23) \]

\[ \rho'_2 + \sqrt{\lambda_2 \rho_4} > 0, \quad (24) \]

\[ 2\rho_2 \sqrt{\lambda_2} + 2\rho'_2 \sqrt{\lambda_1} + \lambda_3 \sqrt{\rho_4} \]

\[ + 2 \left( \sqrt{\lambda_1 \lambda_2 \rho_4} + \sqrt{\left( \lambda_3 + 2\sqrt{\lambda_1 \lambda_2} \right) \left( \rho_2 + \sqrt{\lambda_1 \rho_4} \right) \left( \rho'_2 + \sqrt{\lambda_2 \rho_4} \right)} \right) > 0, \quad (25) \]

\[ 2\rho_2 \sqrt{\lambda_2} + 2\rho'_2 \sqrt{\lambda_1} + (\lambda_3 + \lambda_4 - \lambda_5) \sqrt{\rho_4} + 2 \left( \sqrt{\lambda_1 \lambda_2 \rho_4} \right) \]

\[ + \sqrt{\left( (\lambda_3 + \lambda_4 - \lambda_5) + 2\sqrt{\lambda_1 \lambda_2} \right) \left( \rho_2 + \sqrt{\lambda_1 \rho_4} \right) \left( \rho'_2 + \sqrt{\lambda_2 \rho_4} \right)} \right) > 0. \quad (26) \]

- **Perturbativity**

\[ \Lambda_i < 4\pi. \quad (27) \]
Constraints

- **Experimental Constraints**
- **Collider Constraints**
- The signal strength of SM like Higgs boson $h_1$ is defined as

$$R_1 = \cos^4 \theta \frac{\Gamma^{SM}}{\Gamma}, \quad (28)$$

where

$$\Gamma = \cos^2 \theta \left( \Gamma^{SM} + \Gamma^{inv} \right). \quad (29)$$

$$\Gamma^{inv} (h_1 \rightarrow H_0 H_0) = \left( \frac{g_{h_1 H_0 H_0}}{16\pi m_{h_1}} \right)^2 \left( 1 - \frac{4m_{H_0}^2}{m_{h_1}^2} \right)^{1/2}, \quad m_{H_0} \leq m_{h_1}/2 \quad (30)$$

$$\Gamma^{inv} (h_1 \rightarrow h_2 h_2) = \left( \frac{g_{h_1 h_2 h_2}}{16\pi m_{h_1}} \right)^2 \left( 1 - \frac{4m_{h_2}^2}{m_{h_1}^2} \right)^{1/2}, \quad m_{h_2} \leq m_{h_1}/2 \quad (31)$$

$$g_{h_1 H_0 H_0} = \left( \frac{\lambda_L}{2} \cos \theta - \frac{\lambda_s}{2} \sin \theta \right) v = \lambda_{h_1 H_0 H_0} v, \quad (32)$$

$$\lambda_L = \lambda_3 + \lambda_4 + \lambda_5, \quad \lambda_s = \frac{\rho_1' + 2\rho_2' v_s}{v}, \quad (33)$$
• and
\[ g_{h_1 h_2 h_2} = \frac{1}{2} (6 \lambda_1 v \cos \theta \sin^2 \theta + 2 \rho_1 \cos^2 \theta \sin \theta - \rho_1 \sin^3 \theta + 2 \nu \rho_2 \cos^3 \theta - 4 \nu \rho_2 \cos \theta \sin^2 \theta \\
+ 4 \nu \rho_2 \cos^2 \theta \sin \theta - 2 \nu \rho_2 \sin^3 \theta - 2 \rho_3 \cos^2 \theta \sin \theta - 6 \nu \rho_2 \cos^2 \theta \sin \theta). \] (34)

• The invisible decay branching ratio of SM scalar can be defined as
\[ \text{Br}_{\text{inv}} = \frac{\Gamma_{\text{inv}}}{\Gamma}. \] (35)

• We use the bound \( \text{Br}_{\text{inv}} \leq 24\% \) (for \( m_{h_1} \geq m_{H_0}/2 \)), signal strength of SM Higgs \( R_1 \geq 0.84 \).
LHC \( \sin \theta \leq 0.4 \)

• future collider experiments
  High Luminosity LHC (HL-LHC) \( \cos \theta \geq 0.94 \).
  International Linear Collider (ILC) \( \cos \theta \simeq 0.98 \)
  ILC-1 operating (at \( \sqrt{s} = 250 \) GeV) and \( \cos \theta \simeq 0.99 \) for ILC-3 (at \( \sqrt{s} = 1 \) TeV).
  China Electron Positron Collider (CEPC) or TLEP \( \cos \theta \sim 0.997 \)
Therefore, we consider \( \sin \theta \leq 0.063 \) (\( \cos \theta \geq 0.998 \)),

\[ 14th \ December \ 2020 \ 10 / 24 \]
Relic Density

- The Boltzmann equation is given as

\[
\frac{dn_{H_0}}{dt} + 3H n_{H_0} = -\langle \sigma v \rangle \left[ n_{H_0}^2 - (n_{H_0}^{eq})^2 \right],
\]

(36)

- After suitable transformation this equation reduces to

\[
\frac{1}{Y_0} = \frac{1}{Y_F} + \left( \frac{45G}{\pi} \right)^{-\frac{1}{2}} \int_{T_0}^{T_F} g_*^{1/2} \langle \sigma v \rangle dT,
\]

(37)

where

\[
\langle \sigma v \rangle = \frac{1}{8m_{H_0}^4TK_2^2} \int_{4m_{H_0}^2}^{\infty} ds \left( s - 4m_{H_0}^2 \right) \sqrt{s}K_1 \left( \frac{\sqrt{s}}{T} \right) \sigma(s),
\]

(38)

- Finally the expression for dark matter relic density is

\[
\Omega_{DM}h^2 = 2.755 \times 10^8 \left( \frac{m_{H_0}}{\text{GeV}} \right) Y_0,
\]

(39)

- PLANCK Constraint on Relic Density

\[
0.1172 \leq \Omega_{DM}h^2 \leq 0.1226, \quad h = \frac{H}{100 \text{ Km s}^{-1}\text{Mpc}^{-1}}
\]

(40)
Direct detection

- The spin independent (SI) scattering cross-section

\[
\sigma_{\text{SI}} = \frac{m_N^4}{\pi (m_{H_0} + m_N)^2} f^2 \left( \frac{\lambda_{h_1 H_0 H_0} \cos \theta}{m_{h_1}^2} + \frac{\lambda_{h_2 H_0 H_0} \sin \theta}{m_{h_2}^2} \right)^2,
\]

where

\[
\lambda_{h_1 H_0 H_0} = \left( \frac{\lambda_L}{2} \cos \theta - \frac{\lambda_s}{2} \sin \theta \right),
\]

\[
\lambda_{h_2 H_0 H_0} = \left( \frac{\lambda_L}{2} \sin \theta + \frac{\lambda_s}{2} \cos \theta \right).
\]

- XENON-1T, PandaX-II, LUX and DarkSide-50
Constraining the parameter space

### Table: Parameter Values

<table>
<thead>
<tr>
<th>BP</th>
<th>(m_{H_0}) in GeV</th>
<th>(m_{h_2}) in GeV</th>
<th>(v_s) in GeV</th>
<th>(\sin \theta)</th>
<th>(\rho_1) in GeV</th>
<th>(\rho_3) in GeV</th>
<th>(\lambda_L)</th>
<th>(\lambda_S)</th>
<th>(\lambda_2)</th>
<th>(\Omega_{DM} h^2)</th>
<th>(\sigma_{SI}) [cm^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>100</td>
<td>300</td>
<td>0.01</td>
<td>-3</td>
<td>0.01</td>
<td>0.001</td>
<td>0.0012</td>
<td>0.2</td>
<td>0.1220</td>
<td>(9.41 \times 10^{-48})</td>
</tr>
<tr>
<td>2</td>
<td>68</td>
<td>150</td>
<td>400</td>
<td>0.06</td>
<td>-7</td>
<td>0.2</td>
<td>0.002</td>
<td>0.033</td>
<td>0.031</td>
<td>0.1208</td>
<td>(3.69 \times 10^{-48})</td>
</tr>
<tr>
<td>3</td>
<td>76</td>
<td>200</td>
<td>500</td>
<td>0.03</td>
<td>-1</td>
<td>0.1</td>
<td>0.0016</td>
<td>0.0033</td>
<td>0.01</td>
<td>0.1195</td>
<td>(3.56 \times 10^{-48})</td>
</tr>
</tbody>
</table>

### Diagrams

- **Graph 1**: \(m_{H_0} = 150\) GeV, \(\theta = 0.06, v_s = 400, \rho_1 = -7, \rho_3 = 0.2, \lambda_2 = 0.031\)
- **Graph 2**: \(m_{H_0} = 150\) GeV, \(\theta = 0.06, v_s = 400, \rho_1 = -7, \rho_3 = 0.2, \lambda_2 = 0.031\)

- **Features**: Planck, DarkSide-50, PandaX-II, LUX, XENON-1T
Effective Potential

\[ V_{\text{eff}} = V_{\text{tree-level}} + V_{1-\text{loop}}^{T=0} + V_{1-\text{loop}}^{T\neq0} \]  \hspace{1cm} (44)

\[ V_{1-\text{loop}}^{T=0} = \pm \frac{1}{64\pi^2} \sum_i n_i m_i^4 \left[ \log \frac{m_i^2}{Q^2} - C_i \right], \]  \hspace{1cm} (45)

- \( i \equiv h, H_0, A_0, H^\pm, s, W, Z, t. \)
- \( n_{W^\pm} = 6, n_Z = 3, n_t = 12 \) and \( n_{h,H_0,A_0,H^+,H^-,s} = 1. \)
- We take \( Q = 246.22 \) GeV in our calculations.
- For \( W, Z \) boson the \( C_{W,Z} = 5/6 \) and for the other particles \( C_{h,H_0,A_0,H^+,H^-,s,t} = 3/2. \)

\[ V_{1-\text{loop}}^{T\neq0} = \frac{T^4}{2\pi^2} \sum_i n_i J_\pm \left[ \frac{m_i^2}{T^2} \right], \]  \hspace{1cm} (46)

where

\[ J_\pm \left( \frac{m_i^2}{T^2} \right) = \pm \int_0^\infty dy \ y^2 \log \left( 1 \mp e^{-\sqrt{y^2 + \frac{m_i^2}{T^2}}} \right). \]  \hspace{1cm} (47)
• We include the daisy resummation procedure in $V_{1-loop}^{T\neq 0}$ by adding the temperature dependent terms to the boson masses. Now, the corrected thermal masses are $\mu_1^2(T) = m_1^2 + c_1 T^2$, $\mu_2^2(T) = m_2^2 + c_2 T^2$ and $\mu_3^2(T) = m_s^2 + c_3 T^2$, where

\[
c_1 = \frac{6\lambda_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{16} + \frac{1}{6}\rho_2 + \frac{y_t^2}{4},
\]

(48) \[
c_2 = \frac{6\lambda_2 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{16} + \frac{1}{6}\rho'_2,
\]

(49) \[
c_3 = \frac{1}{8}\rho_4 + \frac{1}{6}\rho_2 + \frac{1}{6}\rho'_2.
\]

(50) • The package CosmoTransitions is used for the computation.
Gravitational Waves Production from the first-order Phase Transition

- The Universe was initially in a state of false vacuum state. At high temperature the Universe begins in the symmetric phase, where the ground state of the theory is located at the origin of the field space and the effective potential has no other minima.

- While the temperature decreases, another minimum forms and when it becomes the global minimum, tunnelling to this state becomes possible.
- The tunneling toward the state of true vacuum could induce the nucleation of bubbles, which expand with constant acceleration, driven by a difference of pressure between the interior true vacuum and the exterior false vacuum.
- The temperature at which the two minima are degenerate is referred to as the critical temperature, $T_c$.
- A first-order phase transition proceeds via nucleation of bubbles of the new phase, which expand, collide and coalesce, leaving eventually the Universe in the electroweak broken phase.
- The phase transition can be completed if at least one bubble is formed per unit of time per Hubble volume. Temperature at which that happens is referred to as nucleation temperature, $T_n$, which is lower than the critical temperature $T_c$.
- The colliding bubbles distort the shape of the bubbles and GWs are emitted.
- GW can be originated from three sources 1) bubble-bubble collisions 2) sound waves induced by the bubbles running through the cosmic plasma 3) turbulence induced by the bubble expansions in the cosmic plasma.
• The total GW intensity $\Omega_{GW} h^2$ as a function of frequency can be expressed as the sum of the contributions from the three components

$$\Omega_{GW} h^2 = \Omega_{\text{col}} h^2 + \Omega_{SW} h^2 + \Omega_{\text{turb}} h^2. \quad (51)$$

$$\Omega_{\text{col}} h^2 = 1.67 \times 10^{-5} \left( \frac{\beta}{H} \right)^{-2} \frac{0.11 v_w^3}{0.42 + v_w^2} \left( \frac{\kappa \alpha}{1 + \alpha} \right)^2 \left( \frac{g_*}{100} \right)^{-\frac{1}{3}} \frac{3.8 \left( \frac{f}{f_{\text{col}}} \right)^{2.8}}{1 + 2.8 \left( \frac{f}{f_{\text{col}}} \right)^{3.8}}, \quad (52)$$

$$\beta = \left[ HT \frac{d}{dT} \left( \frac{S_3}{T} \right) \right] \bigg|_{T_n}, \quad (53)$$

$$S_3 = 4\pi \int dr \ r^2 \left[ \frac{1}{2} \left( \partial_r \phi \right)^2 + V_{\text{eff}} \right], \quad (54)$$

• Nucleation of the bubble occurs at the nucleation temperature $T_n$ if it satisfies the condition $S_3 (T_n) / T_n \approx 140.$

$$v_w = \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1 + \alpha}, \quad (55)$$
\[ \kappa = 1 - \frac{\alpha_\infty}{\alpha}, \]  
\[ \alpha_\infty = \frac{30}{24\pi^2 g_*} \left( \frac{v_n}{T_n} \right)^2 \left[ 6 \left( \frac{m_W}{v} \right)^2 + 3 \left( \frac{m_Z}{v} \right)^2 + 6 \left( \frac{m_t}{v} \right)^2 \right]. \]  
\[ \alpha = \left[ \frac{\rho_{\text{vac}}}{\rho_{\text{rad}}^*} \right] \bigg|_{T_n}. \]  
\[ \rho_{\text{vac}} = \left[ \left( V_{\text{eff}}^{\text{high}} - T \frac{dV_{\text{eff}}^{\text{high}}}{dT} \right) - \left( V_{\text{eff}}^{\text{low}} - T \frac{dV_{\text{eff}}^{\text{low}}}{dT} \right) \right], \]  
and \[ \rho_{\text{rad}}^* = \frac{g_* \pi^2 T_n^4}{30}. \]
• \( f_{\text{col}} \) is the peak frequency produced by the bubble collisions which takes the form

\[
f_{\text{col}} = 16.5 \times 10^{-6} \text{ Hz} \left( \frac{0.62}{v_w^2 - 0.1v_w + 1.8} \right) \left( \frac{\beta}{H} \right) \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}}.
\]

(61)

• The sound wave (SW) component of the gravitational wave is given by

\[
\Omega_{\text{SW}} h^2 = 2.65 \times 10^{-6} \left( \frac{\beta}{H} \right)^{-1} v_w \left( \frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left( \frac{g_*}{100} \right)^{-\frac{1}{3}} \left( \frac{f}{f_{\text{SW}}} \right)^3 \left[ \frac{7}{4 + 3 \left( \frac{f}{f_{\text{SW}}} \right)^2} \right]^\frac{7}{2}.
\]

(62)

\[
\kappa_v = \frac{\alpha_\infty}{\alpha} \left[ \frac{\alpha_\infty}{0.73 + 0.083 \sqrt{\alpha_\infty} + \alpha_\infty} \right].
\]

(63)

\[
f_{\text{SW}} = 1.9 \times 10^{-5} \text{ Hz} \left( \frac{1}{v_w} \right) \left( \frac{\beta}{H} \right) \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}}.
\]

(64)
• The component from the turbulence in the plasma $\Omega_{\text{turb}} h^2$ is given by

$$\Omega_{\text{turb}} h^2 = 3.35 \times 10^{-4} \left( \frac{\beta}{H} \right)^{-1} v_w \left( \frac{\epsilon \kappa v \alpha}{1 + \alpha} \right)^2 \left( \frac{g_*}{100} \right)^{-\frac{1}{3}} \left( \frac{f}{f_{\text{turb}}} \right)^3 \left( 1 + \frac{f}{f_{\text{turb}}} \right)^{-\frac{11}{3}} \left( 1 + \frac{8\pi f}{h_*} \right)^{-1} \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}},$$

(65)

where $\epsilon = 0.1$

$$f_{\text{turb}} = 2.7 \times 10^{-5} \text{ Hz} \left( \frac{1}{v_w} \right) \left( \frac{\beta}{H} \right) \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}}.$$  

(66)

$$h_* = 16.5 \times 10^{-6} \text{ Hz} \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}}.$$  

(67)
Calculations and Results

<table>
<thead>
<tr>
<th>BP</th>
<th>$v_n$ in GeV</th>
<th>$v_c$ in GeV</th>
<th>$T_c$ in GeV</th>
<th>$v_c/T_c$</th>
<th>$T_n$ in GeV</th>
<th>$\alpha$</th>
<th>$\frac{\beta}{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>226.89</td>
<td>220.44</td>
<td>135.68</td>
<td>1.62</td>
<td>119.86</td>
<td>0.24</td>
<td>317.86</td>
</tr>
<tr>
<td>2</td>
<td>191.03</td>
<td>180.67</td>
<td>146.89</td>
<td>1.23</td>
<td>132.14</td>
<td>0.25</td>
<td>402.89</td>
</tr>
<tr>
<td>3</td>
<td>209.95</td>
<td>205.69</td>
<td>170.92</td>
<td>1.20</td>
<td>158.24</td>
<td>0.19</td>
<td>783.65</td>
</tr>
</tbody>
</table>
Summary and Discussions

- We have explored the possible production of GWs from the first-order phase transition of the early Universe from a viable particle dark matter model and its detectability with the future GW detectors (BBO, eLISA, ALIA, DECIGO, U-DECIGO and aLIGO).

- The first-order phase transition that may be initiated by the present extended SM where the scalar sector of the SM is modified by adding an additional inert scalar doublet and a real scalar singlet.

- The model parameter space is constrained using various theoretical and experimental constraints.

- Three benchmark points is chosen and GW intensities are calculated.

- The GW intensity increases as $\beta$ decreases. In addition, the lower value of $\beta$ also lowers the frequency at which the maximum GW intensity is produced.

- We found that the GW signals are detectable by the following future generation detectors namely BBO, U-DECIGO and eLISA (configuration - N2A5M5L6) within the detectable range.
Thank You