Effect of second class currents in $\nu_l(\bar{\nu}_l) - N$ scattering

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Outline

1. Introduction

2. Formalism

3. Results

4. Conclusions
Outline

1 Introduction

2 Formalism

3 Results

4 Conclusions
One of the main aim of the quasielastic $\nu - A$ and $\nu - N$ scattering is to determine the value of $M_A$ which is used in the dipole parameterization of $g_1(Q^2)$ as

\[
g_1(Q^2) = \frac{g_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \quad g_A(0) = D(0) + F(0) = 1.267
\]
### Motivation

Variation in the value of $M_A$ from different experiments

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Recently, in the case of $\nu_\mu (\bar{\nu}_\mu) - N$ scattering, we have shown that the presence of second class currents would affect the determination of $M_A$ [Phys. Rev. D 98, 033005 (2018)]
Introduction

We have studied the quasielastic $\nu_l(\bar{\nu}_l) - N; (l = \mu, \tau)$ scattering cross sections and the polarization observables of the final lepton and nucleon.

The polarization observables of the final lepton and nucleon provide information about the parameters of the weak hadronic current independent of the cross sections.
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The contribution from the second class current form factors as well as the pseudoscalar form factor depends on the lepton mass, thus, the effect of lepton mass on the cross section and polarization observables can be studied from these parameters.
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The present study where we compare the cross section and polarization observables in the $\nu_\mu(\bar{\nu}_\mu)$ and $\nu_\tau(\bar{\nu}_\tau)$ induced reactions will provide a check for the lepton universality.
Outline

1 Introduction

2 Formalism

3 Results

4 Conclusions
Introduction

Formalism

Results

Conclusions

Backup

Scattering cross sections

Differential cross section

\[ \nu_l(\overline{\nu}_l)(k) + N(p) \rightarrow l^\pm(k') + N(p'), \]

(a) \[ \tau^-(k') \]

\[ \nu_{\tau}(k) \]

\[ W^+(q = k - k') \]

(b) \[ \tau^+(k') \]

\[ \bar{\nu}_{\tau}(k) \]

\[ W^-(q = k - k') \]

\[ p(p') \]

\[ n(p) \]

\[ p'(p) \]

\[ n'(p') \]
Scattering cross sections

**Differential cross section**

\[ \nu_l(\bar{\nu}_l)(k) + N(p) \rightarrow l^{\pm}(k') + N(p'), \]

\[ d\sigma = \frac{1}{4M_N E_\nu} (2\pi)^4 \delta^4(k + p - k' - p') \frac{d^3k'}{(2\pi)^3 2E_{k'}} \frac{d^3p'}{(2\pi)^3 2E_{p'}} \sum \sum |M|^2 \]

- \( q = p' - p = k - k' \) is the four momentum transfer
- \( M \) is the transition matrix element
Scattering cross sections

Transition form factors

\[ V_{B'B}(p', p) = f_{1}^{B'}B(Q^2)\gamma_{\mu} + \frac{i\sigma^{\mu\nu}q_{\nu}}{M_{B} + M_{B'}} f_{2}^{B'}B(Q^2) + \frac{2q_{\mu}}{M_{B} + M_{B'}} f_{3}^{B'}B(Q^2) \]

\[ A_{B'B}(p', p) = g_{1}^{B'}B(Q^2)\gamma_{\mu}\gamma_{5} + i\sigma_{\mu\nu}\gamma_{5} \frac{q^{\nu}}{M_{B} + M_{B'}} g_{2}^{B'}B(Q^2) + \frac{2q^{\mu}}{M_{B} + M_{B'}} \gamma_{5} g_{3}^{B'}B(Q^2) \]
Transition form factors

\[ V_{B'BB}(p', p) = f_{1}^{B'BB}(Q^2)\gamma_\mu + \frac{i\sigma^{\mu\nu}q_\nu}{M_B + M_{B'}} f_{2}^{B'BB}(Q^2) + \frac{2q_\mu}{M_B + M_{B'}} f_{3}^{B'BB}(Q^2) \]

Vector FF

\[ A_{\mu}^{B'B}(p', p) = g_{1}^{B'BB}(Q^2)\gamma_\mu\gamma_5 + i\sigma_{\mu\nu}\gamma_5 \frac{q_\nu}{M_B + M_{B'}} g_{2}^{B'BB}(Q^2) + \frac{2q_\mu}{M_B + M_{B'}} \gamma_5 g_{3}^{B'BB}(Q^2) \]

Axial vector FF

Electric FF

Pseudoscalar FF
Symmetry properties

\( \mathcal{T} \) invariance \( \Rightarrow \) form factors are real

\( \mathcal{CVC} \Rightarrow f_3(Q^2) = 0 \)

\( \mathcal{G} \) invariance \( \Rightarrow f_3(Q^2) = 0 \) and \( g_2(Q^2) = 0 \)

\( \mathcal{PCAC} \Rightarrow \) relates \( g_3(Q^2) \) with \( g_1(Q^2) \) through GT relation
Symmetry properties

- \( T \) invariance \( \Rightarrow \) form factors are real

- \( \text{CVC} \Rightarrow f_3(Q^2) = 0 \)

- \( \text{G invariance} \Rightarrow f_3(Q^2) = 0 \) and \( g_2(Q^2) = 0 \)

- \( \text{PCAC} \Rightarrow \text{relates } g_3(Q^2) \text{ with } g_1(Q^2) \text{ through GT relation} \)

✿ \( \text{G violation} \Rightarrow g_2(Q^2) \neq 0 \)

✿ \( \text{T invariance} \Rightarrow \text{Real values of } g_2(Q^2) \)

✿ \( \text{T violation} \Rightarrow \text{Imaginary values of } g_2(Q^2) \)
Scattering cross sections

**Differential and total scattering cross sections**

\[
\frac{d\sigma}{dQ^2} = \frac{G_F^2 \cos^2 \theta_c}{8\pi M^2 E^2} \mathcal{L}_{\mu\nu} \mathcal{J}^{\mu\nu}
\]

\[
\mathcal{L}_{\mu\nu} = \text{Tr} \left[ \gamma_\mu (1 \pm \gamma_5) \Lambda(k') \gamma_\nu (1 \pm \gamma_5) \Lambda(k) \right]
\]

\[
\mathcal{J}^{\mu\nu} = \frac{1}{2} \text{Tr} \left[ \Lambda(p') \Gamma_\mu \Lambda(p) \tilde{\Gamma}_\nu \right]
\]

\[
\Gamma_\mu = V_\mu - A_\mu, \quad \tilde{\Gamma}_\nu = \gamma^0 \gamma_\nu \gamma^0, \quad \Lambda(P) = \slashed{P} + M_P
\]
The vector part of the lepton polarization $\zeta^\tau$ is given by

$$\vec{\zeta} = A^l(Q^2) \vec{k} + B^l(Q^2) \vec{k}'$$
Polarization observables

Polarization vector

The vector part of the lepton polarization $\zeta^\tau$ is given by

$$\zeta = A^l(Q^2) \vec{k} + B^l(Q^2) \vec{k}'$$

$$\zeta = \xi_P \vec{e}_P^l + \xi_L \vec{e}_L^l$$

the independent unit vectors are taken along:

- longitudinal direction, $\vec{e}_L^l = \frac{\vec{k}'}{|\vec{k}'|}$
- transverse direction, $\vec{e}_T^l = \frac{\vec{k}' \times \vec{k}}{|\vec{k}' \times \vec{k}|}$
- perpendicular direction, $\vec{e}_P^l = \vec{e}_L^l \times \vec{e}_T^l$

$$\zeta_L = \zeta \cdot \vec{e}_L^l; \quad \zeta_P = \zeta \cdot \vec{e}_P^l$$
Polarization components of final lepton

\[ \nu_T(\vec{k}) \]

\[ \tau^-(\vec{k}') \]

\[ N(\vec{p}') \]

\[ \hat{e}_T \]

\[ \hat{e}_L \]

\[ \hat{e}_P \]
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1 \textit{Introduction}

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Cross section

\( \sigma \text{ vs } E_{\nu_l(\bar{\nu}_l)} \) for \( e^\pm, \mu^\pm \) and \( \tau^\pm \): first class currents only

\[
\begin{align*}
\nu_e + n &\rightarrow e^- + p \\
\nu_\mu + n &\rightarrow \mu^- + p \\
\nu_\tau + n &\rightarrow \tau^- + p \\
\bar{\nu}_\mu + p &\rightarrow \mu^+ + n \\
\bar{\nu}_e + p &\rightarrow e^+ + n \\
\bar{\nu}_\tau + p &\rightarrow \tau^+ + n
\end{align*}
\]
Cross section

$\sigma$ vs $E_{\nu_l(\bar{\nu}_l)}$ for $\mu^\pm$ and $\tau^\pm$: effect of second class currents
Cross section

\[ \sigma \text{ vs } E_{\nu_\mu}(\bar{\nu}_\mu): \quad g_2^R(0) \text{ vs. } M_A \]

\[ \begin{align*}
\sigma & (10^{-40} \text{ cm}^2) \\
E_{\nu_\mu} (\text{GeV}) & \\

M_A & = 1.026 \text{ GeV}, \quad g_2^R(0) = 0 \\
M_A & = 1.1 \text{ GeV}, \quad g_2^R(0) = 0 \\
M_A & = 1.2 \text{ GeV}, \quad g_2^R(0) = 0 \\
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\end{align*} \]

\[ \begin{align*}
\sigma & (10^{-40} \text{ cm}^2) \\
E_{\bar{\nu}_\mu} (\text{GeV}) & \\

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\end{align*} \]

Cross section

$\sigma$ vs $E_{\nu\tau}(\bar{\nu}_\tau)$: $g_2^R(0)$ vs. $M_A$

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Polarization observables

$M_A$ variation: Polarization components of polarized $\tau^\pm$

$g_2^R(0)$ variation: Polarization components of polarized $\tau^{\pm}$

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The production cross section for $\mu^\pm$ and $\tau^\pm$ are sensitive to the second class current form factor.
Conclusion

The production cross section for $\mu^\pm$ and $\tau^\pm$ are sensitive to the second class current form factor.

For the muon production cross section, the results obtained using $g_R^2(0) = +1$ and $-1$ are exactly the same and higher than the results obtained with $g_R^2(0) = 0$ but this is not true for tauon production.

In the case of tauon production cross section, the results obtained using $g_R^2(0) = +1(-1)$ are higher (lower) than the results obtained with $g_R^2(0) = 0$. This difference (between $\mu$ vs. $\tau$ production) arises only because of the higher mass of the $\tau$ lepton.
The presence of second class currents would reduced the value of $M_A$ for both $\nu_\mu$ and $\nu_\tau$ induced reactions.
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Unlike the cross section, the polarization observables of the final polarized lepton are not much sensitive to the variation in the value of $M_A$ and $g_2^R(0)$.

Although not shown here but the polarization observables of the final polarized nucleon are quite sensitive to the variation in $M_A$ and $g_2^R(0)$ for both $\nu_\mu$ and $\nu_\tau$ induced reactions.
Thank you!
**G parity**

- First used by Weinberg in 1958 to classify first and second class current form factors.

- This is a symmetry of strong interactions, defined as
  \[ G = Ce^{i\pi I_y} \]

- Under G parity, \( V^\mu \) and \( A^\mu \) transforms as
  \[ GV^\mu G^{-1} = V^\mu \quad GA^\mu G^{-1} = -A^\mu \]

- \( f_1(Q^2) \), \( f_2(Q^2) \), \( g_1(Q^2) \) and \( g_3(Q^3) \) transform the same way as above and are termed as first class currents.

- \( f_3(Q^2) \) and \( g_2(Q^2) \) transform in opposite way and are termed as second class currents.
Vector form factors

The assumption of CVC leads to the determination of $f_1(Q^2)$ and $f_2(Q^2)$ in terms of EM form factors of the nucleon $f_N^1(Q^2)$ and $f_N^2(Q^2)$.

<table>
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The EM form factors of the nucleon $f_N^1(Q^2)$ and $f_N^2(Q^2)$ are expressed in terms of the electric and magnetic Sach’s form factors

\[
\begin{align*}
    f_1^{p,n}(Q^2) &= \frac{1}{1 + \frac{Q^2}{4M^2}} \left[ G_E^{p,n}(Q^2) + \frac{Q^2}{4M^2} G_M^{p,n}(Q^2) \right] \\
    f_2^{p,n}(Q^2) &= \frac{1}{1 + \frac{Q^2}{4M^2}} \left[ G_M^{p,n}(Q^2) - G_E^{p,n}(Q^2) \right]
\end{align*}
\]
Axial vector form factors

\[ g_1(Q^2) = \frac{g_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \quad g_A(0) = D(0) + F(0) = 1.267 \]

\[ g_2^{pn}(Q^2) = \frac{g_2(0)}{\left(1 + \frac{Q^2}{M_2^2}\right)^2}, \quad M_2 = 1.026 \text{ GeV}. \]
Axial vector form factors

\[ g_1(Q^2) = \frac{g_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \quad g_A(0) = D(0) + F(0) = 1.267 \]

\[ g_2^{pn}(Q^2) = \frac{g_2(0)}{\left(1 + \frac{Q^2}{M_2^2}\right)^2}, \quad M_2 = 1.026 \text{ GeV}. \]

\[ g_3^{np}(Q^2) = \frac{2M^2g_A(Q^2)}{m_F^2 + Q^2} \]
Helicity projection operator

- Projects spin states of the particle
- Must commute with $\not{p}$
**Helicity projection operator**

- Projects spin states of the particle
- Must commute with $\slashed{p}$

★ A four-vector for spin polarization $\zeta^\mu$ is required

$\zeta^{\mu}$ satisfies the following relations:

- $\zeta^2 = -1$
- $\zeta \cdot p = 0$

$$\Lambda(\zeta) = \frac{1}{2}(1 - \gamma_5 \zeta)(\slashed{p} + M_p)$$
Polarization of the final lepton

Longitudinal component: \( P_L(Q^2) = \frac{m_l}{E_l} \zeta \cdot \vec{e}_L \)

\[
P_L(Q^2) = \frac{m_l}{E_l} \frac{A_L(Q^2) \vec{k} \cdot \vec{k}' + B_L(Q^2) |\vec{k}'|^2}{|\vec{k}'|}
\]

Perpendicular component: \( P_P(Q^2) = \zeta \cdot \vec{e}_P \)

\[
P_P(Q^2) = \frac{A(Q^2)[(\vec{k} \cdot \vec{k}')^2 - |\vec{k}|^2 |\vec{k}'|^2]}{|\vec{k}'| |\vec{k}' \times \vec{k}|}
\]