4 Field Theory Defined on the Tangent Space

The action obtained in step (2) will now be considered as a field theory in the tangent space. It may be noted that the Riemann tensor is expressed, using equations (2), (3), in the form,

\[ R_{\mu\nu\lambda\gamma} = \frac{1}{2} g_{\mu\lambda} \left( \nabla_{\nu} \nabla_{\gamma} - \nabla_{\gamma} \nabla_{\nu} \right) g_{\lambda\nu} - \frac{1}{2} g_{\nu\gamma} \left( \nabla_{\mu} \nabla_{\lambda} - \nabla_{\lambda} \nabla_{\mu} \right) g_{\mu\nu} - \frac{1}{2} g_{\mu\gamma} \left( \nabla_{\nu} \nabla_{\lambda} - \nabla_{\lambda} \nabla_{\nu} \right) g_{\lambda\nu} \]

(18)

where \( \nabla_{\mu} \) denotes a covariant derivative. It may be noted that a non-zero \( \beta \) is a background field and influences the motion through \( \beta \), second term of (19) are already in a form that has such a correspondence, and first term of (19) is already in a form that has such a correspondence. Also, before we can import the EMT (19) to the local patch at \( P \) and express the equation of motion for the \( \phi \) auxiliary field theory in these coordinates as

\[ T^\mu_\nu - \Theta^\mu_\nu = 0, \quad \Theta^\mu_\nu = \frac{\delta}{\delta \phi} \left( \int d^4x \sqrt{-g} \left( \frac{1}{2} \left( g^{\mu\nu} \frac{\partial}{\partial \phi} \phi \frac{\partial}{\partial \phi} \phi^* - \frac{1}{2} M_2^2 \phi^* \phi \right) \right) \right) \]

(26)

Looking back at our equation (24) it is easy to see that it agrees with (28) which is the form obtained in the locally inertial coordinates. This agreement assures that, like the standard general relativity, the dynamically constructed EMT serves as the source even for non-minimally coupled gravity theories. In fact, it shows that only the covariantly conserved EMT has dynamical roles and should be identified as source.

5 Conclusion

We have developed a novel algorithm to obtain the EMT (28) of scalar field theories coupled with gravity. The method rests on the correspondence that the EMT is non-dynamical. Locally inertial coordinates are adapted where the first derivative of the metric variables is in infinitesimal. We have then defined a field theory in the tangent space by the same action as the one obtained in the locally inertial coordinates. This theory has translation symmetry and the conserved Noether current gives an improved EMT identifying it as the EMT of the original theory in the locally inertial coordinates, we generalized the same to curved coordinates.

When the coupling of the scalar field with gravity is minimal, the standard form of the EMT is reproduced from the expression (24) derived here. More interesting area of application is in context of the non-minimally coupled scalar field theories where different conventions [9, 10, 11, 12, 13, 14, 15, 16, 27] are advocated, since no systematic method exists to write the EMT. The dynamical property here is to provide a particular form of the EMT which allows one to write the equation of motion (25) in the same way as the minimally coupled theories. It is expected that the methodology adopted here will have wider applicability, e.g., in general scalar-tensor theories.

6 References