Study of Rutherford Scattering

1) Aim of the experiment:

- Study the Rutherford scattering using Am^{241} α-source
- Determine the nuclear charge number of Aluminium

2) Theory:

If α-particles are allowed to strike a thin gold foil, they are deflected from their path ("scattering"), each by an angle θ. The majority of α-particles are scattered by angles less than 1° (Figure 1).

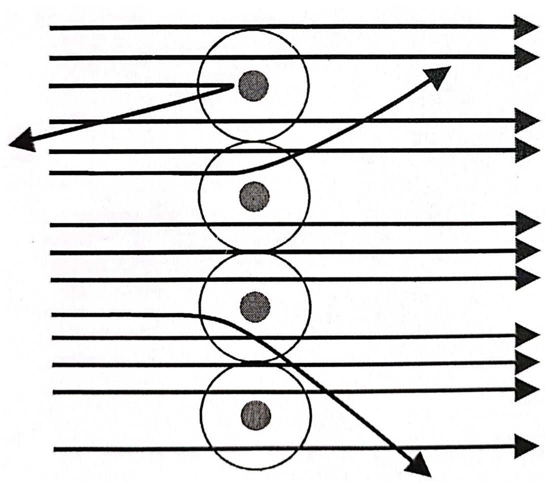

Figure 1. Scattering of α-particles on a monolayer of atoms.

A few particles, however, show substantially large scattering angles θ, in the extreme case up to 180° ("back scattering"). These initially qualitative observations can only be explained by assuming that the gold atoms have a very small nucleus, containing practically the whole atomic mass, and being positively charged.

On the basis of this idea, Rutherford calculated the angular distribution of the scattering rate N(θ). The scattering rate is the number of particles which are scattered during the time unit in a determined interval dθ around an average angle θ. The result of this calculation is "Rutherford's scattering formula":

$$N(\theta) = N_0 . c_F . d_F \frac{Z^2 . e^4}{(8\pi\varepsilon_0 E_\alpha)^2 . sin^4(\theta/2)} \qquad (1)$$

N_0: particle rate in the foil

c_F: atomic concentration in the foil

d_F: thickness of the foil

Z: nuclear charge number of the scattering material

E_α: energy of the α-particles
e: elementary charge (e = $1.6021*10^{-19}$ As)
ε_0: dielectric constant (ε0 = $8.8524*10^{-12}$ As/Vm)

Recording the scattering rate as function of the angle:
In this case we need not care about the proportionality factors in equation (1) which are kept constant in this experiment. The relevant shape of this angular distribution curve is described by the function:

$$f(\theta) = \frac{1}{\sin^4(\theta/2)} \qquad (2)$$

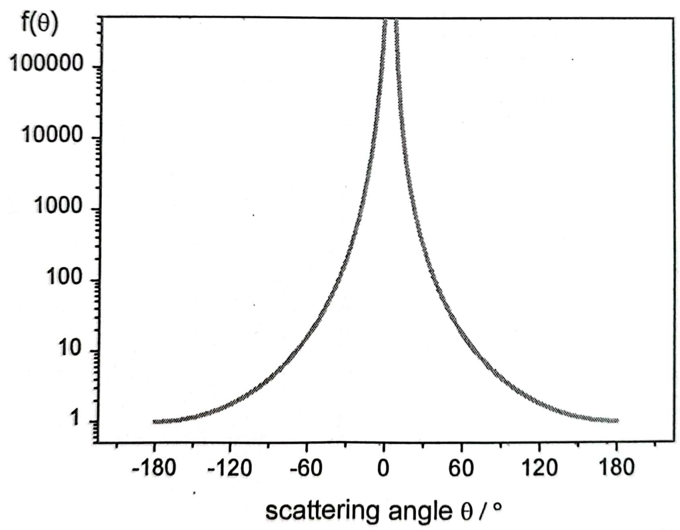

Figure 2. Theoretical slope of the scattering rate according to Equation (2).

The values of f(θ) decrease rapidly with increasing scattering angle θ. Hence, in Figure 2 the graphical representation of f(θ) is plotted in a logarithmic scale over θ. A singularity spot becomes visible at θ = 0°. Therefore, compare measuring results with the theoretical slope only outside of this region, i.e. for values |θ| > 5°.
As for higher scattering angles θ the counting rates become very small. The gate times t(θ) for determining the counting rate N(θ) have to be increased with increasing angle θ to obtain an acceptable accuracy. For keeping the total measuring time sustainable the angular range can be restricted to |θ| ≤ 30°.
Because of the very low range of α-particles in the air this experiment must be carried out in a closed chamber under vacuum (Figure 3). Figure 4 shows the arrangement of the components on the lid of the scattering chamber.

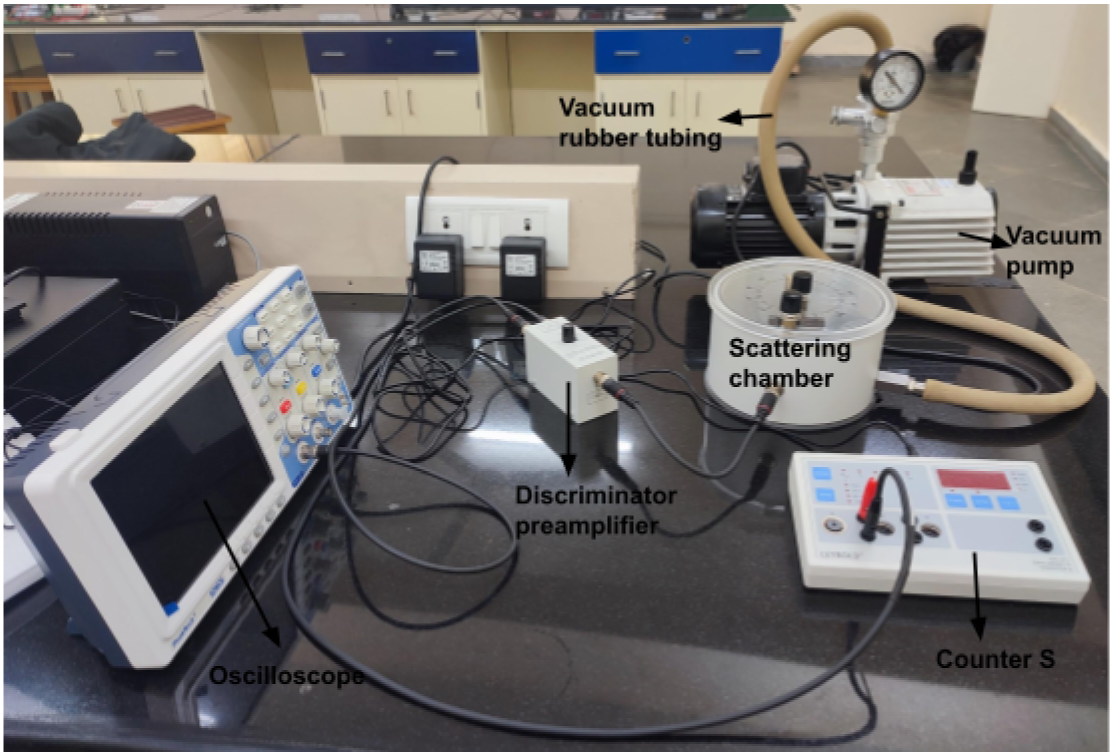

Figure 3. Experimental setup for the Rutherford Scattering Experiment.

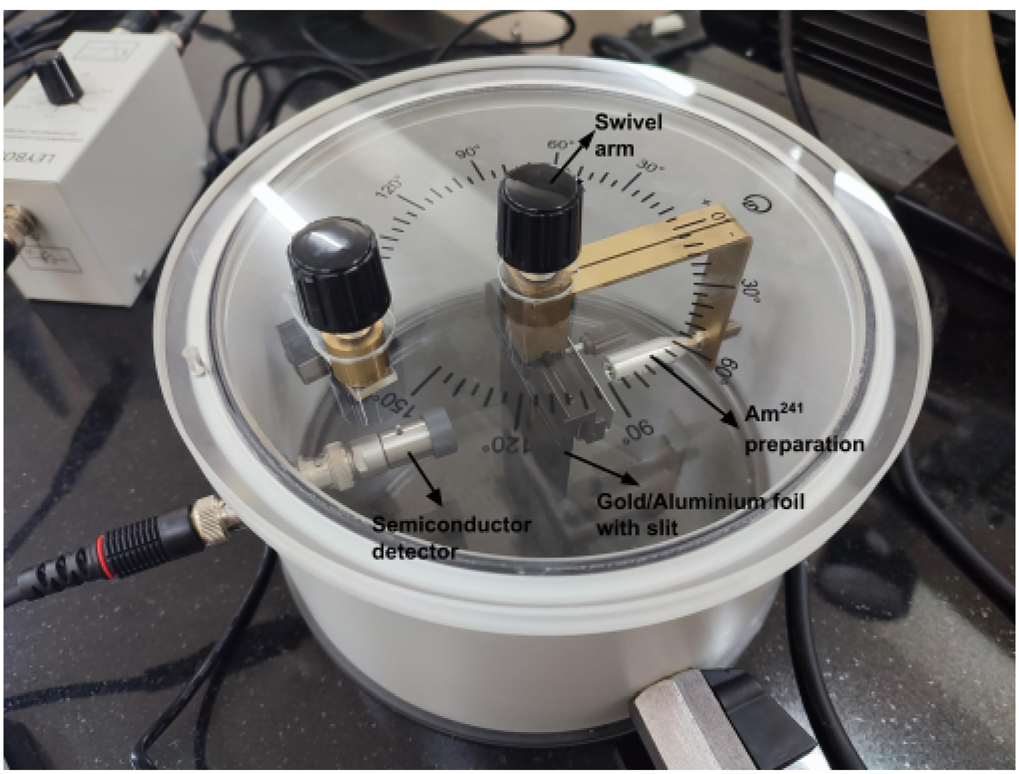

Figure 4. The scattering chamber.

The α-particles emitted from the Am^{241} preparation fall through a slit aperture of 5 mm width onto the Gold foil and leave this gold foil with various scattering angles. The scattered α-particles are identified with a semiconductor detector. By swinging the detector in steps of 5°, for example, the scattering rate can be determined for all scattering angles from 5° to 60°. In the current setup we are going to use, the detector is not swung but rather the preparation, slit and gold foil, which are attached on a common swivel arm. The detector is firmly attached to the side wall of the chamber.
The 5 mm wide slit is a good choice for getting acceptable count statistics in reasonable time, but at small angles below 20° the angular precision might be increased by using a 1 mm slit and scaling the results appropriately. On the other hand, this scaling can introduce further errors, so it has not been used in the measurement described later.

Determining the nuclear charge number of Aluminium:
If we compare the scattering rates between two different foil materials (e.g. Au and Al) at the same angle θ, we can derive from the scattering formula (1):

$$\frac{N_{Au}}{N_{Al}} = \frac{c_{Au} d_{Au} Z^2_{Au}}{c_{Al} d_{Al} Z^2_{Al}} \qquad (3)$$

Hence the nuclear charge number of Aluminium Z_{Al} can be determined by scattering experiments as following:

$$Z_{Al} = \sqrt{\frac{N_{Al}(\theta)\, c_{Au} d_{Au} Z^2_{Au}}{N_{Au}(\theta)\, c_{Al} d_{Al}}} \qquad (4)$$

3) Equipment required:
- Scattering chamber
- Semiconductor detector
- Am^{241} preparation
- Gold and Aluminium foils
- Discriminator preamplifier (559 931)
- Counter S (575 471)
- Vacuum pump
- Vacuum rubber tubing

4) Procedure:

Preparing the scattering chamber:
Vent the scattering chamber (when using at first time) and take off the lid (Figure 4). Insert the Am^{241} preparation into the 4 mm socket of the main swivel arm down to the

stop. Place the 5 mm slit aperture and the plastic sheet containing the Gold foil on top of one another (with the Gold foil between them) and insert both of them into the holder so that the slit points towards the preparation. Move the small swivel arm (not used here) close to the chamber's side wall so that the measurements are not disturbed.
Make sure that the detector is fixed on the BNC socket at the inner wall of the chamber with the sides of the silicon chip parallel to the lid, and that the detector's plastic aperture slit is perpendicular to the lid (with the mark at the top). Fit vacuum tubing to the hose nozzle. Close the chamber by placing its lid onto the housing. Ensure the correct positioning by positioning the pin in the borehole. Then evacuate the chamber. Firmly press the lid on, if necessary. While the vacuum builds up, the lid is pressed onto the rubber seal and the deep black stripe where they touch gets two to three mm wide.
During the measurement, the vacuum pump can either run continuously or the valve of the Rutherford chamber can be closed and the pump switched off. The rubber tubing must be vented through the inlet valve to avoid oil creeping back into the vacuum.

Preparing the electrical counting components:

- Connect the scattering chamber and discriminator preamplifier using the given short 25 cm BNC to BNC cable.
- Connect the discriminator preamplifier and counter S using BNC to two banana connectors cable.
- Set the discriminator to zero.
- Connect the power cord of the discriminator preamplifier to the power supply.
- Connect the power cord of the counter S to the power supply.
- Connect the discriminator preamplifier analogue output to the oscilloscope.

See these connections in Figure 3.

Adjusting the discriminator level:

- Set up the experiment with the 5 mm slit, the Gold foil, Am^{241} preparation and detector, and move the holder to a high angle, 30°. Only very few α-particles will reach the detector in this position.
- Evacuate the chamber for 1 - 2 mn.
- The potentiometer sets the discriminator level, a setting of zero (fully counterclockwise) will not discriminate anything, but let every noise signal pass through. The counter will overload instantly and display "----". Increasing the discriminator setting will reduce these noise counts. To find the right position, increase the discriminator level by approximately a quarter turn and reset the counter. By varying the discriminator level in both directions search for the setting where the noise count rate just drops to zero. Note that position.

- Now swivel the preparation to 0° position, a lot of alpha radiation will hit the detector. The counter will start to count something between 10 to 100 counts per second. Increase the discriminator level again until the count rate just starts to drop. Note this position too.
- The correct position for the discriminator level is midway between both positions, the one where the noise is masked out and the other where the alpha count rate starts to drop.

The counter S normally starts counting the pulses with activated $N_{A,E}$. and gate time set to manual. Press START to start counting and ➡0⬅ to reset the counter when necessary.

Recording scattering rate as function of the angle:
During measurements protect the sensitive detector from light (e.g. especially from fluorescent light of ceiling lamps) If necessary, cover the scattering chamber during measurements with a black cloth.

- Prepare the counter S for pulse counting by pressing the push button MODE to activate $N_{A,E}$.
- Select gate time t(θ) = 100 s by pressing the toggle button GATE three times Note: t(θ) = 100 s is useful for small angles, i.e. angle up to +/- 15°. By pressing GATE + MODE, longer gate times can be adjusted, i.e. up to 9999 s (MODE upwards, GATE downwards).
- Count at least 50 particles (n(θ)>50) at the angles: θ = +/-5°, +/-10°, +/-15°, +/-20°, +/-25° and +/-30°.
- Use gate times t(θ) as given in Table 1. Repeat the counting three times for all angles.

Determining the nuclear charge number of Aluminium:

- Carefully vent the chamber by gradually releasing the knob, take off the lid and remove the 5 mm slit with the Gold foil.
- Put the 1 mm slit together with the Gold foil back into the holder, put the lid on the chamber and evacuate.
- Set the swivel arm first to a position of +15°, count for 100 s, then go to –15° and count again for 100 s. Tabulate the values in Table 2.
- Carefully vent the chamber, take off the lid and remove the slit with the Gold foil. Put the 1 mm slit together with the Aluminium foil back into the holder, put the lid on the chamber and evacuate.
- Count again at the same position of +15° and –15°, but for a much longer time (1000 s), as the aluminum foil scatters less particles.

5) Observations:

Table 1. Measured counts with Gold foil and slit d = 5 mm:

Angle (θ in °)	Gate time (t(θ) in s)	Pulse counts (single values) n(θ)	Pulse counts (mean value) $n_m(\theta)$	Counting rate (directly) $N_d(\theta)$ in 1/s	Counting rate (space corrected) N(θ) in 1/s
-30	900	51 41 51	47.66666667	0.05296296296	0.1663037037
-25	600	62 70 81	71	0.1183333333	0.3140617176
-20	200	97 78 86	85	0.425	0.9128517625
-15	100	204 233 220	219	2.19	3.559590091
-10	100	1533 1563 1566	1554	15.54	16.94653404
-5	100	3505 3598 3369	3490.666667	34.90666667	19.10574737
+5	100	3730 3641 3669	3680	36.8	20.14204077
+10	100	2258 2269 2173	2233.333333	22.33333333	24.35473575
+15	100	532 493 513	512.6666667	5.126666667	8.332799939
+20	200	151 123 147	140.3333333	0.7016666667	1.507100361

+25	600	120 127 125	124	0.2066666667	0.5485021546
+30	900	82 66 58	68.66666667	0.0762962963	0.2395703704

Table 2. Measured counts with Gold foil and slit d = 1 mm:

Angle (θ in °)	Gate time (t(θ) in s)	Pulse counts (single values) n(θ)	Pulse counts (mean value) n_m(θ)	Counting rate (directly) N_d(θ) in 1/s	Counting rate (space corrected) N(θ) in 1/s
-15	100	12 11 9	10.67	0.1067	0.1734
+15	100	15 23 25	21	0.21	0.341

Table 3. Measured counts with Aluminium foil and slit d = 1 mm

Angle (θ in °)	Gate time (t(θ) in s)	Pulse counts (single values) n(θ)	Pulse counts (mean value) n_m(θ)	Counting rate (directly) N_d(θ) in 1/s	Counting rate (space corrected) N(θ) in 1/s
-15	1000	11 16 8	11.67	0.01167	0.019
+15	1000	4 3 11	6	0.006	0.01

6) Calculations:

Recording the scattering rate as function of the angle:

After recording the pulse counts n(θ) the mean values $n_m(\theta)$ can be determined. Using the mean values $n_m(\theta)$ the scattering rates $N_d(\theta)$ are calculated by:

$$N_d(\theta) = \frac{n_m(\theta)}{t(\theta)} \qquad (5)$$

These measuring results $N_d(\theta)$ are typical for a plane scattering geometry which is given by the transparent construction of the chamber used in this experiment. The theoretical function (according to Rutherford's formula), however, is related to a three-dimensional geometry. The relation between these different aspects can be considered by the following concept (Figure 5):

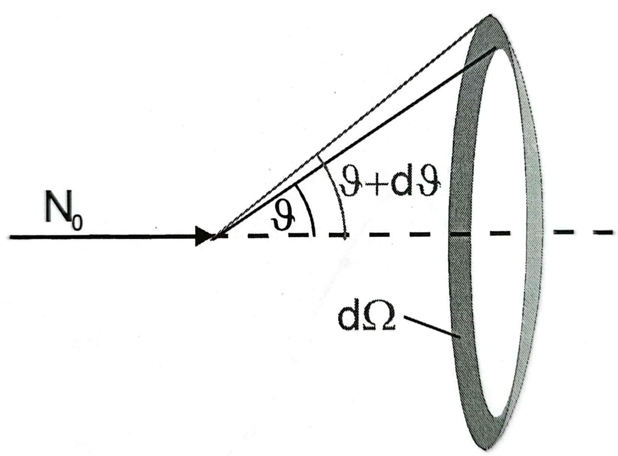

Figure 5. The α-particles are scattered into the angular region ϑ + dϑ.

Each plane angle θ corresponds in space to a cone with an aperture of 2·θ (produced by rotation of the plane structure around the incident beam axis). In the same way the plane angular differential dθ corresponds in three dimensions to a spatial angular differential dΩ given by:

$$d\Omega = 2 \cdot \pi \cdot \sin(\theta)\, d\theta \qquad (6)$$

This geometrical correction allows to derive a relation between the plane scattering rate $N_d(\theta)$ and the spatial scattering rate N(θ):

$$N(\theta) = 2 \cdot \pi \cdot \sin(\theta) \cdot N_d(\theta) \qquad (7)$$

Calculate the corresponding spatial values N(θ), tabulate them in Table 1, and plot the space corrected values as a function of scattering angle.

The measuring value pairs {θ / N(θ)} can be compared with the shape of the theoretical curve of equation (2):

$$f(\theta) = \frac{p0}{\sin^4(\theta - p1/2)} \qquad (8)$$

The proportionality factor p0 represents a vertical shift (at logarithmic scale). The coefficient p1 is representing a small displacement along the horizontal angular scale.

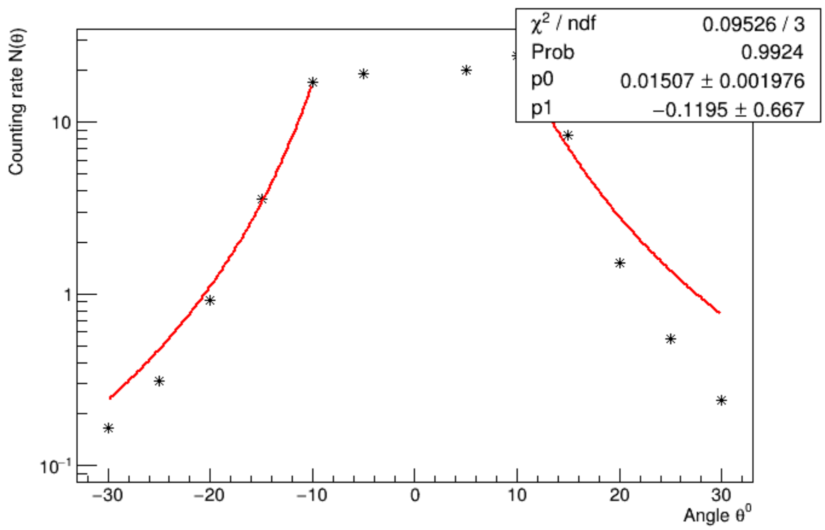

Figure 6. Counting rate N(θ) as a function of scattering angle. The data is fitted with Equation (8).

Determining the nuclear charge number of Aluminium:

Results for the counting rates of Gold and Aluminium foils are (-15°/15°, 1 mm slit):
N_{Au}(-15°/15°) = 0.1734 /s at -15°
N_{Al}(-15°/15°) = 0.019 /s at -15°

With d_{Au} = 2 μm, d_{Al} = 8 μm, $c_{Au} \approx c_{Al}$ and Z_{Au} = 79, with equation (4):

Z_{Al} = 13.075
(exact value: Z_{Al} = 13).

7) Conclusions:

References:

(1) Manual from the supplier (LD-didactic)