Study of Compton Scattering

1) Aim of the experiment:

- Determine the change in wavelength of the scattered gamma radiation as a function of the scattering angle.
- Determine the differential cross-section using Klein-Nishina formula and calculation of the calibration factor.

2) Theory:

Compton scattering is an example of inelastic scattering of light by a charged particle, where the wavelength of the scattered light is different from that of the incident radiation. In 1920, Arthur Holly Compton observed scattering of x-rays from electrons in a carbon target. He found that the scattered x-rays have a longer wavelength than the incident x-rays. The shift of the wavelength increased with scattering angle according to the Compton formula:

$$\lambda_\theta - \lambda_0 = \Delta\lambda = \frac{h}{m_e c}(1 - cos\theta) \qquad (1)$$

Where,

λ_0 is the incident wavelength of photon, λ_θ is the scattered wavelength of photon, h is Planck's constant, m_e is the rest mass of electron, c is the velocity of light and θ is the scattering angle of the photon.

The Compton scattering process is illustrated in Figure 1. In the figure, Φ is the scattering angle of the recoiled electron.

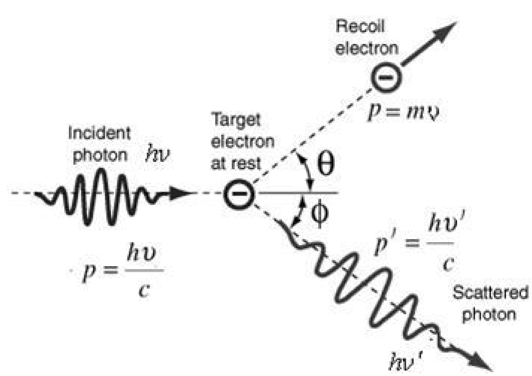

Figure 1. Compton scattering

The value $\frac{h}{m_e c}$ = 0.02426 Å is called Compton wavelength of electron. In terms of energy Equation (1) can be rewritten as

$$E_\theta = E_0 \frac{1}{1+(\gamma.(1-cos\theta))} \quad (2)$$

where E_0 and E_θ are energy of incident and scattered photon, respectively, and $\gamma = \frac{E_0}{m_e c^2}$. For high energy photons with (λ << 0.02 Å or E >> 511 keV), the wavelength of the scattered radiation is always of the order of the Compton wavelength whereas for low energy photons (E << 511 keV), the Compton shift is very small. In other words, in a non-relativistic energy regime, Compton scattering results approach the results predicted by classical Thompson scattering.

Compton's experiment had a lot of significance that time since it gave clear and independent evidence of particle-like behaviour of light. Compton was awarded the Nobel Prize in 1927 for the "discovery of the effect named after him".

The differential Compton scattering cross section was correctly formulated by Klein-Nishina in 1928 using quantum mechanical calculations. This formula is famously known as Klein-Nishina formula which is expressed as follows:

$$\frac{d\sigma}{d\Omega} = r_0^{\ 2} \frac{1+cos^2\theta}{2} \frac{1}{[1+\gamma(1-cos\theta)]^2} [1 + \frac{\gamma^2(1-cos\theta)^2}{(1+cos^2\theta)(1+\gamma(1-cos\theta))}] \quad (3)$$

Here, $r_0 = \frac{e}{4\pi\varepsilon_0 m_e c^2}$ is the classical electron radius and has the value r_0 = 2.8179 × 10^{-15} m. This result is for the cross section averaged over all incoming photon polarizations. By integrating Equation (3) over all angles, the total cross section can be obtained.

In this experiment gamma rays from a Cs^{137} source are used as the source of photons that are scattered. Difference in the incident and scattered energy and wavelength of the photons is determined by a calibrated scintillation detector placed at different scattering angles. The relative intensities I_θ of the scattered radiation peaks can be compared with the predictions of the Klein-Nishina formula for the differential effective cross section $\frac{d\sigma}{d\Omega}$ by calculating the calibration factor C using the formula below:

$$C = \frac{1}{n}\sum_{\theta=0}^{n} \frac{I_\theta}{(\frac{d\sigma}{d\Omega})} \quad (4)$$

3) Equipment required:

1. Mixed nuclide preparation BB-4772 (559 845) ⟶ Am^{241}+Cs^{137}
2. Cs^{137} preparation BC-8774 (559 809)
3. Lead block source holder
4. Additional Lead block to block the direct gammas from the source hitting the counter
5. Aluminium scatterer

6. Brass scatterer
7. NaI(TI) Scintillation counter (559 901)
8. Scintillation counter holder (559 800)
9. Detector output stage (559 912)
10. High-voltage power supply unit (521 68)
11. SENSOR-CASSY 2 (524 013)
12. MCA BOX (524 058)
13. PC with Windows operating system
14. CASSY Lab 2 software

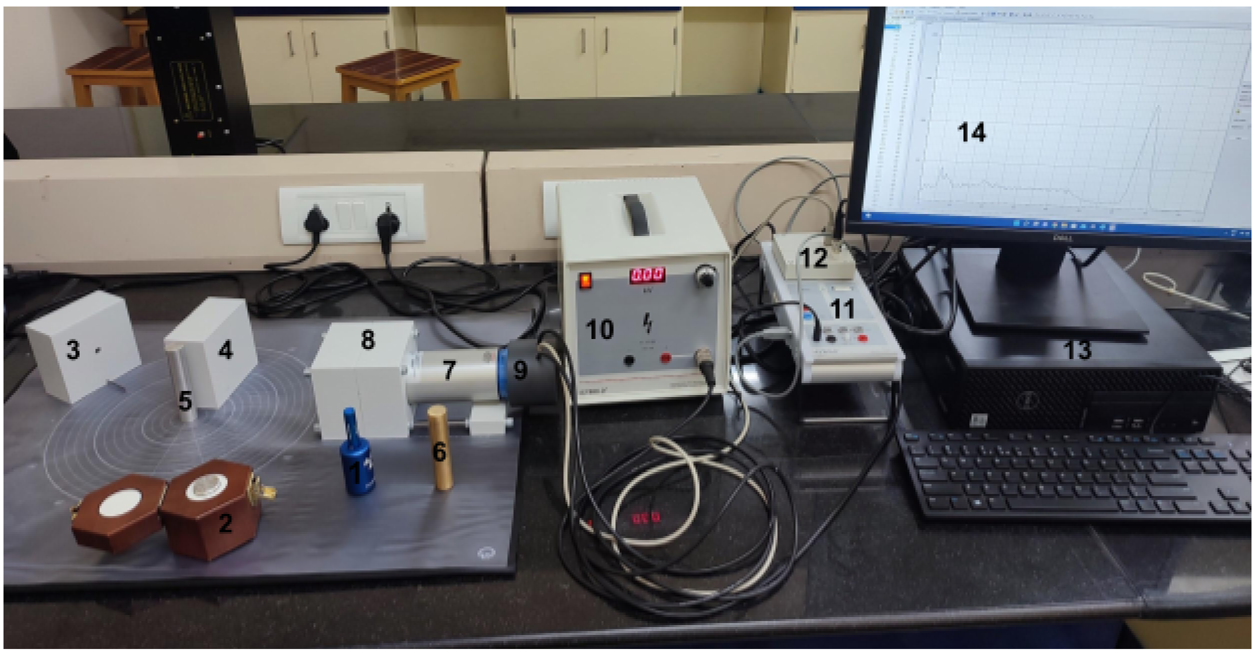

Figure 2. Experimental setup Compton scattering experiment. Numbering is given to the equipment according to the above list.

The complete experimental set up is shown in Figure 2, which can be visualized in the following sequence. A radioactive Cs^{137} source produces 661.66 keV γ-rays which can escape the shielded cavity only through a small hole. The beam is collimated and reaches an Aluminium rod (scatterer). Some portion of the γ-rays are scattered by the electrons in the target which are detected and counted by the scintillator detector. The detected signal is further processed by an MCA and the complete spectrum is displayed on the computer. By placing the source at different angles on the experimental angular panel, the scattered radiations are collected to study the angular dependence of Compton scattering.

4) Procedure:

Assemble all the accessories to set up the experiment as shown in Figure 2.

- From the detector output stage there are three cables. Connect the cable with bulkhead connector to High-voltage power supply unit, cable with BNC connector to MCA BOX INPUT and the cable with six pin connector to MCA BOX PREAMPLIFIER.
- Connect the power card of the High-voltage power supply unit to power supply and power on the unit.
- Connect the 12 V power card of SENSOR-CASSY 2 to power supply and power on the unit.
- Connect the cable with USB type-B connector on one end and USB type-A connector on other end from SENSOR-CASSY 2 to the computer.
- Set the operating voltage for the Scintillation counter at an optimized value of 0.72 kV (recommended operating voltage is from 0.7 to 0.8 kV).
- Load CASSY Lab 2 software on the computer.
- Select “Help” option (with ? symbol), CASSY Lab 2 “Help” wizard populates. Select Experiment examples ⟶ Physics ⟶ Atomic and Nuclear Physics ⟶ Quantitative observation of Compton effect.
- Opt for “Load settings” with which all are set for collecting the data, with the tabs Energy Calibration, Measurement, Scattering Spectra and Evaluation.

Calibration of MCA channels:

- Insert the mixed nuclide preparation into the Lead block source holder, align it at 0° mark and put it quite close to the Scintillation counter.
- In the CASSY Lab 2 software, select the Energy Calibration tab and record the spectrum for 100 s. Two peaks can be seen, the smaller one on the left side is for 59.54 keV Am^{241} and the larger on the right side is for 661.66 keV Cs^{137}.
- Select the “Settings” option, right side of the spectra “Settings” wizard populates. Select CASSYs ⟶ Sensor-CASSY 2 ⟶ Input A_1 (MCA box, 524058) ⟶ Channel n_A . Another wizard populates, opt no.
- In the settings wizard, under “Energy Calibration” two rows can be seen, one for 661.66 keV Cs137 and another for 59.54 keV Am241. Currently, the Channel numbers are not assigned for these two energies.
- Right click on the spectra, from the dropdown menu opt for “Calculate Peak Center”. Select the range for each peak one after another, these centers will appear on the “Channel” tab of the “Settings” wizard for each energy peak.
- Drag the energy value “E_A” into the diagram, and now the spectrum is energy calibrated.
- Put the mixed nuclide preparation into its storage box.

Measuring the energy of scattered gamma rays as a function of θ:

- Replace the mixed nuclide preparation in the Lead block source holder with the Cs^{137} preparation, align it at 0° mark and put it at the end of angular scale. Record the spectrum for 100 s. A peak can be seen at 661.66 keV for Cs^{137}.
- Put the source at the angles 30°, 60°, 90°, 120° and 45°, respectively and record the spectra for 500 s, with the Aluminium scatterer (by putting it at designated location) and without the scatterer. These spectra are recorded in the "Measurement" tab.
- While recording these spectra align the Additional Lead block so that the direct gammas from the source will not hit the detector.
- The software provides the difference between with and without scatterer spectra, which will be the peak energy of the scattered gammas (E_θ). These spectra can be seen in the "Scattering Spectra" tab.
- Repeat the same procedure with a Brass scatterer and record the spectra.

5) Observations:

<u>For Aluminium scatterer:</u>

- Energy calibration spectra

- Measurement spectra

- Scattering spectra

<u>For Brass scatterer:</u>

- Energy calibration spectra

- Measurement spectra

- Scattering spectra

6) Calculations:

<u>For Aluminium scatterer:</u>

Fit the scattered energy with Compton formula:

- In the “Scattering Spectra" tab, for each peak, right click and select Fit Function ⟶ Gaussians of equal Widths from the dropdown menu. Then, select the range for fit. The distribution will be fitted with the Gaussian function and the fit parameters can be seen at the bottom left of the screen. Do this for all spectra.
- Collect the means of each Gaussian peak from the fit parameters and note down them in the “Evaluation” tab as a function of the corresponding angle, as designated on the left side of the wizard. These means as a function of angle are automatically stored on the diagram.
- Right click on the diagram, select Fit Function ⟶ Free Fit and fit with the function $\frac{661.66}{[1+\frac{661.66}{511}(1-cosx)]}$ and see the distribution follows this Compron formula.
- **To identify electron mass**: In the above formula replace 511 keV with a free parameter, for example A, and give an initial value for A, and fit. The electron rest mass value can be seen at bottom left of the screen.

Change in energy and wavelength as a function of scattering angle:

For the corresponding scattering angle and the change in energy, the change in wavelength can be calculated using the formula:

$$\Delta E = \frac{hc}{\Delta\lambda}$$

$$\Delta\lambda = \frac{hc}{\Delta E} \quad (5)$$

Table 1. Change in energy and wavelength of the scattered photons as a function of scattering angle:

Scattering angle (θ)	Change in energy (keV)	Change in wavelength (Δλ in m)
30	556.4	2.229×10^{-12}
45	476.8	2.6×10^{-12}
60	407.3	3.044×10^{-12}
90	292.2	4.244×10^{-12}
120	227.1	5.46×10^{-12}

Evaluation of differential cross-sections and relative intensities:

- The differential cross-sections as a function of scattering angle can be calculated using Equation (3).
- The relative intensities (I_θ) can be obtained from the "Scattering Spectra" tab as follows. Right click on each fit, from dropdown menu select Calculate Integral → Peak Area. Then select the appropriate range to get the intensity.

Table 2. Differential cross-section and relative intensities as a function of of scattering angle:

Scattering angle (θ)	Differential cross-section ($\frac{d\sigma}{d\Omega}$ in m^2)	Relative intensities (I_θ)
30	6.075×10^{-30}	824
45	3.348×10^{-30}	538
60	2.2×10^{-30}	482
90	1.3047×10^{-30}	373
120	0.5881×10^{-30}	327

Calibration factor:

- The calibration factor can be calculated using the Equation (4).

$C = 2.7147 \times 10^{32}$

<u>For Brass scatterer:</u>

Change in energy and wavelength as a function of scattering angle:

Scattering angle (θ)	Change in energy (keV)	Change in wavelength (Δλ in m)
30	552.4	2.245×10^{-12}
45	474.7	2.612×10^{-12}
60	406.2	3.053×10^{-12}
90	292	4.247×10^{-12}
120	229.1	5.412×10^{-12}

Evaluation of differential cross-sections and relative intensities:

Scattering angle (θ)	Differential cross-section ($\frac{d\sigma}{d\Omega}$ in m^2)	Relative intensities (I_θ)
30	6.075×10^{-30}	1775
45	3.348×10^{-30}	1331
60	2.2×10^{-30}	987
90	1.3047×10^{-30}	680
120	0.5881×10^{-30}	565

Calibration factor:

- The calibration factor can be calculated using the Equation (4).

$C = 5.2406 \times 10^{32}$

7) Conclusions:

References:

(1) Manual from the supplier (LD-didactic)
(2) http://web.mit.edu/rbowru/Public/Jlab/BowensRubin_Compton.pdf
(3) https://aapt.scitation.org/doi/pdf/10.1119/1.11222