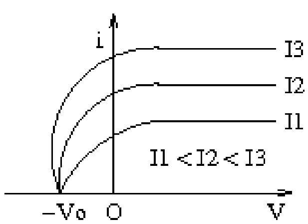
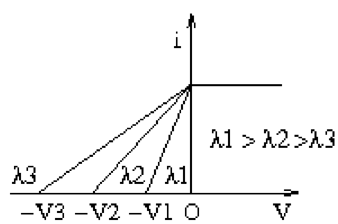
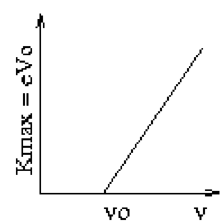
Einstein's equation for photoelectric effect

Photoelectric effect: The ejection of electrons from a metal surface by light is called the photoelectric effect. It has been observed that

1. there is a minimum or cut-off or threshold frequency ν_0, specific to the metal surface, below which no emission of electrons takes place, no matter what the intensity of the incident radiation is or for how long it falls on the surface,
2. the maximum kinetic energy of the emerging electrons is independent of intensity of incident radiation but depends linearly on the frequency of the radiation,
3. electrons start emitting immediately after the light shines on surface without detectable time delay,
4. for a given frequency of incident radiation, above ν_0, the number of electrons emitted per unit time is proportional to the intensity of incident radiation.







Classical wave theory of light, however in direct conflict with above observations, tells us

1. photoelectric effect should occur for any frequency of light provided only the light is intense enough to give energy needed to eject photoelectrons,
2. kinetic energy of the emitted electrons should depend on intensity of incident radiation since more intensity implies more imparted energy and
3. if the incident light is too feeble there would be a measureable time lag between incidence of light and ejection of photoelectrons, since electrons cannot emit unless it has absorbed enough energy.

Planck restricted energy discretization to the oscillators, representing the source of electromagnetic fields, that can radiate electromagnetic energy in quantum, which once radiated spreads as wave. **Einstein proposed (1905) discrete quanta for electromagnetic field itself, which later came to be called *photon*, each carrying energy $h\nu$ as it moves away from source with velocity c. Einstein also assumed that in the photoelectric process one photon** (of appropriate frequency) **is completely absorbed by one electron in photo-cathode or none at all.** The maximum kinetic energy of the emitted electrons is, therefore,

$$K_{\max} = e\,V_0 = h\nu - W = h\,(\nu - \nu_0), \tag{20}$$

where W is the characteristic energy of the metal called *work function* and is defined as the minimum energy needed by an electron to liberate from the metal, $K_{\max} =$

$0 \Rightarrow W = h\nu_0$ where ν_0 is the *cut-off frequency*. The V_0 is the *stopping potential*, the reverse potential at which photoelectric current goes to zero. The present day accepted value of Planck's constant is $h = 6.62 \times 10^{-34}$ joule-sec. Intensity of light beam is merely the number of photons in the beam, doubling the light intensity simply doubles the number of photons and thus doubles photoelectric current. It does not change the energy $h\nu$ of the individual photons. The photon hypothesis thus explains all the features of photoelectric effect.

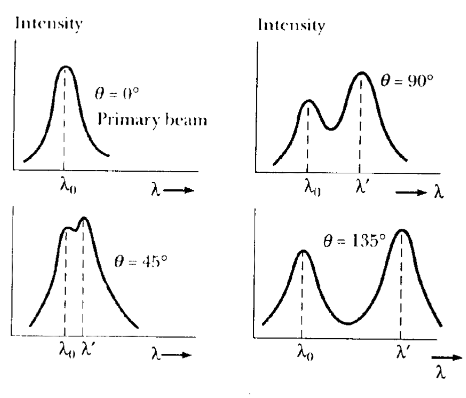
1. It follows from eqn. (20) if the frequency of incident radiation is reduced below ν_0, the individual photons, no matter how many of them there are (*i.e.* no matter how intense the radiation is), will not have enough energy individually to liberate photoelectrons.

2. $K_{\max}$ is completely independent of intensity and depends linearly only on the frequency of the incident radiation.

3. When the photons above cut-off frequency strike the metal, there is either hit or no-hit with the electrons and when hit, the photon will be absorbed immediately leading to immediate emission of photoelectron.

The Compton effect

Another important demonstration of corpuscular nature of radiation is the Compton effect (1923). Upon incident on a block of material, the x-ray of wavelength λ_0 scatters and the intensity of scattered radiation is found to peak at **two** wavelengths – one is the same as the incident wavelength λ_0 while the other is λ_1, where $\lambda_1 > \lambda_0$. The shift $\Delta\lambda = \lambda_1 - \lambda_0$ is called **Compton shift** and depends only on the scattering angle and *not* on the initial wavelength λ_0 and material of the target.



Classically, the oscillating electric field of the incident radiation, of specific frequency $\nu_0 = c/\lambda_0$ interacts with the electrons contained in the atoms of the target and forces them to vibrate with same frequency, thus scattering at the same wavelength λ_0 as the incident x-ray. Hence, classical picture cannot explain the presence of larger wavelength λ_1.

Compton and Debye regarded the incident x-ray beam as a collection of photons, and not as waves, each of energy $E_0 = h\nu_0 = hc/\lambda_0$. They suggested that λ_1 could be attributed to scattering of x-ray photons from loosely bound electrons in the atom of the target, where they loose some of its energy in the inelastic collision, $E_1 < E$. Therefore, their frequency is reduced implying larger wavelength $\lambda_1 = c/\nu_1 = hc/E_1$. Since the electrons participating in the scattering process are treated almost free and initially stationary (binding energy of the electrons are small compared to the energy of the x-ray photons) and does not involve entire atoms, this kind of explains why $\Delta\lambda$ is independent of the material of the scatterer.

To calculate the Compton shift, let a photon of total energy E_0 and momentum p_0 is incident on a stationary electron of rest mass energy m_0c^2,

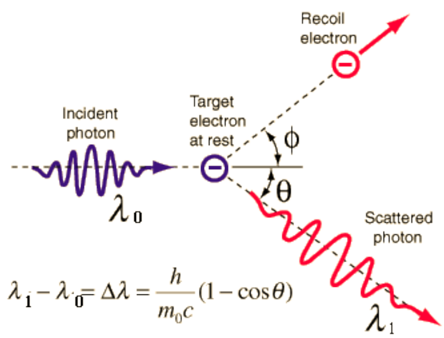
$$E_0 = h\,\nu_0 = \frac{hc}{\lambda_0} \quad \text{and} \quad p_0 = \frac{E_0}{c} = \frac{h}{\lambda_0}. \tag{21}$$

After the collision, the photon is scattered at an angle θ and moves off with total energy E_1 and momentum p_1,

$$E_1 = h\,\nu_1 = \frac{hc}{\lambda_1} \quad \text{and} \quad p_1 = \frac{E_1}{c} = \frac{h}{\lambda_1}. \tag{22}$$

and electron recoils at an angle ϕ with kinetic energy K, total energy E and momentum p,

$$E^2 = p^2c^2 + m_0^2c^4 \quad \text{and} \quad K = E - m_0\,c^2. \tag{23}$$



Momentum conservation leads to,

$$\begin{aligned} p_0 &= p_1\,\cos\theta + p\,\cos\phi \\ 0 &= p_1\,\sin\theta - p\,\sin\phi. \end{aligned}$$

Squaring and adding the above two equations, we get

$$p^2 = p_0^2 + p_1^2 - 2p_0p_1\,\cos\theta. \tag{24}$$

From conservation of energy in the collision, it follows that

$$E_0 + m_0c^2 = E_1 + E \;\Rightarrow\; E = (E_0 - E_1) + m_0c^2 \tag{25}$$

and using equations (21) and (22), we obtain

$$\left(p^2c^2 + m_0^2c^4\right)^{1/2} = c(p_0 - p_1) + m_0c^2 \tag{26}$$

which upon squaring gives us,

$$p^2 \;=\; (p_0 - p_1)^2 + 2m_0c(p_0 - p_1). \tag{27}$$

Comparing equations (24) and (27), we have

$$(p_0 - p_1)^2 + 2m_0c(p_0 - p_1) \;=\; p_0^2 + p_1^2 - 2p_0p_1\,\cos\theta$$

which reduces to

$$\frac{1}{p_1} - \frac{1}{p_0} \;=\; \frac{1}{m_0c}\,(1 - \cos\theta). \tag{28}$$

Multiplying through by h and applying (21) and (22) we obtain the Compton equation

$$\Delta\lambda \;=\; \lambda_1 - \lambda_0 \;=\; \lambda_c\,(1 - \cos\theta) \tag{29}$$

where, λ_c is the Compton wavelength defined as,

$$\lambda_c \;\equiv\; \frac{h}{m_0c} \;=\; 0.0243\mathring{A}. \tag{30}$$

A few more lines of calculation gives us the relation between scattering and recoil angle and kinetic energy of the recoiled electron (using $\alpha = \lambda_c\nu_0/c = \lambda_c/\lambda_0$),

$$\cot\phi \;=\; (1+\alpha)\,\tan\frac{\theta}{2}, \tag{31}$$

$$K \;=\; h\nu_0\,\frac{\alpha(\cos\theta - 1)}{1 + \alpha(\cos\theta - 1)}. \tag{32}$$

To explain the presence of peak at unchanged photon wavelength λ_0, we observed that if the electron involved in scattering are particularly strongly bound to the atom in the target then the whole atom recoils. Therefore, the electron rest mass m_0 in Compton equation (29), has to be replaced by mass of the atom $M \gg m_0$ and hence the Compton shift becomes way too small, $\Delta\lambda \sim 1/M$.