
National Institute of Science Education and Research

Bhubaneswar

Mid-Semester Examination 2016-17

M101- Mathematics I, Maximum Marks: 30

Date: 22-09-2016, Time: 10.00 -12.00

1. Let M be the set of all months in a year. Let A be the subset of M containing all the months starting with the letter "J". Let B be the subset of M containing all the months ending with the letter "Y". Write the elements of the following sets:

$$A \cup B, A \cap B, A - B. \quad [3]$$

2. Prove the following by the method of induction:

For every natural number n , [3]

$$1.2 + 2.3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

3. Define the relation on the set of natural number \mathbb{N} :

$a \sim b$ if a and b have a common factor greater than 1.

(a) Find five natural numbers which are related to 2. [2]

(b) Check if this relation \sim is reflexive, symmetric and transitive. Justify your answers. [2+2+2]

4. Find six rational numbers q such that $q > 1.7$ and $q^2 < 3$. [3]

5. Let $S = \{a_1, a_2, \dots, a_n, a_{n+1}\}$ be a set with $n+1$ elements and $T = S - \{a_{n+1}\}$. Let A be the collection of all subsets of S having a_{n+1} as an element and B be the collection of all subsets of T . Find a bijection (a function which is one-one and onto) between A and B . [4]

6. For a set A , let $P(A)$ be the power set of A (i.e., the collection of all subsets of A). Prove by the method of induction that if A has n elements then $P(A)$ has 2^n elements. [4]

7. Let $A = \{x \in \mathbb{Q} : x^2 < 2\}$. Show that A has no least upper bound in \mathbb{Q} . [6]

8. Recall that a set A is said to be countable if A is a finite set or there is a bijection between \mathbb{N} and A .

Prove that every subset of a countable set is also countable. [6]