Viscosity of hadron gas

Maneesha Sushama Pradeep

School of physical sciences
Project guide-Dr.Bedangadas Mohanty
NISER

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Overview

Viscosity

- Bulk phenomenon that characterises the resistance offered to the free flow of a fluid due to the interaction between the various layers of itself
- Inverse of fluidity
- $$F_x = \eta A \frac{dv_x}{dz}$$

  In principle η a 3×3 tensor.
- Newtonian Fluids:η is a constant;eg:Water
- Non-Newtonian fluids:Stress is NOT linearly proportional to strain rate.eg:paint,cement

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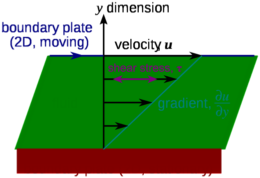
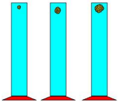
Figure: Pitch

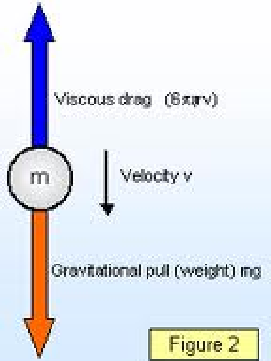
Figure: Laminar Flow of fluid between two plates



Measuring viscosity-First year lab experiment

- A spherically uniform body was dropped
- Terminal velocity assumed
- Stokes'Law,$F = 6\pi\eta r v$ where η is the viscosity, r is the radius of the sphere, v its velocity and F is the viscous force acting on it.





Temperature dependence of viscosity in liquids and gases

Liquid viscosity:

- In a liquid, every molecule has to overcome the forces due to its neighbours to move
- Inversely proportional to mobility
- Probability that a molecule has the required energy at a given temperature $\propto e^{-\frac{E_a}{RT}}$
- Mobility $\propto e^{-\frac{E_a}{RT}}$
- $\eta = Ae^{\frac{E_a}{RT}}$
- $ln\eta = lnA + \frac{E_a}{RT}$
- Liquid viscosity decreases with increase in temperature

Gas viscosity

- $\eta = \frac{1}{3}nl < |p| >$,$\eta$-viscosity,l-Mean free path,$< |p| >$-expectation value of momentum,n-number density
- $l = \frac{1}{\sqrt{2}\sigma}$,$\sigma$-Effective crossection for collisions
- $< |p| >= \sqrt{\frac{8mk_BT}{\pi}}$,m-mass of gas molecule,T-temperature of the system,k_B-Boltzmann's constant
- $\eta = \frac{2}{3}\sqrt{\frac{mk_BT}{\pi\sigma^2}}$
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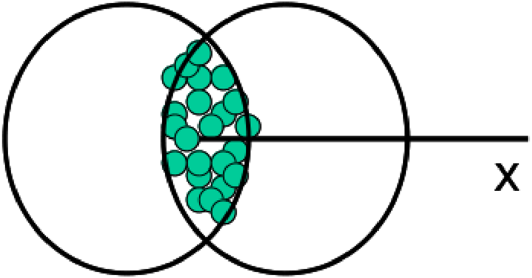
Why should we study viscosity of hadron gases

- Elliptic flow in high enery heavy ion collisions-signature of fluid like behaviour of particles formed?
- Initial anisotropy of the particles produced in position space leading to an anisotropy in the momentum space-indicating strongly interacting medium?

Figure: Non-central collision seen in the transverse plane: the overlap area, where particles are produced, is not a circle



My regime of study

- Two distinct phases:Quark Gluon plasma and Hadrons
- Two stages:Chemical freeze out and thermal freeze out
- My regime:Hadronic stage,only elastic collisions,between chemical freeze out and thermal freeze out

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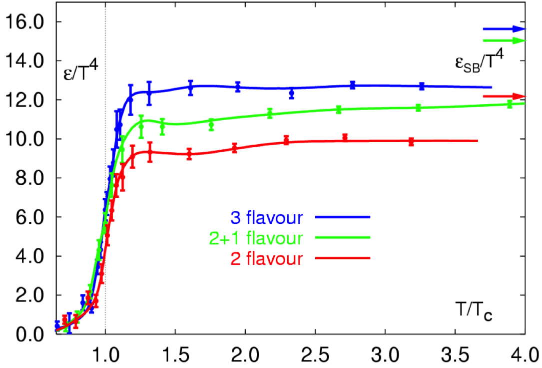
Figure: $\frac{\epsilon}{T^4} V s \frac{T-T_c}{T_c}$



A system of ideal hadron gases

- Baryons and mesons, composite relativistic particles
- Only interaction through elastic collisions which are instantaneous
- Particles are point-like, volume occupied much less compared to distance between them

Figure: Proton

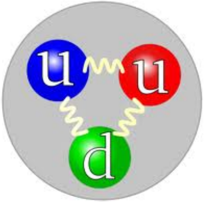
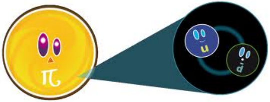


Figure: Pion



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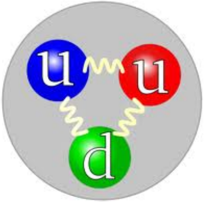

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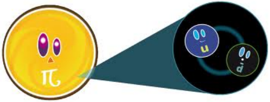

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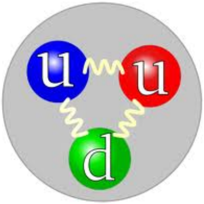

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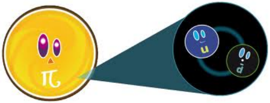

A System of Volume excluded Vanderwaal gases

Particles have a finite radius, two particles cannot occupy the same volume.

Figure: Excluded Volume Vanderwaal Gas

— EXCLUDED VOLUME —

$$\text{Excluded vol per molecule} = \frac{1}{2}\left(\tfrac{4}{3}\pi\sigma^3\right)$$

$$\text{Actual vol of a molecule} = \tfrac{4}{3}\pi\left(\frac{\sigma}{2}\right)^3 = \frac{1}{8}\left(\tfrac{4}{3}\pi\sigma^3\right)$$

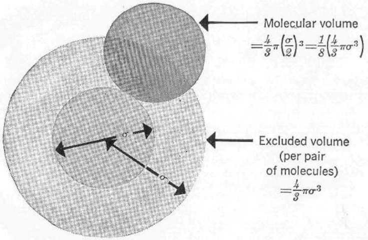

Fig. 2-8. The excluded volume (light shade) for a pair of molecules according to van der Waals' treatment.

Viscosity of a system of relativistic gases

- $\eta \propto nl < |p| >$
- $< |p| >= \frac{\int_0^\infty p^3 n(p)dp}{\int_0^\infty p^2 n(p)dp}$ where $n(p)$ is the average number of particles having momentum p
- For a system of identical particles,

$$\eta = \frac{5}{64\sqrt{\pi}} \frac{\sqrt{mT}}{r^2} \frac{K_{\frac{5}{2}}(\frac{m}{T})}{K_2(\frac{m}{T})}$$

where K_ν is the modified Bessel function of order ν.

- For a system of x different hadrons(assuming same radius),

$$\eta = \frac{5}{64\sqrt{\pi}} \frac{\sqrt{T}}{r^2} \Sigma_{i=0}^{x} \sqrt{m_i} \frac{K_{\frac{5}{2}}(\frac{m_i}{T})}{K_2(\frac{m_i}{T})}$$

where m_i is the mass of the i^{th} hadron.

Why $\frac{\eta}{s}$(Ratio of viscosity to entropy density)?

- Analogous to kinematic viscosity
- Entropy density is a factor times number density
- Dimensionless quantity
- This quantity facillitates a better comparison of the viscosities of two isentropic samples.

Finding entropy density of a system of volume excluded relativistic hadrons at a temperature T

- Aid of statistical mechanics
- Which ensemble to choose?
- Grand canonical pressure ensemble-Pressure, temperature and chemical potential are the conserved quantities
- $\mu_i = b_i\mu_B + s_i\mu_S + q_i\mu_Q$, $\mu_B >> \mu_Q, \mu_S, \mu_Q \approx \mu_S \approx 0$
- Determining the appropriate statistics
- Calculate entropy density using the commonly used expressions in Statistical Mechanics

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Statistical Analysis

- Grand Canonical Pressure Partition function for a general system:

$$D(z,p,T) = \int_0^\infty dV e^{-Vp} Z(V,z,T)$$

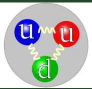
- Grand canonical pressure partition function for a system of volume excluded gases:

$$D(z,s,T) = \Sigma_{N=0}^{\infty} \frac{\phi^N}{N!} \int_{vN}^\infty dV e^{-(s(V-vN))} (V-vN)^N$$

, where ϕ is the ideal gas number density and v is the excluded volume per molecule.

- Pressure is given by the transcedental equation:

$$p = e^{-\frac{vp}{T}} T\phi$$



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- Number density:

$$n_i = \frac{e^{-\frac{vp}{T}}\phi_i}{1+ve^{-\frac{vp}{T}}\phi}$$

,where ϕ is ideal gas number density

- Entropy density:

$$s = \frac{x}{1+vx}\left(1+vx+\frac{T}{\phi}\frac{\partial\phi}{\partial T}\right)$$

, where $x = e^{-\frac{vp}{T}}T\phi$

- $s = An$,where n is the total number density of the system and $A = 1 + vx + \frac{T}{\phi}\frac{\partial\phi}{\partial T}$.

Relating with experimental data

Furthermore, the viscosity of the medium consisting of the particles produced at RHIC, SPS, AGS and SIS energies can be found out as follows:

- The experiment gives the particle densities as the observed quantity.
- These values are then fit into the expressions for particle densities obtained using statistical analysis and the temperature and chemical potential of the system produced at those energies can be computed.
- Viscosity is calculated accordingly.

Computation

- For a system of pions,

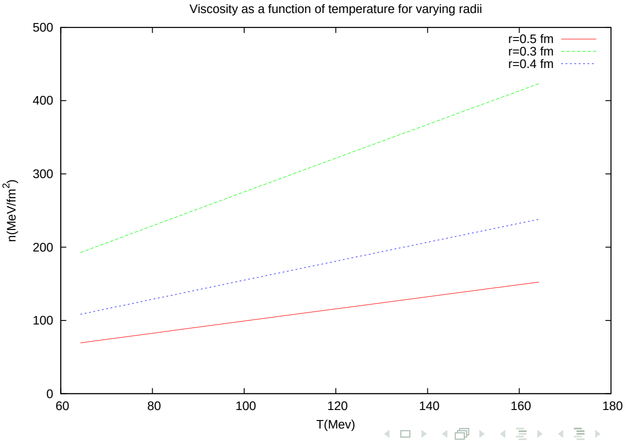
$$\eta = \frac{5}{64\sqrt{\pi}} \frac{\sqrt{mT}}{r^2} \frac{K_{\frac{5}{2}}(\frac{m}{T})}{K_2(\frac{m}{T})}$$

where K_ν is the modified Bessel function of order ν,m is the mass of pion, T is the temperature of the system.

- $s = An$,where n is the total number density of the system and $A = 1 + vx + \frac{T}{\phi}\frac{\partial\phi}{\partial T}$($\phi$ is the ideal gas number density).

Computational results for a system of pion gases

Figure: Viscosity as a function of temperature for varying radii



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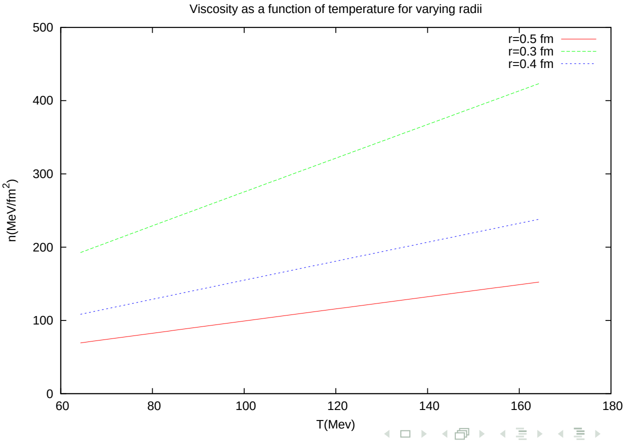

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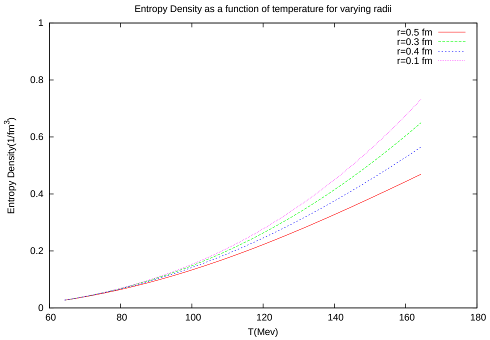
Figure: Entropy Density as a function of temperature for varying radii



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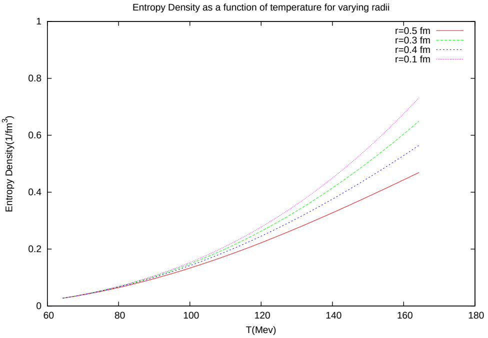
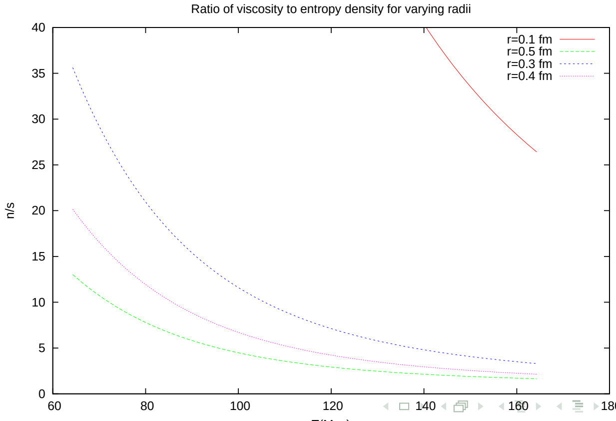


Computational results for a system of pion gases

Figure: Ratio of Viscosity Vs Entropy Density as a function of temperature for varying radii



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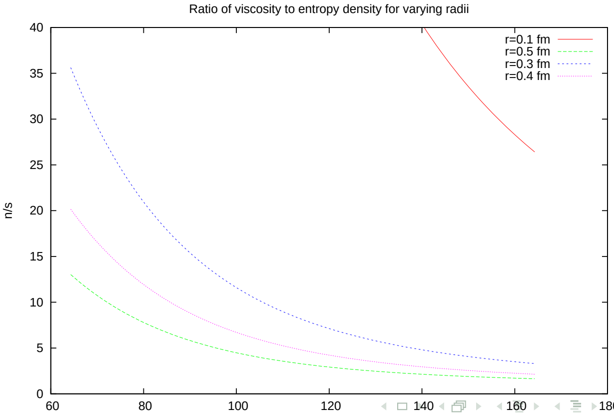
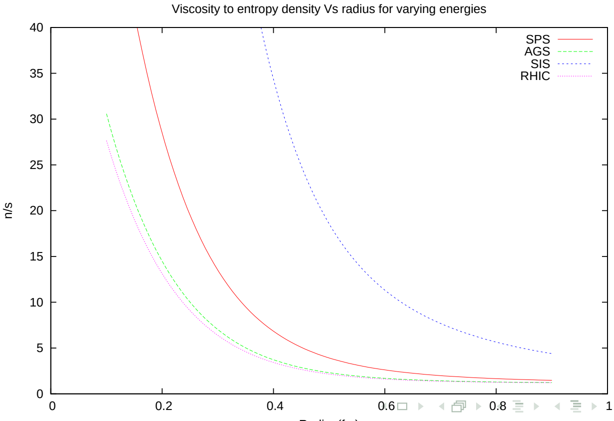


Computational results for a system of pion gases

Figure: Ratio of Viscosity Vs Entropy Density as a function of radius for varying beam energies



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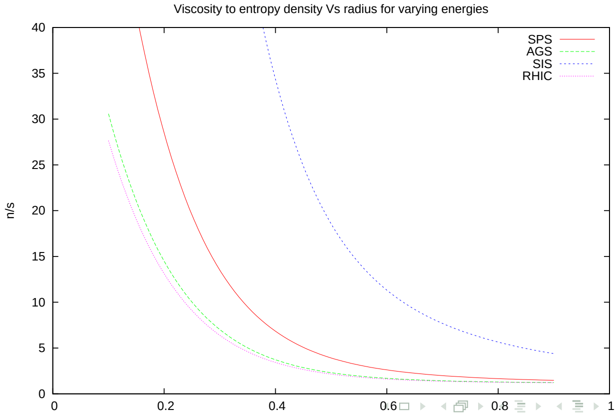
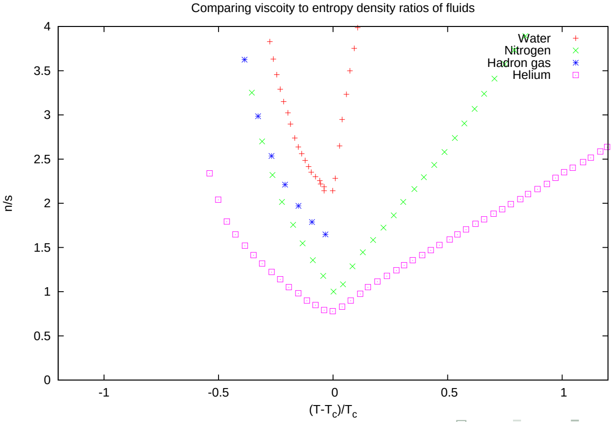


Table: Temperature and chemical potential of systems of different beam energies

	Beam energy(GeV)	T(MeV)	μ_B
SIS	2.32	64.3	800.8
AGS	4.86	116.5	562.2
SPS	17.3	154.4	228.6
RHIC	200	161.1	23.5

A comparison between viscosity measurements of He, N_2, Water and Pion gas with $r = 0.5fm$

Figure: $\frac{\eta}{s}$ Vs $\frac{T-T_c}{T_c}$ for different fluids



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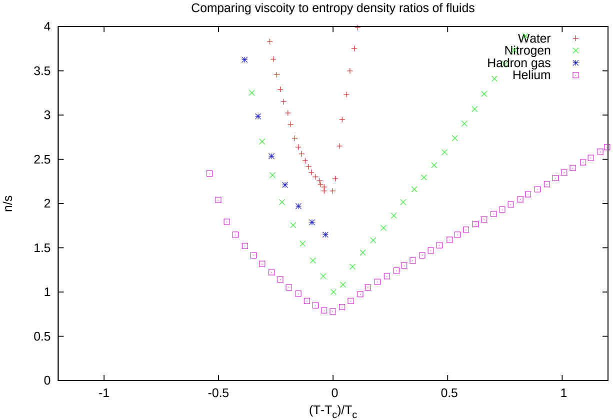

Summary

- The viscosity of the pion gas system was found to increase with rise in temperature and decrease with increase in radius.
- As the temperature increases entropy density decreases.
- The ratio of viscosity to entropy density of pion was found to decrease with temperature.
- The ratio of viscosity to entropy density was found to be of the same order as Helium, Nitrogen and Water for pion gas with radius 0.5 fm at high temperatures.

What more?

I am currently working on computationally determing the viscosity and $\frac{\eta}{s}$ of a system consisting of all the discovered and proposed hadrons which have mass less than 2 GeV. Since the experiments have shown close to a perfect liquid like behaviour, the $\frac{\eta}{s}$ expected for the system is considerably low.

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THANK YOU