Shear viscosity of a hadron gas

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Table of Contents

List of Tables

List of Figures

Chapter 1

Elementary particles

The elememtary particles of which the world is made up of can be broadly classified as gluons, leptons and quarks. Leptons are found in the three generations, they are spin 1/2 particles(fermions). They carry -e charge and each of them has a positive compliment with the same mass called antileptons. Neutrinoes are chargeless and also have corresponding antiparticles. */$\mu\nu_\mu\nu_\tau(0.18); e\nu_e\nu_\tau(0.17); \pi\nu_\tau(0.1); \rho\nu_\tau(0.22)$.

Table 1.1: Leptons

Particle	Antiparticle	Mass($\frac{MeV}{c^2}$)	Charge	Spin	Lifetime(s)	Decay modes
e^-	e^+	0.511	±1	$\frac{1}{2}$	Stable	
ν_e	$\bar{\nu_e}$	$< 0.46 * 10^{-6}$	0	$\frac{1}{2}$	Stable	
μ^-	μ^+	105.66	±1	$\frac{1}{2}$	$2.20 * 10^{-6}$	$e\nu_e\nu_\mu$
ν_μ	$\bar{\nu_\mu}$	< 0.50	0	$\frac{1}{2}$	Stable	
τ_-	τ_+	1784	±1	$\frac{1}{2}$	$3.4 * 10^{-13}$	Many Channels*
ν_τ	$\bar{\nu_\tau}$	< 164	0	$\frac{1}{2}$	Stable	

Table 1.2: Quarks

Name	Symbol	Mass(in GeV)	Isospin	Charge(e)	Lifetime(s)
Down	d	0.3	$\frac{1}{2}$	$-\frac{1}{3}$	Stable
Up	u	0.3	$\frac{1}{2}$	$\frac{2}{3}$	Stable
Strange	s	0.5	0	$-\frac{1}{3}$	$10^{-8} - 10^{-12}$
Charm	c	1.5	0	$\frac{2}{3}$	$10^{-12} - 10^{-13}$
Bottom	b	4.5	0	$-\frac{1}{3}$	$10^{-12} - 10^{-13}$
Top	t	171	0	$\frac{2}{3}$	10^{-25}

1.1 Quarks

Unlike leptons, quarks are never found in their free state. They are found confined in the hadrons. Hadrons are of two types-baryons and mesons. Baryons are composed of three quarks wheareas mesons are made up of a quark and an antiquark. There are 6 types of quarks each having a finite mass and carrying certain charge and are spin half(fermions) particles. The quarks with their properties like mass, charge, isospin, strangeness, charm, beauty and lifetimes is tabled below.

1.1.1 Baryon list

Since baryons are composed of three quarks, they can have a spin angular momentum of either $\frac{3}{2}$ or $\frac{1}{2}$. A baryon with a higher total angular momentum although having the same quark composition is considered to be a different particle as it is heavier. The strangeness of an s quark is -1 and that of its antiquark +1. The strangess of a baryon is calculted by adding the strangeness number of all its constituents. Isospin tells about the number of particles of its type with different charge. The degrees of freedom is $(2Isospin + 1)(2Spin + 1)$.

Table 1.3: Baryons

Baryon(Mass in MeV)	Spin	Strangeness	Isospin	Degrees of freedom
N(1440)	$\frac{1}{2}$	0	$\frac{1}{2}$	4
N(1990)	$\frac{7}{2}$	0	$\frac{1}{2}$	16
N(1520)	$\frac{3}{2}$	0	$\frac{1}{2}$	8
N(1680)	$\frac{5}{2}$	0	$\frac{1}{2}$	4
N(1700)	$\frac{3}{2}$	0	$\frac{1}{2}$	8
N(1710)	$\frac{1}{2}$	0	$\frac{1}{2}$	4
N(1720)	$\frac{3}{2}$	0	$\frac{1}{2}$	8
N(1860)	$\frac{5}{2}$	0	$\frac{1}{2}$	12
N(1875)	$\frac{3}{2}$	0	$\frac{1}{2}$	8
N(1900)	$\frac{3}{2}$	0	$\frac{1}{2}$	8
Δ(1232)	$\frac{3}{2}$	0	$\frac{3}{2}$	16
Δ(1600)	$\frac{3}{2}$	0	$\frac{3}{2}$	16
Δ(1620)	$\frac{1}{2}$	0	$\frac{3}{2}$	8
Δ(1700)	$\frac{1}{2}$	0	$\frac{3}{2}$	8
Δ(1750)	$\frac{1}{2}$	0	$\frac{3}{2}$	8
Δ(1900)	$\frac{1}{2}$	0	$\frac{3}{2}$	8
Δ(1905)	$\frac{5}{2}$	0	$\frac{3}{2}$	24
Δ(1910)	$\frac{1}{2}$	0	$\frac{3}{2}$	8
Δ(1920)	$\frac{3}{2}$	0	$\frac{3}{2}$	16
Δ(1930)	$\frac{5}{2}$	0	$\frac{3}{2}$	24
Δ(1940)	$\frac{3}{2}$	0	$\frac{3}{2}$	16
Δ(1950)	$\frac{7}{2}$	0	$\frac{3}{2}$	32

Baryon(Mass in MeV)	Spin	Strangeness	Isospin	Degrees of freedom
$\Lambda(1405)$	$\frac{1}{2}$	-1	0	2
$\Lambda(1520)$	$\frac{3}{2}$	-1	0	4
$\Lambda(1600)$	$\frac{1}{2}$	-1	0	2
$\Lambda(1670)$	$\frac{1}{2}$	-1	0	2
$\Lambda(1690)$	$\frac{3}{2}$	-1	0	4
$\Lambda(1800)$	$\frac{1}{2}$	-1	0	2
$\Lambda(1810)$	$\frac{1}{2}$	-1	0	2
$\Lambda(1820)$	$\frac{5}{2}$	-1	0	6
$\Lambda(1830)$	$\frac{5}{2}$	-1	0	6
$\Lambda(1890)$	$\frac{3}{2}$	-1	0	4
$\Sigma(1385)$	$\frac{3}{2}$	-1	1	12
$\Sigma(1580)$	$\frac{3}{2}$	-1	1	12
$\Sigma(1620)$	$\frac{1}{2}$	-1	1	6
$\Sigma(1660)$	$\frac{1}{2}$	-1	1	6
$\Sigma(1670)$	$\frac{3}{2}$	-1	1	12
$\Sigma(1750)$	$\frac{1}{2}$	-1	1	6
$\Sigma(1770)$	$\frac{1}{2}$	-1	1	6
$\Sigma(1775)$	$\frac{5}{2}$	-1	1	18
$\Sigma(1840)$	$\frac{3}{2}$	-1	1	12
$\Sigma(1880)$	$\frac{1}{2}$	-1	1	6
$\Sigma(1915)$	$\frac{5}{2}$	-1	1	18
$\Sigma(1940)$	$\frac{3}{2}$	-1	1	12
$\Xi(1530)$	$\frac{3}{2}$	-2	$\frac{1}{2}$	8
$\Xi(1820)$	$\frac{3}{2}$	-2	$\frac{1}{2}$	8

1.1.2 Meson list

The following table shows the eight combinations of mesons(u,d and s) and their properties. The mesons have either spin 0 or spin 1. The mesons having spin 1 are represented by the same symbol as its spin 0 meson but with an asterix superscript to denote excited state.

Table 1.4: Mesons

Meson(Mass in MeV)	Tot.ang.momentum	Strangeness	Isospin	Degrees of freedom
$f_0(500)$	0	0	0	1
$f_0(980)$	0	0	0	1
$f_0(1370)$	0	0	0	1
$f_0(1500)$	0	0	0	1
$f_0(1710)$	0	0	0	1
$f_1(1285)$	1	0	0	3
$f_1(1420)$	1	0	0	3
$f_1(1510)$	1	0	0	3
$f_2(1270)$	2	0	0	5
$f_2(1430)$	2	0	0	5
$f_2'(1525)$	2	0	0	5
$f_2(1565)$	2	0	0	5
$f_2(1640)$	2	0	0	5
$f_2(1810)$	2	0	0	5
$f_2(1910)$	2	0	0	5
$f_2(1950)$	2	0	0	5
$b_1(1235)$	1	0	1	9
$a_0(980)$	0	0	1	3
$a_0(1450)$	0	0	1	3
$a_1(1260)$	1	0	1	9
$a_1(1640)$	1	0	1	9
$a_2(1320)$	2	0	1	15
$a_2(1700)$	2	0	1	15
$\Pi(1300)$	0	0	1	3
$\Pi(1800)$	0	0	1	3
$\Pi_1(1400)$	1	0	1	9
$\Pi_1(1600)$	1	0	1	9

Meson(mass in MeV)	Tot.ang.momentum	Strangeness	Isospin	Degrees of freedom
$\Pi_2(1670)$	2	0	1	15
$\Pi_2(1880)$	2	0	1	15
$\eta(1405)$	0	0	0	1
$\eta(1295)$	0	0	0	1
$\eta(1475)$	0	0	0	1
$\eta(1760)$	0	0	0	1
$\eta_2(1645)$	2	0	0	5
$\eta_2(1870)$	2	0	0	5
$\rho(1450)$	1	0	1	9
$\rho(1570)$	1	0	1	9
$\rho(1700)$	1	0	1	9
$\rho(1900)$	1	0	1	9
$\rho_3(1690)$	3	0	1	21
$\rho_3(1990)$	3	0	1	21
$h_1(1595)$	1	0	0	3
$\Omega(1650)$	1	0	0	3
$\Omega_3(1670)$	3	0	0	7
$\Phi(1680)$	1	0	0	3
$\Phi_3(1850)$	3	0	0	7
K_s^0	0	±1	$\frac{1}{2}$	2
K_l^0	0	±1	$\frac{1}{2}$	2
$K_0^*(100)$	0	±1	$\frac{1}{2}$	2
$K^*(892)$	1	±1	$\frac{1}{2}$	6
$K_1(1270)$	1	±1	$\frac{1}{2}$	6
$K_1(1470)$	1	±1	$\frac{1}{2}$	6
$K^*(1410)$	1	±1	$\frac{1}{2}$	6
$K_0^*(1430)$	0	±1	$\frac{1}{2}$	2
$K_0^*(1950)$	0	±1	$\frac{1}{2}$	2
$K_2^*(1430)$	2	±1	$\frac{1}{2}$	10
$K(1460)$	0	±1	$\frac{1}{2}$	2
$K_2(1580)$	2	±1	$\frac{1}{2}$	10
$K_2(1770)$	2	±1	$\frac{1}{2}$	10
$K_3^*(1430)$	3	±1	$\frac{1}{2}$	14
$K^*(1680)$	1	±1	$\frac{1}{2}$	6
$K_3^*(1780)$	3	±1	$\frac{1}{2}$	14
$K_2(1820)$	2	±1	$\frac{1}{2}$	10
$K_3(1830)$	0	±1	$\frac{1}{2}$	2

1.2 Gluons

The last of the elementary particles gluons are of eight types and are massless particles and act as mediators for the interactions between quarks and leptons. They are spin-1 particles.

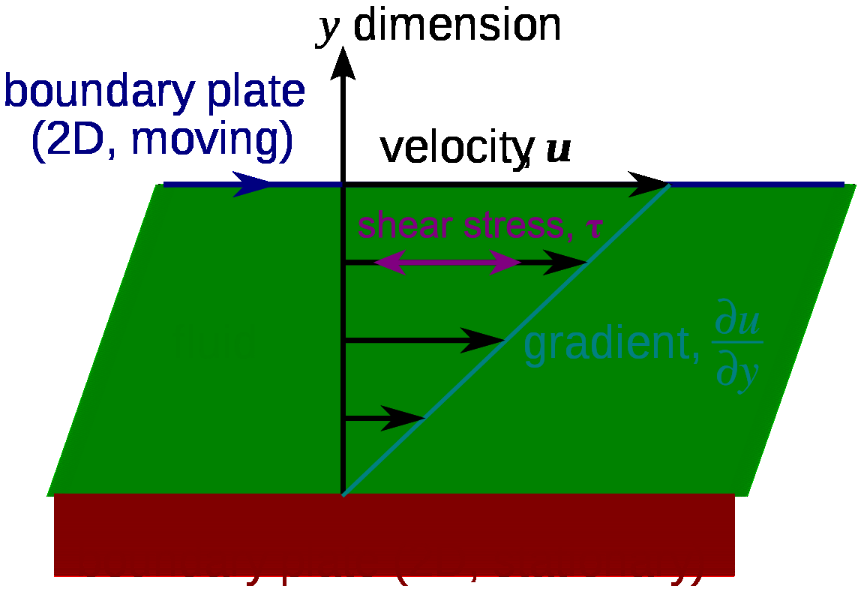
Chapter 2

Viscosity

2.1 An introduction to viscosity

A many body system which shows a collective behaviour of movement or flow is called a fluid. For such a system, the mean free path of the molecules is negligible when compared to the system size. Viscosity or the internal resistance is a relevant parameter in the study of fluids. It can be concieved as the inverse of fluidity.The shear viscosity of a fluid expresses its resistance to shearing flows, where adjacent layers move over one another with different speeds. Consider a layer of fluid trapped between to parallel plates. when the top plate is moved, the layer just below it would move with it. Ideally, if there were no internal friction, the layer just below it should have moved along with the top layer with the same velocity. But this doesnot occur in realistic fluids. The drag force due to the layers below lead to a velocity gradient in the direction perpendicular to applied shearing force. This internal friction is called the viscous force and viscosity is a measure of this friction in a fluid.

Figure 2.1: Laminar Shear of fluid between two plates



$$\frac{\partial u}{\partial y} = \frac{\partial(\frac{\partial x}{\partial t})}{\partial y} \tag{2.1}$$

$$= \frac{\partial(\frac{\partial x}{\partial y})}{\partial t} \tag{2.2}$$

$$\tag{2.3}$$

Velocity Gradient=Tangential strain rate

In Newtonian fluids, the shear stress is proportional to the strain rate and the coefficient of proportionality is called viscosity.

$$F_x = \eta_{xz} A \frac{dv_x}{dz} \tag{2.4}$$

$$\tag{2.5}$$

In principle η can be a 3×3 tensor of rank 2.

2.2 An analysis of temperature dependence of viscosity in liquids and gases

2.2.1 Viscosity in liquids

The two broad classification amongst fluids, liquids and gases, show differences in their viscous behaviour. The underlying reason lies in the difference in the origin of this internal resistance in both of them. In case of liquids, the molecules are quite closely packed.There exists a frictional force between the layers of the liquid which oppose any motion of one over the other. Therefore in order to move the molecules of a layer should have a minimum energy(say E_A) to overcome this frictional force. At any temperature T, the probability that a molecule has energy E_a is $e^{-\frac{E_a}{RT}}$. Hence viscosity should be inversely proportional to this term.

$$\eta = Ae^{\frac{E_a}{RT}} \tag{2.6}$$

$$ln\eta = lnA + \frac{E_a}{RT} \tag{2.7}$$

$$\tag{2.8}$$

Therefore, viscosity of a liquid decreases with an increase in temperature.

2.2.2 Viscosity in Gases

In case of gases, the molecules are so separated from each other that its not possible to attribute the frictional forces to the observed velocity gradient in the system. The only means for a momentum transfer between two layers of molecules is due to intermolecular collisions. In the process of a collision the motion of the molecules in one layer can be deflected such that now they might happen to undergo a collision with some molecule in another layer, thereby imparting some momentum to it. The layers between which the momentum transfer is discussed therefore, in this case would be those placed a mean free path apart. Since we expect higher velocity particles to impart more momentum on collisions, viscosity which is a ratio of stress per strain rate should be directly proportional to velocity and mean free path for the reason explained in the previous statement. It should also be proportional to the number density of particles.

$$
\begin{aligned}
\eta &= \frac{F_x}{A\frac{dv_x}{dz}} && (2.9)\\
&= \frac{\frac{d}{dt}(mv_x)}{A\frac{dv_x}{dz}} && (2.10)\\
&= \frac{v_x\frac{d}{dt}(\rho V)}{A\frac{v_x}{\lambda}} && (2.11)\\
&= \frac{v_x\rho\frac{dz}{dt}}{\frac{v_x}{\lambda}} && (2.12)\\
&= \rho v_z\lambda && (2.13)\\
&= \frac{1}{3}\rho v\lambda && (2.14)\\
&= \frac{1}{3}n < |p| > \lambda && (2.15)
\end{aligned}
$$

$$\eta \propto n\lambda < |p| > \quad (2.16)$$

$$(2.17)$$

F_x is the shear force in the x direction on the molecules,v is the average velocity of the system, m is the mass of a molecule,ρ is the mass density of the system.n refers to number density,$< |p| >$ stands for average momentum of gas and λ refers to mean free path of the gas molecule.

The average momentum of a gas in a Maxwell Boltzmanian Ensemble($< |p| >$) is $\sqrt{\frac{8MT}{\pi}}$. assuming the gas to have the same velocity in all directions, momentum in x direction is one third of this value.

The mean free path of an ideal gas can be derived as follows. Let us consider one of the molecules to be of radius r. If it travels a distance vt in time t, assuming the colliding molecules to be point particles and the number of molecules that our candidate molecule can collide with is proportional to the volume it has travelled,the mean free path would be given as $\frac{vt}{vt\pi r^2 n}$ where n is the number of molecules per unit volume. Here we have assumed our candidate molecule to be a hard rigid sphere whereas the other colliding molecules to be point particles, along with the assumption that its velocity doesn't change after each collision which is generally not true.So the viscosity of a gas doesn't depend on its number density. Since velocity has a $T^{\frac{1}{2}}$ dependence, viscosity of a gas increases with temperature.

In a gas of particles having a radius 'r', the effective collision area around a molecule is $4\pi r^2$. Assuming the target molecules to be point masses, the mean free path can be calculated as follows.

Distance covered by the molecule in time t=vt

Effective volume swept by the molecule in time t=Velocity of the molecule with respect to target molecules $\times$ t $\times 4\pi r^2$

Since particles move in random directions with nearly identical velocity, relative velocity of the molecule= $\sqrt{2}v$.

Effective volume swept by the molecule in time t= $4\sqrt{2}r^2vt$

Number of target molecules in this volume= $n * 4\sqrt{2}r^2vt$ where n stands for number density.

Distance between two consecutive collisions(λ)=vt$\frac{}{4\sqrt{2}nr^2vt}$= $\frac{1}{4\sqrt{2}nr^2}$.

$$\eta = \frac{1}{6r^2}\sqrt{\frac{MT}{\pi}}$$

Table 2.1: Viscosity of common fluids

Substance	Viscosity (Pa s)
Air (at 18 oC)	1.9×10^{-5}
Water (at 20 oC)	1×10^{-3}
Canola Oil at room temp.	0.1
Motor Oil at room temp.	1
Corn syrup at room temp.	8
Pahoehoe lava	100 to 1,000
A'a lava	1000 to 10,000
Andesite lava	$10^6 to 10^7$
Rhyolite lava	$10^{11} to 10^{12}$

Chapter 3

High energy heavy ion collisions

In high energy heavy ion collisions, nuclei with large atomic number are stripped off their electrons and are made to collide at high energies. Following the laws of conservation of energy and momentum, a large number of different hadrons are produced as an outcome. Hadrons are composed of quarks,antiquarks and gluons which are confined within them. At low temperatures, the a hadron gas behaves as an ideal gas, when T increases to nearly 100 MeV pion creation sets in and with furthere temperature rise, the ratio of other hadrons in the system also starts increasing. At temperatures, close to 170 Mev the hadrons come together to form a strongly interacting state of matter called quark-gluon plasma. This exotic phase transition has been observed through the following signatures. The energy density of the system is found to saturate at this stage.

Experimentally, the ratio of viscosity to entropy density ($\frac{\eta}{s}$) is found to have a minimum at this temperature.

My regime of study would be confined to the hadron phase where the particles would be treated as hard rigid spheres which interact among themselves only through elastic collisions. A statistical model of the system would be studied.

Figure 3.1: Energy density as a function of temperature

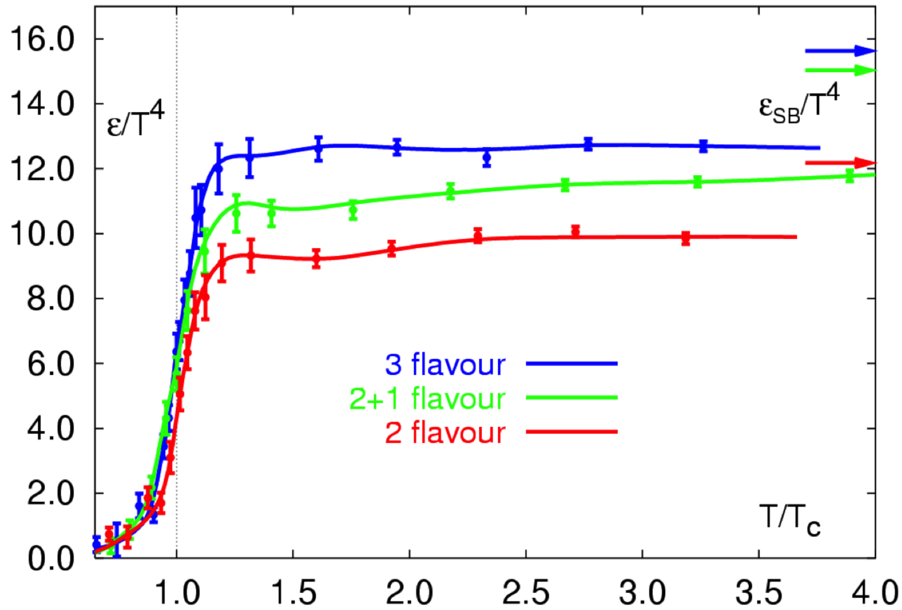
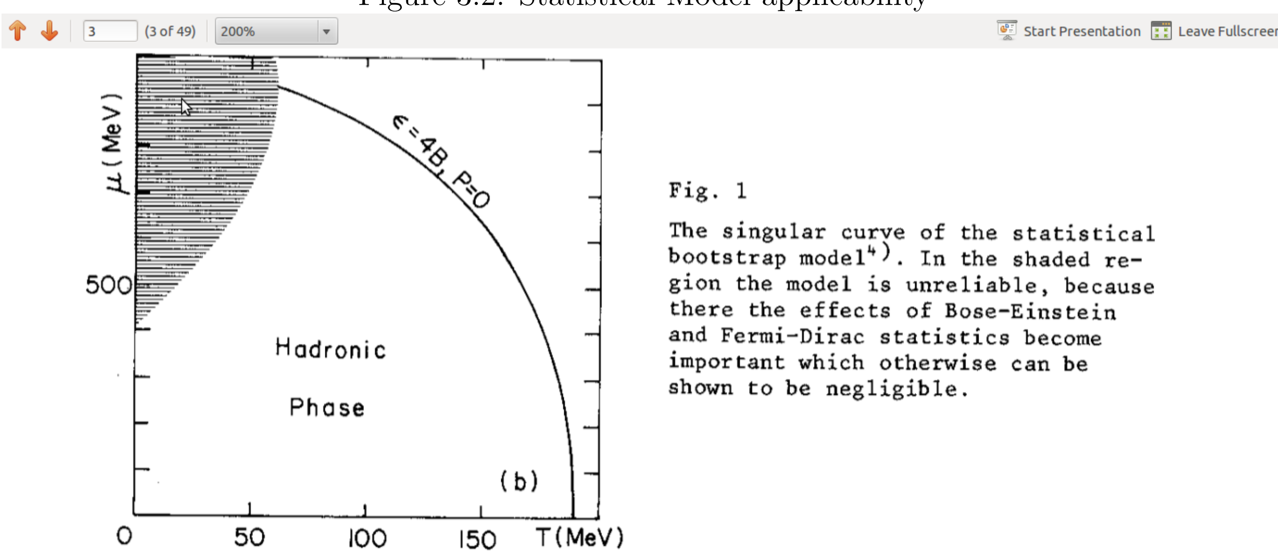


Figure 3.2: Statistical Model applicability



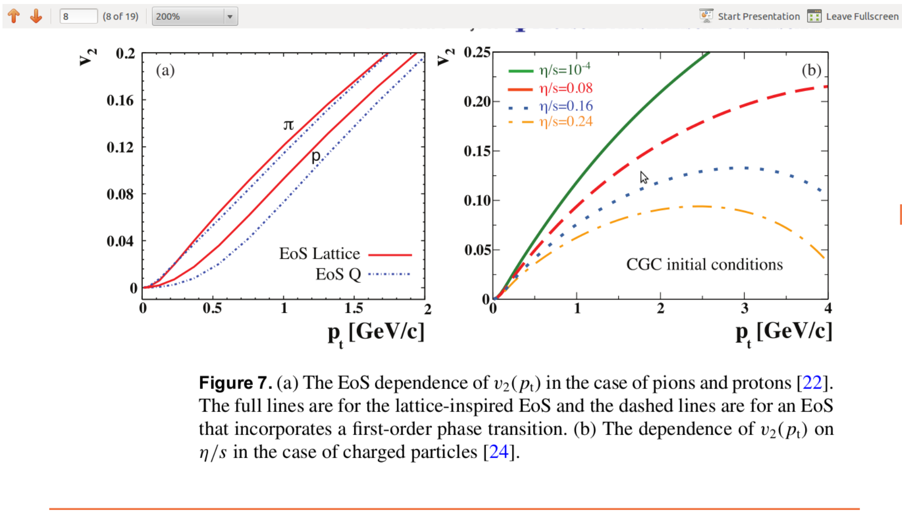
3.1 Motivation to study about viscosity of hadron gas

Heavy ions are particles with large surface area. When the impact factor(b) is more than zero, the centrality of the collision is given by $\frac{\pi b^2}{2\pi R_A^2}$ with R_A being the nuclear radius. The non-centrality of the collision makes the initial geometry of the participating nucleons in the transverse plane to be elliptical. Assuming complete participation in the overlap region, the initial geometry of the particles produced would also be elliptical in this plane. If the particles were non-interacting, this ellipse would have retained its form over time. On the other hand, if they were interacting the pressure gradient along the minor axis of the ellipse would be more(assuming particle production was uniform throughout the ellipse). Therefore, as Euler equation suggests, the particles would be accelerated more towards this direction(anisotropy in momentum space). This should eventually lead to a circular distribution of the particles at equilibrium. This gradual deformation is governed by the strength of the interaction forces and hence will be quicker for a strongly interacting system. This is known as elliptic flow.

Therefore a measure of the eccentricity of the geometry at a certain stage of the process would tell us about the viscosity of the medium produced. The lesser the eccentricity, more is the viscosity. Theoretically, the minor axis of the initial geometry is assigned as the x axis and $v_2 =< cos(2\phi) >$, where ϕ is the angle made by the postion vector of a particle with the x axis is recorded. This value is then averaged over all particles and all events. A zero value would mean infinite viscosity and any non-zero value would mean a finite viscosity. Experiments have shown a non-zero value implying finite viscosity and therefore a fluid(liquid) like behaviour.

For a much more organised way of researching, the physicists have coined two terms to classify different stages of the process. After the collision, at a certain stage it is found that the number of particles produced no more changes and the inelastic processes begin to cease. This stage is called 'chemical freeze-out'(chemical composition is frozen). A latter stage is the 'thermal freeze-out' when the interactions between the particles terminate and the elastic processes also come to a halt(momentum distribution of the system is frozen). The primary objective of my work would be to find the viscosity of a system lying in the regime between chemical freeze out and the thermal freeze out where I would be treating them as ideal hadron gases interacting only through elastic collisions. The dependence of viscosity on temperature,baryon

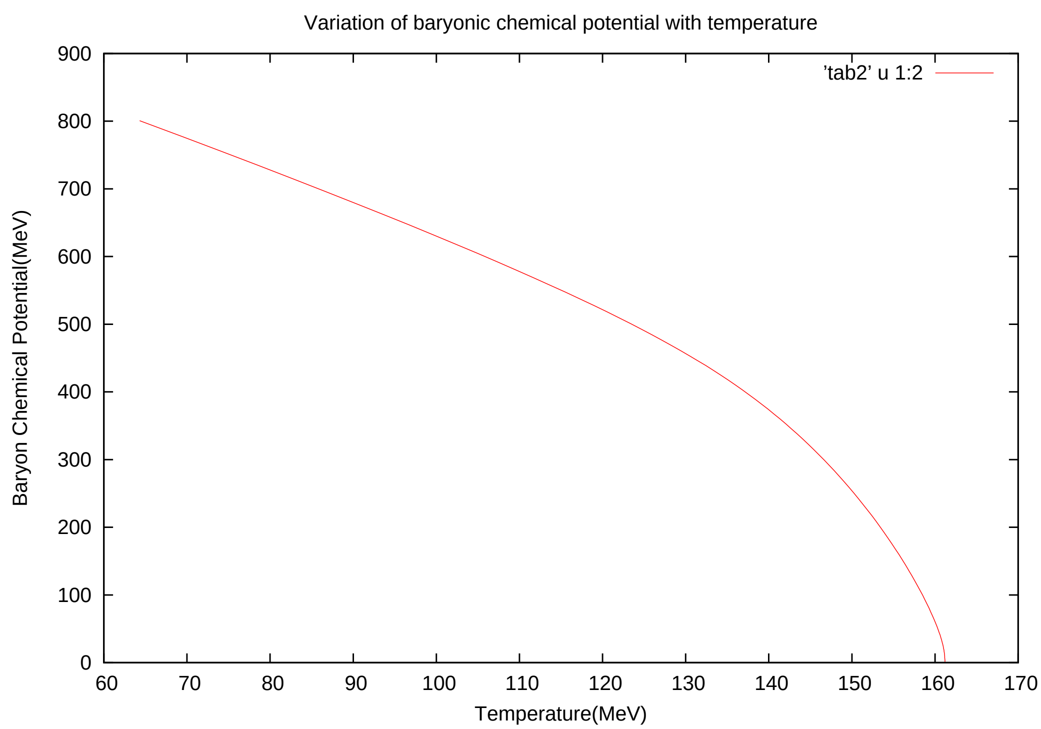
Figure 3.3: v_2 as a function of p_t for different $\frac{\eta}{s}$



chemical potential and centre of mass energies would be studied. Furthermore, the relevance of a more significant quantity, namely the ratio between the viscosity and the entropy density would also be studied.

The model considered demands the particle ratios at different temperature and chemical potential as inputs which is obtained by fitting the observed particle ratios at the given centre of mass energy in the thermal model.Since the baryonic chemical potential is very large when compared to strangeness chemical potential and charge chemical potential, the latter two are neglected. The following is a plot of baryonic potential as function of temperature.

Figure 3.4: Chemical potenial as a function of temperature



Chapter 4

Statitsical Analysis of ideal relativistic hadron gases

4.1 An Overview

A statistical description of an ideal relativistic gas would give us an estimate of the expectation values of the physical quantities like number density, energy density, pressure and entropy density of a system in equillibrium with a large reservoir. A microcanonical ensemble is employed for description when the energy, temperature, volume and the number of particles in the system is fixed. When the system can exchange energy with the reservoir, a canonical ensemble is used. In cases where there is an exchange of particles as well a grand canonical ensemble is chosen which has chemical potential, temperature and volume as its conserved quantities.

In the following chapter I will be listing the details of the properties of a system of ideal gases studied statistically. The constituents are called ideal gases as they exhibit the following properties:

1. The collision times between them are negligibly small and this is the only means of interaction amongst them.
2. They are treated as point particles of fixed mass.

3. The collisions between them are purely elastic.
4. Any number of particles can be accomodated in a finite volume(implied from 2).

The system we are considering is an identical indistinguishable many particle system. Classically, this indistinguishability has to be artificially introduced into the treatment, whereas the indistinguishbility of of a system of two non-interacting particles in quantum mechanics is reflected naturally in its wave function. Depending on the spin, the particles are classified as either fermions(half odd integer spin) or bosons(integer spin). The Pauli exclusion principle forbades two fermions from having the same set of quantum numbers. This brings a distinction in the distribution function applicable for fermions and bosons. The classical treatment with an additional condition to account for indistinguishability is called Maxwell Boltzmann distribution.

Since my motivation for studying this system is to deal with such a system produced in high energy collisions where large number of particles of varying energies are invloved, I will be discussing here about the canonical and grand canonical ensemble, in particular. Let us consider the Maxwell Boltzmann description fo a system of ideal relativistic gases.The canonical partition function is given by:

$$
\begin{aligned}
Q(V,T,N) &= (\Sigma_{\epsilon} e^{-\beta\epsilon})^N && (4.1)\\
&= (\frac{4}{h^3}\int_0^{\infty} e^{-\beta(p^2+m^2)^{\frac{1}{2}}} p^2 dp)^N && (4.2)\\
& && (4.3)
\end{aligned}
$$

The grand canonical partition function is given by:

$$\Sigma_{N=0}^{\infty}\frac{1}{N!}(z\Sigma e^{-\beta\epsilon})^N = e^{z\Sigma_{\epsilon} e^{-\beta\epsilon}}$$

$$lnD(V,z,T) = zQ_1(V,T)$$

For a relativistic gas molecule, the canonical partition function is given by:

$$Q_1 V, T = \frac{4\pi V}{h^3} \int_0^\infty e^{-\frac{m}{T}(p^2+m^2)^{\frac{1}{2}}} p^2 dp$$

Evaluating the above integral;

$$\begin{aligned}
\int_0^\infty e^{-\frac{m}{T}(p^2+m^2)^{\frac{1}{2}}} p^2 dp &= m^3 \int_0^\infty e^{-\frac{m}{T} coshx} sinh^2 x coshx dx && (4.4)\\
&= \frac{m^3}{4}[\int_0^\infty e^{-\frac{m}{T} coshx} cosh3x dx - \int_0^\infty e^{-\frac{m}{T} coshx} coshx dx] && (4.5)\\
&= \frac{m^3}{4}[K_3(\frac{m}{T}) - K_1(\frac{m}{T}) && (4.6)\\
&= m^2 T K_2(\frac{m}{T}) && (4.7)\\
& && (4.8)
\end{aligned}$$

$$Q_1(V,T) = \frac{m^2 T 4\pi V}{h^3} K_2(\frac{m}{T})$$

The Bose-Einstein Grand Canonical Partition function is given by:$\Pi_\epsilon \frac{1}{1-ze^{-\beta\epsilon}}$ where $ze^{-\beta\epsilon} << 1$.

The Fermi Dirac grand canonical partition function is given as:$\Pi_\epsilon 1 + ze^{\beta\epsilon}$.

4.2 Estimating physical quantities

The number density of the system, $< n >= \frac{1}{V}\Sigma_\epsilon \frac{1}{z^{-1}e^{\beta\epsilon}+a}$, where value of a is 0,1 and -1 for MB, FD and BE statistics respectively. Let us first find out the number density for a MB gas.

$$< n > = \frac{4\pi}{h^3}\int_0^\infty ze^{-\beta(p^2+m^2)^{\frac{1}{2}}}p^2dp \tag{4.9}$$
$$= \frac{m^4\pi z}{4h^3T}K_2(\frac{m}{T}) \tag{4.10}$$
$$\tag{4.11}$$

Therefore,

$$D(z,V,T) = \Sigma_{N=0}^\infty \frac{1}{N!}(V < n >)^N \tag{4.12}$$
$$\tag{4.13}$$

The number density for the other distributions have to be numerically determined.

The pressure of the system can be calculated from the integral:

$$P = \frac{kT}{a}\int_0^\infty ln[1+aze^{-\beta\epsilon}]\frac{4\pi p^2}{h^3}dp \tag{4.14}$$
$$= \frac{4\pi kT}{ah^3}[\frac{p^3}{3}ln[1+aze^{-\beta\epsilon}]|_0^\infty + \int_0^\infty p^3\frac{aze^{-\beta\epsilon}}{1+aze^{-\beta\epsilon}}\beta\frac{d\epsilon}{dp}dp] \tag{4.15}$$
$$\tag{4.16}$$

The entropy density follows from the Maxwell's relations and is given by $\frac{P}{T}$.

Chapter 5

Excluded hadron gas model

5.1 Excluded Volume van der Waals gas

An extension to the ideal gas picture based on the van der Waals(VDW) excluded volume procedure is suggested to phenomenologically take into account the repulsive interactions between hadrons. For a two identical particle system, the presence of the other molecule restricts the volume available for it. The volume restricted for a pair of molecules is $\frac{4}{3}\pi(2r)^3$. Therefore, excluded volume for one molecule is $4 * \frac{4}{3}\pi r^3$.

5.2 Statistical analysis of an excluded volume VDW hadron gas

The canonical ensemble treatment requires local conservation of physical quantities like strangeness, charge and baryon number. Since this need not be the case in actual high energy collisions, canonical ensemble is not preferred for describing a hadron gas system. To add to it, as the number of particles produced can be considerably large the grand canonical ensemble is a better

Figure 5.1: Excluded Volume Vanderwaal Gas

— EXCLUDED VOLUME —

$$\text{Excluded vol per molecule} = \frac{1}{2}\left(\tfrac{4}{3}\pi\sigma^3\right)$$

$$\text{Actual vol of a molecule} = \tfrac{4}{3}\pi\left(\frac{\sigma}{2}\right)^3 = \frac{1}{8}\left(\tfrac{4}{3}\pi\sigma^3\right)$$

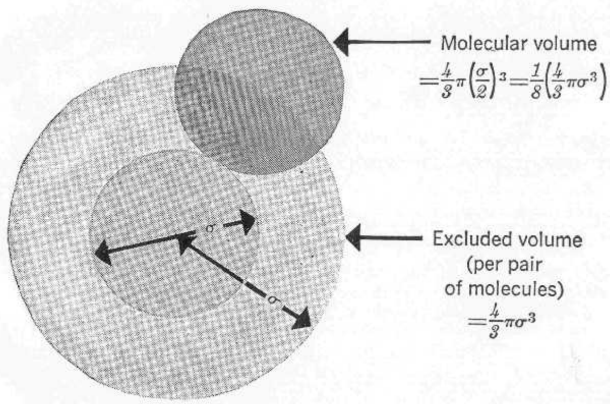


Fig. 2-8. The excluded volume (light shade) for a pair of molecules according to van der Waals' treatment.

choice. However, the additional requirement of considering the non zero radius of the hadron makes it difficult for the incorporating a grand canonical ensemble framework. This makes the number of molecules that can be accomodated in a finite volume bounded which is incompatible with the propositions of the grand canonical ensemble. Furthermore, a momentary glance at the physical phenomenon would tell us that in fact during the process, we donot confine the particles to a fixed volume, the particles are produced and as they increase in number, the volume expands alongwith. Its apparently the pressure that is preserved. Therefore, we chose the grand canonical pressure partition function to describe our system. This function would involve an integration over all volumes also.

5.3 Grand Canonical Pressure Partition function

The general grand canonical pressure partition function is given by:

$$K(P,z,T) = \int_0^\infty dV e^{-PV} D(z,V,T) \tag{5.1}$$

$$= \int_0^\infty dV e^{-V(P+\frac{lnD(z,V,T))}{V}} \tag{5.2}$$

$$\tag{5.3}$$

where D(z,V,T) is the Grand canonical partition function.Its noteworthy to mention that a singularity exists for the function when $P = \lim_{V\to\infty} \frac{D(z,V,T)}{V}$.

For a gas of excluded volume VDW hadron, the grand canonical pressure partition function is given by:

$$K(s,z,T) = \int_{v_jN}^\infty dV e^{-sV} D(z_j,V,T) \tag{5.4}$$

$$= \Sigma_{N=0}^\infty \frac{< n_j >^N}{N!} \int_{v_jN}^\infty dV e^{-(s(V-v_jN))} (V - v_jN)^N \tag{5.5}$$

$$\tag{5.6}$$

Evaluating the above integral:

$$\int_{v_jN}^{\infty} dV e^{-(s(V-v_jN))}(V-v_jN)^N = \frac{-1}{s}e^{-(V-v_jN)s}(V-v_jN)^N|_{v_jN}^{\infty} + \int_{v_jN}^{\infty} dV \frac{N}{s}e^{-Vs}(V-v_jN)^{N-1} \tag{5.7}$$

$$= \frac{N!e^{-v_jN}}{s^{N+1}} \tag{5.8}$$

$$\tag{5.9}$$

$$K(s,z,T) = \tfrac{1}{s} \quad \Sigma_{N=0}^{\infty}\frac{(< n_j > e^{-v_js})^N}{s^N} \tag{5.10}$$

$$= \frac{1}{s}\frac{1}{1-\frac{<n_j>e^{-v_js}}{s}} \tag{5.11}$$

$$= \frac{1}{s-e^{-v_js} < n_j >} \tag{5.12}$$

5.3.1 Pressure in the system

Pressure=$\lim_{V\to\infty} T\frac{lnD(V,z,T)}{V}$=$Ts^*$ where s* is the pole singularity of the grand canonical partiition function. Hence, the pressure can be found to obey a transcedental equation given by:

$$p = e^{-vp/T}T\phi \tag{5.14}$$

$$\tag{5.15}$$

where $\phi = \Sigma_i\phi_i$(Here on ϕj means ideal gas number density of j^{th} gas and $\phi = \Sigma_j\phi_j$.n would be used for denoting the modified number density.) is the total ideal hadron gas number density derived above.

5.3.2 Number density

$$n_i = \frac{\partial p}{\partial \mu_i}$$

Taking the derivative of the transcedental equation for pressure with respect to μ_i on both sides,

$$\frac{\partial p}{\partial \mu_i} = -v\phi e^{\frac{-vp}{T}} \frac{\partial p}{\partial \mu_i} + e^{\frac{-vp}{T}} \phi_i$$

Let $x_i = e^{\frac{-vp}{T}} \phi_i$ and $x = \Sigma_i x_i$, then

$$n_i = \frac{x_i}{1 + vx}$$

5.3.3 Entropy density

$$s = \frac{\partial p}{\partial T}$$

Taking the derivative of the transcedental equation for pressure with respect to temperature on both sides,

$$\frac{\partial p}{\partial T} = p\frac{v}{T} e^{-\frac{vp}{T}} \phi - ve^{-\frac{vp}{T}} \phi \frac{\partial p}{\partial T} + e^{-\frac{vp}{T}} \phi + e^{-\frac{vp}{T}} T \frac{\partial \phi}{\partial T}$$

Since $p = e^{-\frac{vp}{T}} \phi T$ and retaining the definition for x,

$$s = \frac{x}{1 + vx}(1 + vx + \frac{T}{\phi} \frac{\partial \phi}{\partial T})$$

(5.15)

$s = An$,where n is the total number density of the system and $A = 1 + vx + \frac{T}{\phi}\frac{\partial \phi}{\partial T}$.As mentioned earlier entropy density is a multiplicative factor times number density.

Chapter 6

Viscosity of the hadron gas

The viscosity a system of hadrons can be determined in a manner similar to the one carried out for ordinary gases. Viscosity is assumed to depend on number density, mean free path and average momentum of the distribution. The only difference lies in the fact that the system is relativistic and the particles are either fermions or bosons. Depending on the nature of particles, Fermi-Dirac, Bose-Einstein or Maxwell Boltzmann distribution may be chosen.

6.1 Deriving an expression for viscosity of hadron gas

Average momentum of the system=$\frac{\Sigma_p p\times Number of particles with momentum'p'}{Total number of particles}$.

For a Maxwell Boltzmanian Distribution, it is given as:

$$< |p| >= \frac{\int_0^\infty dpp^3 e^{-\frac{1}{T}(m^2+p^2)^{\frac{1}{2}}}}{\int_0^\infty dpp^2 e^{-\frac{1}{T}(m^2+p^2)^{\frac{1}{2}}}} \tag{6.1}$$

The integral in the numerator was evaluated previously, and was expressed in terms of

Modified Bessel function of 2nd kind.

$$\int_0^\infty dpp^2 e^{-\frac{1}{T}(m^2+p^2)^{\frac{1}{2}}} = m^2 T K_2(\frac{m}{T})$$

$$\begin{aligned}
\int_0^\infty dpp^3 e^{-\frac{1}{T}(m^2+p^2)^{\frac{1}{2}}} &= m^4 \int_0^\infty dx sinh^3 x coshx e^{-\frac{m}{T}coshx} && (6.2)\\
&= T^4 \frac{2^{\frac{3}{2}}}{\sqrt{\pi}}(\frac{3}{2}z^{\frac{3}{2}}K_{\frac{3}{2}}(\frac{m}{T}) - K'_{\frac{3}{2}}(\frac{m}{T})) && (6.3)\\
&= T^4 \frac{2^{\frac{3}{2}}}{\sqrt{\pi}}\frac{m}{T}^{\frac{5}{2}}(\frac{3T}{2m}K_{\frac{3}{2}}(\frac{m}{T}) - K'_{\frac{3}{2}}(\frac{m}{T})) && (6.4)\\
&= T^4 \frac{2^{\frac{3}{2}}}{\sqrt{\pi}}(\frac{m}{T})^{\frac{5}{2}}\frac{1}{2}(K_{\frac{5}{2}}(\frac{m}{T}) - K_{\frac{1}{2}}(\frac{m}{T}) + K_{\frac{5}{2}}(\frac{m}{T}) + K_{\frac{1}{2}}(\frac{m}{T})) && (6.5)\\
&= T^4 \frac{2^{\frac{3}{2}}}{\sqrt{\pi}}(\frac{m}{T})^{\frac{5}{2}}K_{\frac{5}{2}}(\frac{m}{T}) && (6.6)
\end{aligned}$$

$$\begin{aligned}
< |p| > &= \frac{T^4 \frac{2^{\frac{3}{2}}}{\sqrt{\pi}}(\frac{m}{T})^{\frac{5}{2}}K_{\frac{5}{2}}(\frac{m}{T})}{m^2 T K_2(\frac{m}{T})} && (6.7)\\
&= \sqrt{\frac{8mT}{\pi}}\frac{K_{\frac{5}{2}}(\frac{m}{T})}{K_2(\frac{m}{T})} && (6.8)
\end{aligned}$$

As noted earlier, $\lambda \propto \frac{1}{r^2}$.

The final expression for the viscosity of a relativistic gas of identical hadrons is:

$$\eta = \frac{5\sqrt{mT}}{64\sqrt{\pi}r^2}\frac{K_{\frac{5}{2}}(\frac{m}{T})}{K_2(\frac{m}{T})} \tag{6.9}$$

For a gas containing a mixture of different hadrons of same radius;

$$\eta = \frac{5\sqrt{T}}{64\sqrt{\pi}r^2}\Sigma_i \sqrt{m_i}\frac{K_{\frac{5}{2}}(\frac{m_i}{T})}{K_2(\frac{m_i}{T})} \tag{6.10}$$

6.2 Ratio of Viscosity to entropy density($\frac{\eta}{s}$)

In case of ordinary fluids, kinematic viscosity which is the ratio of absolute viscosity to mass density is preferred for comparing of viscous behaviour of different samples. This quantitiy would give a better estimate of the strength of the interaction between the molecules if their intermolecular separation were same. Analogous to kinematic viscosity, we introduce a quantity which is the ratio of viscosity to entropy density for describing the fluidity of the media created in heavy ion collisions. This is because entropy density is number density times a multiplicative factor and the ratio in natural units is dimensionless.The ratio of viscosity to entropy density gives the viscosity of the system which has an entropy density of $1fm^{-3}$.$\frac{\eta}{s}$ values which are viscosities of isoentropic systems give a better insight in comparing the fluidity of two different samples.

Chapter 7

Computational Results

7.1 Characteristics of pion

SPIN:0(BOSON) (7.0)

ISOSPIN:3(π^0, π^+, π^-) (7.1)

CONSTITUENT QUARKS:π^0(u$\bar{u}$),$\pi^+(u\bar{d})$,$\pi^-(d\bar{u})$ (7.2)

MASS=134.98 MeV (7.3)

$\mu_B = \mu_S = \mu_Q = 0$ (7.4)

$\gamma_S = 0$ (7.5)

7.2 Computation

The expressions derived for viscosity, entropy density and their ratio using the one component Van der waal excluded volume gas model have been numerically determined using c++ code. The average momentum of the system was determined using numerical integration technique and the other specified parameters plugged into the expression to calculate viscosity. The pressure of the system was calculated to an error of $0.01\frac{Mev}{fm^3}$ by allowing the simultaneous evaluation of the functions $y(x) = x$ and $g(x) = e^{-\frac{vx}{T}}T\phi$ and thereby observing their equivalence at some point.The temperature derivative of ideal gas number density was also obtained using numerical integration. The program also takes into account the requisite factors to ensure proper unit conversions. As mentioned earlier, the program assumes that the particles have hard core radii. Further, it would be assumed that the radii is the same for all particles. The differences in the study using Maxwell Boltzmanian statistics and Bose Einstein statistics is tabulated. The results are given in natural units.

7.3 Mixture of pion gas in a Maxwell Boltzmanian ensemble

Table 7.1: Pion gas with radius=0.5 fm

T(MeV)	K	P($\frac{MeV}{fm^3}$)	$\phi(fm^{-3})$	R	$n(fm^{-3})$	$\eta(GeV^3 fm^{-3})$	s(fm^3)	$\frac{\eta}{s}$
64.3	1.51232	0.4	0.00508051	0.976808	0.00496268	70.2769	0.0257762	13.84
74.3	1.58493	0.7	0.00914451	0.962408	0.00880075	79.1715	0.0437774	9.18
84.3	1.65554	1.3	0.0149065	0.939839	0.0140097	88.0884	0.0675118	6.623
94.3	1.72424	2.1	0.0226297	0.913165	0.0206647	97.0326	0.0972868	5.063
104.3	1.79111	3.2	0.0325696	0.881437	0.0287081	106.005	0.132885	4.049
114.3	1.85624	4.8	0.0449765	0.843142	0.0379216	115.006	0.173439	3.366
124.3	1.91973	6.7	0.060097	0.803052	0.048261	124.034	0.218974	2.875
134.3	1.98168	9.2	0.0781754	0.758818	0.0593209	133.087	0.267866	2.522
144.3	2.04215	12.1	0.0994546	0.714249	0.0710354	142.163	0.320085	2.254
154.3	2.10124	15.6	0.124176	0.668608	0.0830252	151.26	0.374118	2.052
164.3	2.15902	19.6	0.152581	0.623802	0.0951803	160.376	0.429681	1.895

Table 7.2: Pion gas with radius=0.3 fm

T(MeV)	K	P($\frac{MeV}{fm^3}$)	$\phi(fm^{-3})$	R	$n(fm^{-3})$	$\eta(GeV fm^{-3})$	s(fm^3)	$\frac{\eta}{s}$
64.3	1.51232	0.4	0.00508051	0.994912	0.00505466	195.214	0.0262124	37.804
74.3	1.58493	0.7	0.00914451	0.991666	0.0090683	219.921	0.0449754	24.821
84.3	1.65554	1.3	0.0149065	0.986449	0.0147045	244.69	0.0705141	17.615
94.3	1.72424	2.2	0.0226297	0.979588	0.0221678	269.535	0.103586	13.208
104.3	1.79111	3.4	0.0325696	0.971274	0.031634	294.46	0.144865	10.318
114.3	1.85624	5.1	0.0449765	0.960876	0.0432168	319.462	0.194793	8.325
124.3	1.91973	7.3	0.060097	0.948691	0.0570135	344.54	0.253789	6.891
134.3	1.98168	10.2	0.0781754	0.93433	0.0730417	369.686	0.321961	5.829
144.3	2.04215	13.8	0.0994546	0.918139	0.0913132	394.897	0.399439	5.018
154.3	2.10124	18.2	0.124176	0.900148	0.111777	420.166	0.486113	4.388
164.3	2.15902	23.5001	0.152581	0.880436	0.134338	445.489	0.581714	3.887

Table 7.3: Pion gas with radius=0.1 fm

T(MeV)	K	P($\frac{MeV}{fm^3}$)	$\phi(fm^{-3})$	R	$n(fm^{-3})$	$\eta(GeV fm^{-3})$	s(fm^3)	$\frac{\eta}{s}$
64.3	1.51232	0.4	0.00508051	0.999811	0.00507955	1756.92	0.0263303	338.712
74.3	1.58493	0.7	0.00914451	0.999689	0.00914167	1979.29	0.045303	221.777
84.3	1.65554	1.3	0.0149065	0.999492	0.0148989	2202.21	0.0713505	156.674
94.3	1.72424	2.2	0.0226297	0.999231	0.0226123	2425.82	0.105442	116.782
104.3	1.79111	3.4	0.0325696	0.99891	0.0325341	2650.14	0.148532	90.57
114.3	1.85624	5.2	0.0449765	0.998487	0.0449084	2875.16	0.201556	72.41
124.3	1.91973	7.5	0.060097	0.997987	0.059976	3100.86	0.265449	59.30
134.3	1.98168	10.5	0.0781754	0.997388	0.0779712	3327.18	0.341129	49.51
144.3	2.04215	14.4	0.0994546	0.996673	0.0991238	3554.07	0.429502	42.004
154.3	2.10124	19.2	0.124176	0.995852	0.123661	3781.5	0.531471	36.118
164.3	2.15902	25.1001	0.152581	0.994909	0.151804	4009.4	0.647917	31.412

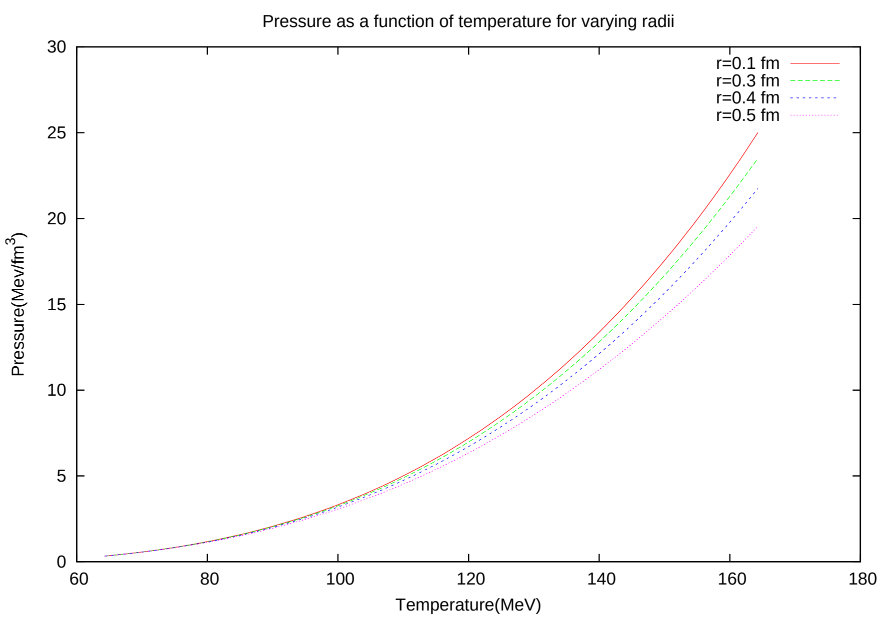


Figure 7.1: Variation of pressure with temperature for different radii

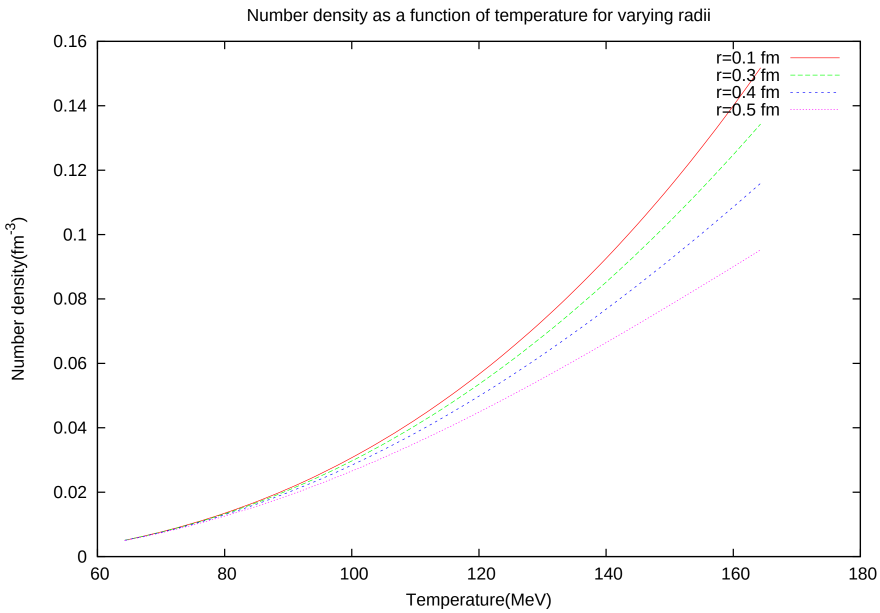


Figure 7.2: Variation of number density with temperature for varying radii

Table 7.4: Temperature and chemical potential of systems of different beam energies

	Beam energy(GeV)	T(MeV)	μ_B
SIS	2.32	64.3	800.8
AGS	4.86	116.5	562.2
SPS	17.3	154.4	228.6
RHIC	200	161.1	23.5

Table 7.5: Variation of viscosity with radius

r(fm)	K	P($\frac{MeV}{fm^3}$)	$\phi(fm^{-3})$	R	$n(fm^{-3})$	$\eta(GeV fm^{-3})$	s(fm^3)	$\frac{\eta}{s}$
0.1	1.51232	0.4	0.00508051	0.999811	0.00507955	1756.92	0.0263303	66726.3
0.3	1.51232	0.4	0.00508051	0.994912	0.00505466	195.214	0.0262124	7447.37
0.5	1.51232	0.4	0.00508051	0.976808	0.00496268	70.2769	0.0257762	2726.43
0.7	1.51232	0.4	0.00508051	0.938472	0.00476792	35.8556	0.0248488	1442.95

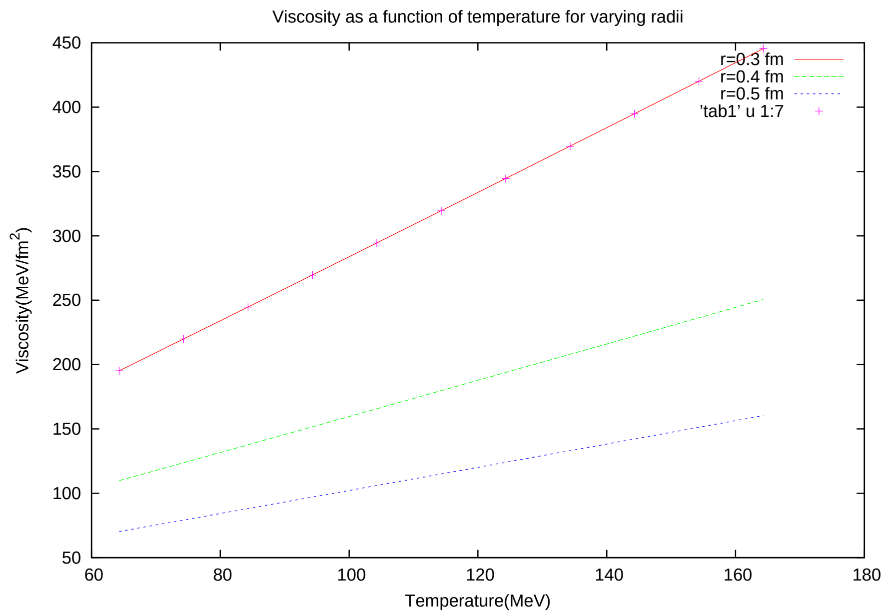


Figure 7.3: Variation of viscosity with temperature for different radii

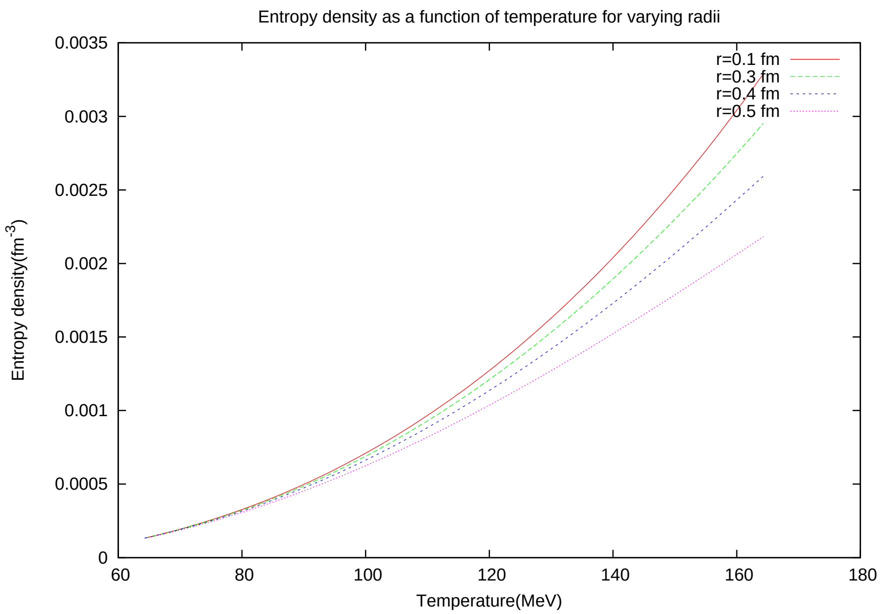


Figure 7.4: Variation of entropy density with temperature for different radii

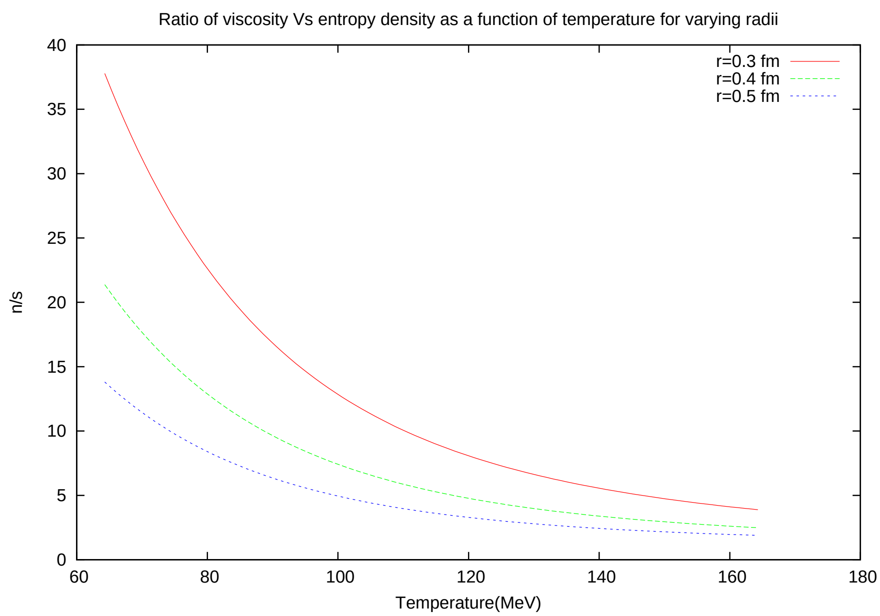
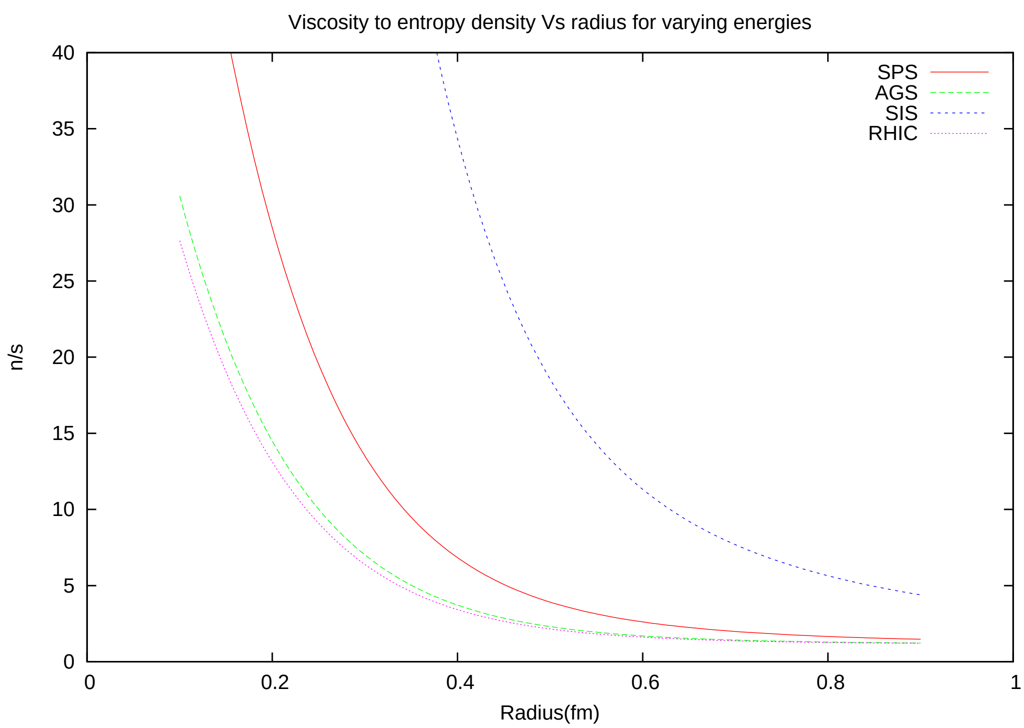


Figure 7.5: Variation of $\frac{\eta}{s}$ with temperature for different radii

Figure 7.6: Ratio of Viscosity Vs Entropy Density as a function of radius for varying beam energies



7.4 Dependence of viscosity on temperature and radius of pion

The pressure of the system wa found to increase with increase in temperature. The plots show that while the viscosity and the entropy density of the system increase with increase in temperature, the ratio of both decreases which is due to the faster increase of the denominator(entropy density) with temperature.

On increaing the radius of the particles,the viscosity, entropy density and the ratio were found to decrease. Although not evident from the table placed above, the pressure of the system also reflected a decreasing dependence on radius which could not be spotted with a program of accuracy only upto 0.1. Further magnification of the pressure-radii dependence produced the plot that has been shown.

The positive rate of change of number density and thereby the entropy density is because the system has a non-negative chemical potential.

Table 7.6: Difference in pressure calculations using both statistics(r=0.5 fm)

T(MeV))	p(BE)($MeV fm^{-3}$)	p(MB)($MeV fm^{-3}$)	Relative error
64.3	0.333999	0.323999	0.0308639
74.3	0.695995	0.666995	0.043478
84.3	1.283	1.22	0.0516417
94.3	2.16203	2.04003	0.0597986
104.3	3.40094	3.18695	0.0671439
114.3	5.05982	4.71584	0.0729403
124.3	7.19466	6.6757	0.077739
134.3	9.85035	9.10905	0.0813806
144.3	13.0626	12.0492	0.0841057
154.3	16.8564	15.5216	0.0859911
164.3	21.248	19.5429	0.0872472

7.5 Mixture of pions using Bose-Einstein Statistics

Due to the small value of pion mass, the Maxwell Boltzmanian Statitistics might not be a good approximation to the actual statistics displayed, the Bose Einstein one. So, in this section I analyse the differences in the magnitude of the quantities studied at various temperatures using both statistics.

The relative error obtained is quite significant for a pion gas. $e^{\frac{m}{T}}$ decreases as m increases. When, $e^{\frac{m}{T}} >> 1$, MB statistics is a good approximation for the system. I expect the error to decrease as I include more particles of higher masses.

Table 7.7: Difference in number density using both statistics(r=0.5 fm)

T(MeV))	$n(fm^{-3})(MB)$)	$n(fm^{-3})(BE)$	Relative error
64.3	0.00497477	0.00513757	0.0316878
74.3	0.0088086	0.009179	0.040353
84.3	0.0140363	0.0147437	0.0479745
94.3	0.0206902	0.0218771	0.0542527
104.3	0.0287143	0.0305173	0.0590808
114.3	0.0379738	0.0405083	0.0625684
124.3	0.048277	0.0516197	0.0647577
134.3	0.0593928	0.063579	0.065842
144.3	0.0710765	0.0761002	0.0660142
154.3	0.0830948	0.0889112	0.0654188
164.3	0.0952326	0.10177	0.0642383

Table 7.8: Differences in viscosity calculations

T(MeV))	v(BE)($MeV fm^{-2}$)	w(MB)($MeV fm^{-2}$)	Relativeerror
64.3	69.3287	70.2767	-0.0136743
74.3	77.7466	75.9947	0.0225326
84.3	86.112	82.1833	0.0456239
94.3	94.4418	88.6756	0.0610557
104.3	102.747	95.3694	0.0718055
114.3	111.036	102.201	0.0795731
124.3	119.314	109.13	0.0853533
134.3	127.586	116.128	0.089803
144.3	135.853	123.186	0.0932443
154.3	144.119	130.281	0.096016
164.3	152.384	137.413	0.098247

Table 7.9: Differences in entropy density using both statistics

T(MeV))	s(MB)(fm^{-3})	s(BE)(fm^{-3})	Relative error
64.3	0.0258388	0.0270187	0.0436688
74.3	0.0438161	0.0463298	0.0542573
84.3	0.0676414	0.0722105	0.0632747
94.3	0.0974089	0.104805	0.0705723
104.3	0.132912	0.143871	0.0761726
114.3	0.173685	0.188833	0.0802175
124.3	0.219047	0.238846	0.0828917
134.3	0.268202	0.292916	0.0843755
144.3	0.320277	0.349998	0.0849168
154.3	0.374449	0.409082	0.0846615
164.3	0.429933	0.469261	0.0838082

Table 7.10: Differences in $\frac{\eta}{s}$ using both statistics

T(MeV))	$\frac{\eta}{s}(MB))$	$\frac{\eta}{s}(BE)(fm^{-3})$	Relative error
64.3	2719.81	2565.95	-0.0599615
74.3	1734.4	1678.11	-0.0335447
84.3	1214.98	1192.51	-0.0188431
94.3	910.343	901.116	-0.0102393
104.3	717.536	714.16	-0.00472727
114.3	588.426	588.014	-0.000700633
124.3	498.205	499.546	0.00268415
134.3	432.988	435.57	0.00592762
144.3	384.622	388.154	0.00910017
154.3	347.927	352.297	0.0124047
164.3	319.615	324.732	0.0157596

Chapter 8

What more?

I am currently working on computationally determing the viscosity and $\frac{\eta}{s}$ of a system consisting of all the discovered and proposed hadrons which have mass less than 2 GeV. Since the experiments have shown close to a perfect liquid like behaviour, the $\frac{\eta}{s}$ expected for the system is considerably low.

Chapter 9

References

- Physical Review C 77,024911(2008),Viscosity in the excluded volume hadron gas model,M.I.Gorenstein,M.Hauer and O.N.Moroz
- The pressure ensemble as a tool for describing hadron-quark phase transition, R.Hagedorn,RE.Th-3392,CERN
- New Journal of Physics 13(2011) 055008(18pp),Elliptic Flow:A brief review,Raimond Snellings
- Heat and Thermodynamics,Richard H Dittman Mark W Zeemansky,TMH Publications
- Statistical Mechanics,R.K.Pathria Paule d.Beale,Elsevier Publications
- Introductory Nuclear Physics,Krane,Wiley India Edition