Class

(1)

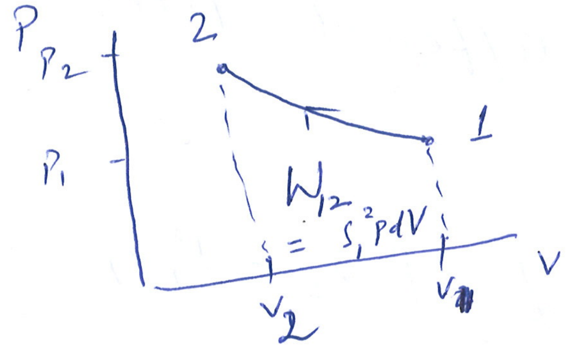
A gas is compressed from state (P_1, V_1) to (P_2, V_2) via two different paths, A and B:

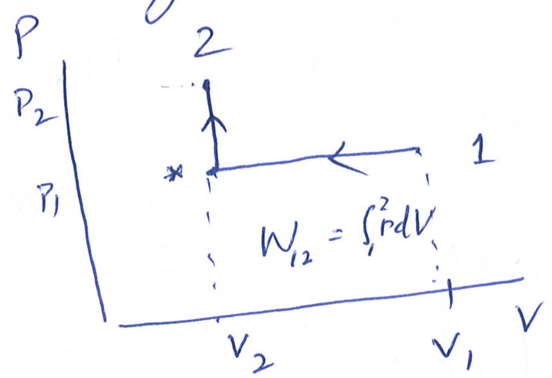
Path A: A polytropic process in which

$$PV^n = c$$

Path B: An isobaric process to $(P_1\ V_1)$ followed by an isochoric process compression to $P_2 V_2$

Determine the work along both paths.





First consider Path A:

on path A we have $PV^n = c = P_1 V_1^n = P_2 V_2^n$

$$P = c\frac{1}{V^n}$$

So

work is $W_{12} = \int_1^2 p\,dV = c\int_{V_1}^{V_2} \frac{dV}{V^n} = c\left[\frac{V^{1-n}}{1-n}\right]_{V_1}^{V_2}$

$$= c\left(\frac{V_2^{1-n} - V_1^{1-n}}{1-n}\right)$$

using $c = P_1 V_1^n = P_2 V_2^n$

$$\boxed{W_{12} = \frac{P_2 V_2 - P_1 V_1}{1-n}}$$

Now for Path B, we denote intermediate state as * and calculate

$$W_{12} = \int_1^2 P\,dV = \int_1^* P\,dV + \int_*^2 P\,dV$$

Now the state $*$ has $V_* = V_2$

$$W_{12} = \int_1^2 P\,dV = \int_{V_1}^{V_2} P\,dV + \cancel{\int_{V_2}^{V_2} P\,dV} \to 0$$

In the first part of the process - isobaric

$P = P_1$

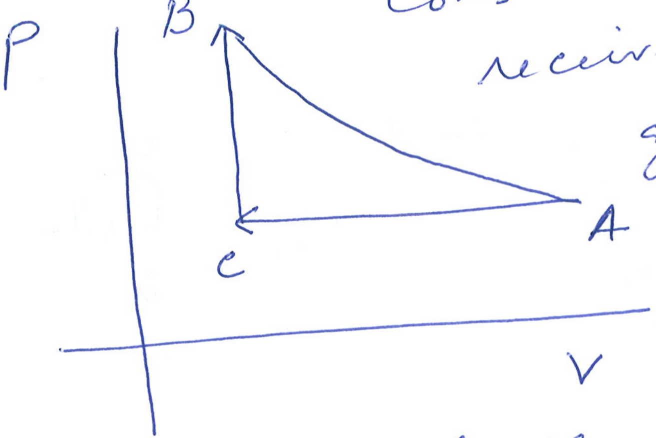
$$W_{12} = P_1 \int_{V_1}^{V_2} dV = P_1 (V_2 - V_1) = \boxed{P_1 V_2 - P_1 V_1}$$

The work done for the different paths is different

$$\frac{P_2 V_2 - P_1 V_1}{1-n} \neq P_1 V_2 - P_1 V_1$$

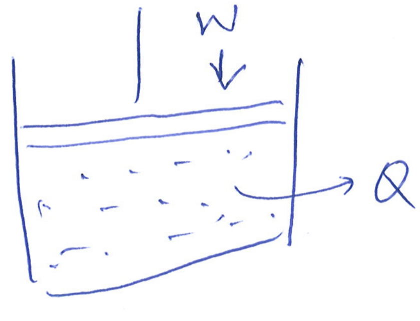
(2)

A gas is confined in a cylinder by a piston. It is taken from a state A to state B along the path ACB as shown on the PV diagram below. The process A to C is constant pressure, and the system receives 50 J of work and gives up 25 J of heat to the surroundings. The process from C to B is



constant volume and the system receives 75 J of heat. The return path B to A is adiabatic. How much work is exchanged with the surroundings for the adiabatic path? Assume all processes are reversible.

<u>Solu^n</u>



system : gas in cylinder

Known: $Q_{AC} = -25\,J$

$W_{AC} = 50\,J$

$Q_{CB} = 75\,J$

$Q_{BA} = 0$

$\Delta U_{BA} = -\Delta U_{AB}$ &

As U is a state function, we know that we can get the same ΔU by following different path so,

$\cancel{\Delta U_{BA} = -\Delta U_{AB}}$

$$\Delta U = 0 = \Delta U_{AC} + \Delta U_{CB} + \Delta U_{BA}$$

$$\Delta U = \Delta Q + \Delta W$$

$$\Delta U_{BA} = Q_{BA} + W_{BA}$$

Now from $\Delta U_{AC} + \Delta U_{CB} + \Delta U_{BA} = 0$

$$\Delta U_{BA} = -\Delta U_{AC} + \Delta U_{CB} = -\Delta U_{AB}$$

$$\Delta U_{AC} = Q_{AC} + W_{AC}$$

$$= -25J + 50J = 25J$$

$$W_{CB} = \int_{V_C}^{V_B} p\,dV = 0$$

$$\Delta U_{CB} = Q_{CB} + \underset{\overset{\shortparallel}{0}}{W_{CB}}$$

$$= 75J$$

So

$$\Delta U_{BA} = -\Delta U_{AC} - \Delta U_{CB}$$

$$= -25J - 75J = -100J$$

$$\Delta U_{BA} = \underset{\overset{\shortparallel}{0}}{\cancel{\Delta} Q_{BA}} + W_{BA}$$

$$= W_{BA} = -100J$$

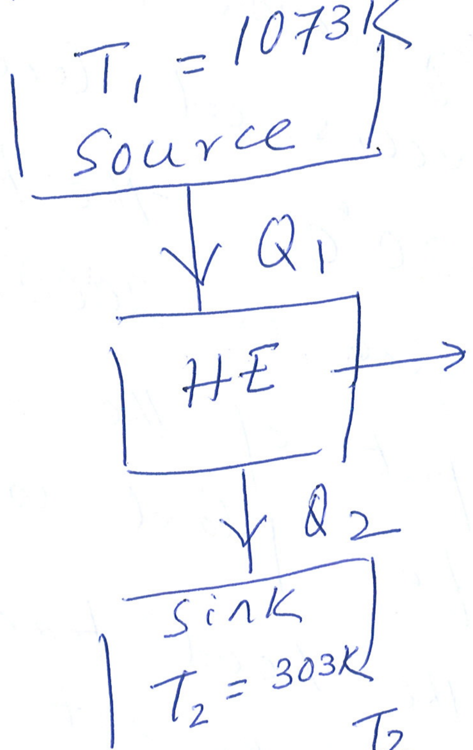
work done by system

Class

(3)

A cyclic heat engine operates between a source temperature of 800 °C and a sink temperature of 30°C. What is the least rate of heat rejection per kW net output of the engine?

Soln For a reversible engine, the rate of heat rejection will be minimum



$$W = Q_1 - Q_2 = 1\,kW$$

$$\eta_{max} = \eta_{rev} = 1 - \frac{T_2}{T_1} = 1 - \frac{30+273}{800+273}$$

$$= 1 - 0.282 = 0.718$$

$$\frac{W_{net}}{Q_1} = \eta_{max} = 0.718 \Rightarrow Q_1 = \frac{1\,kW}{0.718}$$

$$= 1.392\,kW$$

Now $Q_2 = Q_1 - W_{net}$

$= 1.392 - 1$

$= 0.392$ kW

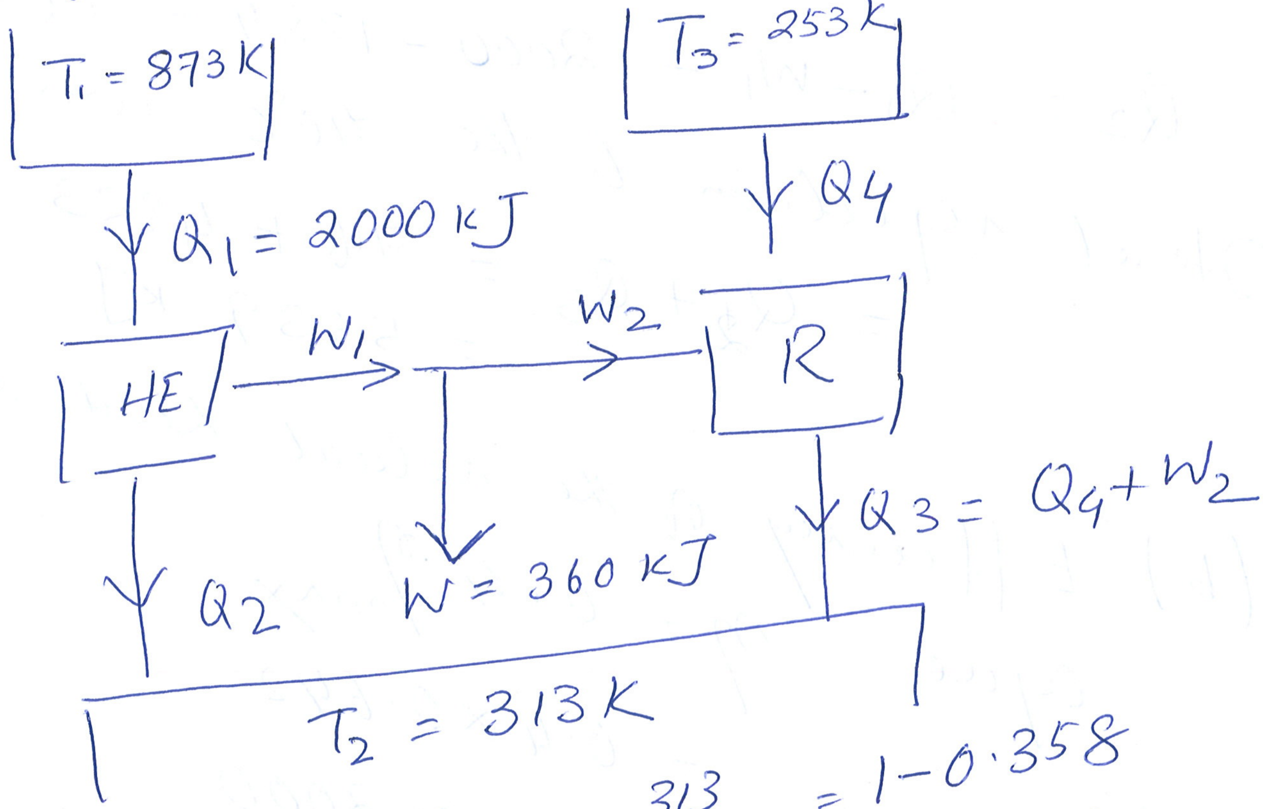
This is the least rate of heat rejection.

(4) A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40°C and −20°C. The heat transfer to the heat engine is 2000 kJ and the net work output of the combined engine transfer plant is 360 kJ.

(a) Evaluate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C

(b) Reconsider (a) given that the efficiency of the heat engine and COP of the refrigerator are 40% of their maxm possible values.

<u>Soln</u> : Max^{m} efficiency of the heat engine Cycle is given by



$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{313}{873} = 1 - 0.358 = 0.642$$

Again $\dfrac{W_1}{Q_1} = 0.642 \Rightarrow W_1 = 0.642 \times 2000 = 1284\,kJ$

Max^{m} COP of the refrigerator cycle

$$(COP)_{max} = \frac{T_3}{T_2 - T_3} = \frac{253}{313 - 253} = 4.22$$

Also $COP = \dfrac{Q_4}{W_2} = 4.22$

As $W_1 - W_2 = W = 360\,kJ$

$W_2 = W_1 - W = 1284 - 360 = 924\,kJ$

So $Q_4 = 4.22 \times 924 = 3899\,kJ$

$Q_3 = Q_4 + W_2 = 924 + 3899$
$= 4823 \text{ kJ}$

$Q_2 = Q_1 - W_1 = 2000 - 1284 = 716 \text{ kJ}$

Heat rejection to the 40°C reservoir
$= Q_2 + Q_3 = 716 + 4823$
$= 5539 \text{ kJ}$

(b) Efficiency of the actual heat engine cycle $\eta = 0.4 \eta_{max}$
$= 0.4 \times 0.642$

$W_1 = 0.4 \times 0.642 \times 2000$
$= 513.6 \text{ kJ}$

$W_2 = 513.6 \text{ kJ} - 360 \text{ kJ}$
$= 153.6 \text{ kJ}$

COP of the actual refrigerator $= \frac{Q_4}{W_2} = 0.4 \times 4.22 = 1.69$

Therefore $Q_4 = 153.6 \times 1.69 = 259.6 \text{ kJ}$
$Q_3 = 259.6 + 153.6 = 413.2 \text{ kJ}$
$Q_2 = Q_1 - W_1 = 2000 - 513.6 = 1486.4 \text{ kJ}$

Heat rejected to reservoir 40°C
$= Q_2 + Q_3 = 413.2 + 1486.4 = 1899.6 \text{ kJ}$