Introduction to the cylindrical co-ordinate system

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Cylindrical co-ordinate system

Idea: Given Cartesian Co-ordinates (x, y, z), apply Polar co-ordinates to x and y and leave the z co-ordinate alone.

Cylindrical co-ordinates

$$x = r\cos\phi$$
$$y = r\sin\phi \qquad (1)$$
$$z = z$$

r = Distance from 0 to P
ϕ = Angle between (x, y) and x-axis
z = Height of (x, y, z)

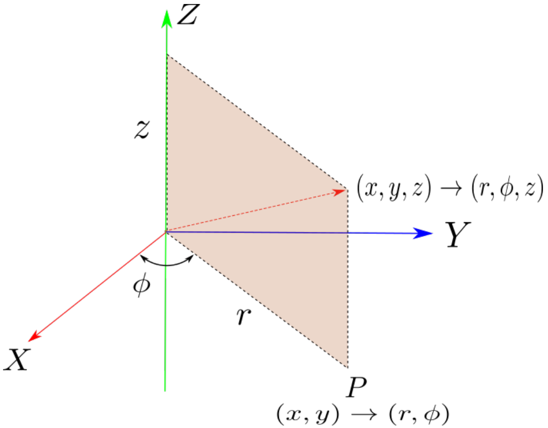
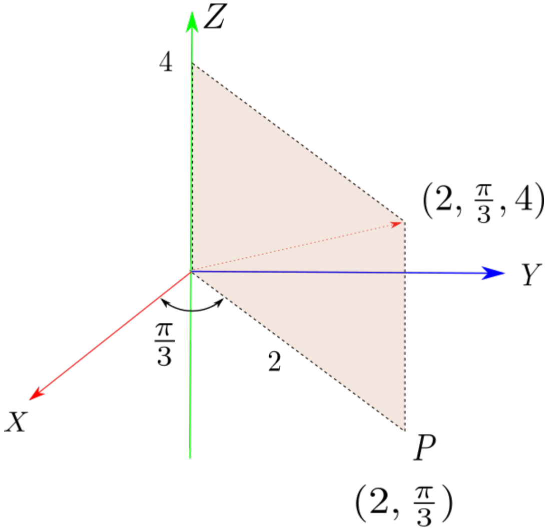


Figure: Cylindrical co-ordinate system

Example 1

Important:
$x^2 + y^2 = r^2$
(Used to simplify calculation)

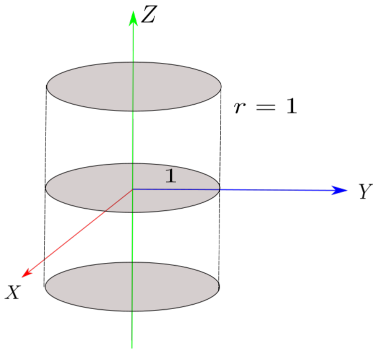


Example 2

Draw $r = 1 \rightarrow \sqrt{x^2 + y^2} = 1$ (Cylinder)

Point r is much simpler than saying $x^2 + y^2 = 1$

Use in problem cylindrical coordinate whenever you see $x^2 + y^2$ or a cylinder or a cone



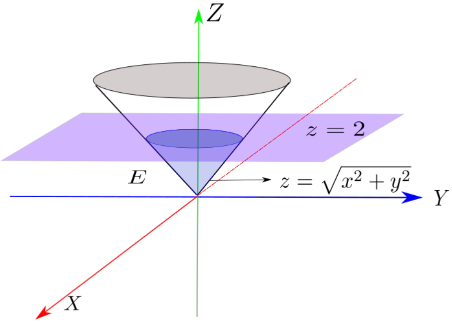
Example 3

$$\iiint_E y^2 dx\, dy\, dz$$

E region between cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 2$.

z values from r to 2. Intersect $z = 2$ and $z = r$ where $r = 2$ is the radius of the disc.

r values from 0 to 2 and ϕ from 0 to 2π.



$$\iiint y^2 dx\,dy\,dz = \int_0^{2\pi} \int_0^2 \int_r^2 (r\sin\phi)^2 dz\, r\, dr\, d\phi$$

$$\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \\ dx &= \cos\phi dr - r\sin\phi d\phi \\ dy &= \sin\phi dr + r\cos\phi d\phi \end{aligned}$$

Jacobian for the transformation (x, y) to (r, ϕ)

$$\begin{aligned} dx\,dy &= dr\,d\phi \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \phi} \end{vmatrix} \\ &= dr\,d\phi \begin{vmatrix} cos\phi & -r sin\phi \\ sin\phi & r cos\phi \end{vmatrix} \\ &= r|\cos^2\phi + \sin^2\phi| dr\,d\phi \\ &= r\,dr\,d\phi \end{aligned}$$

$$\int_0^{2\pi}\int_0^2\int_r^2 r^3(sin\phi)^2 dzdrd\phi$$

$$= \int_0^{2\pi}(sin\phi)^2 d\phi \int_0^2 r^3 dr \int_r^2 dz$$

$$= \int_0^{2\pi}\int_0^2 (2-r)r^3(sin\phi)^2 drd\phi$$

$$= \int_0^{2\pi}(sin\phi)^2 d\phi \int_0^2 (2-r)r^3 dr = \frac{8\pi}{5}$$