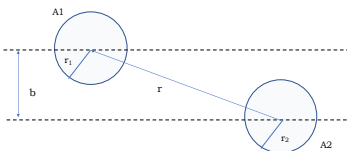


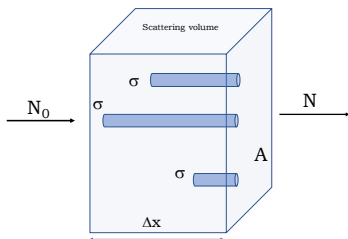
## Collision cross section and mean free path



### Some definitions

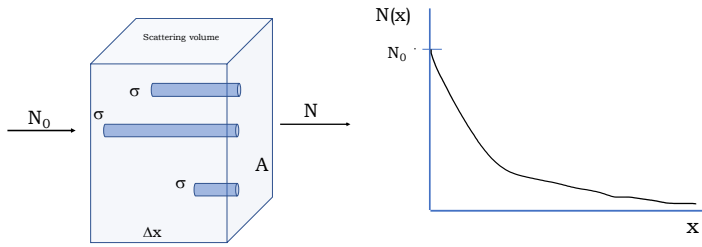
- Impact parameter  $b$ : Perpendicular distance between the center of two molecules
- $r_1$  and  $r_2$  are the radii of two molecules.
- Collision takes place when  $b \leq (r_1 + r_2)$ . The closest distance between the centre of molecule:  $d = r_1 + r_2$ .
- Collisional cross section is the area exposed by one particle to the other particle for interaction. Denoted by  $\sigma$  and has unit of area.
- The circular area around the centre of A2, when particles A1 come and will get deflected (interact/collide):  $\sigma = \pi (r_1 + r_2)^2$ . Collision takes place when  $b \leq (r_1 + r_2)$

# Particle Flux



- For simplicity let us consider 1D model along X-axis. Let us calculate a fraction of total cross section to total area.
- $\frac{\sum \sigma}{A} = \frac{n \sigma \Delta x A}{A} = n \sigma \Delta x$ .
- where  $n$  is the number density and  $\Delta x A = V$  is the volume.
- If  $N$  molecules impinge per second the area  $A$  of Volume  $V$ , then the fraction that suffers collision after a path length  $\Delta x$  :  
 $\frac{\Delta N}{N} = \frac{\sum \sigma}{A} = n \sigma \Delta x$ .

## Particle Flux Contd...



- In differential form and taking negative sign as particles are deflected or lost:  $\frac{dN}{N} = -n \sigma \Delta x$ .
- Integrating:  $N(x) = N_0 e^{-n\sigma x}$
- This is the particle flux  $N(x)$  after a path length  $x$  through the collision volume.

Kinetic Theory of Gases

Thermodynamics

State Variables

Zeroth law of thermodynamics

Reversible and Irreversible processes

First law of Thermodynamics

Second law of Thermodynamics

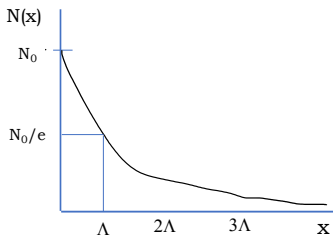
Entropy

Thermodynamic Potential

Third Law of Thermodynamics

Phase diagram

## Mean free path



- Mean free path ( $\Lambda$ ) is the path length which a particle A1 passes on an average without a collision.
- $$\Lambda = \frac{\int_0^{\infty} xN(x)dx}{\int_0^{\infty} N(x)dx} = \frac{\int_0^{\infty} xe^{-n\sigma x}dx}{\int_0^{\infty} e^{-n\sigma x}dx} = \frac{1}{n\sigma}$$
- Now,  $N(x) = N_0 e^{-n\sigma x} = N_0 e^{-x/\Lambda}$
- Hence, mean free path  $\Lambda$  represents that path length after which the number of particles in the incident beam has decreased to  $1/e$  of its initial value.

## Average time between two successive collisions

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- $\tau = \frac{\Lambda}{\langle v \rangle} = \frac{1}{n\sigma\langle v \rangle}$
- If  $v_1$  and  $v_2$  are velocities of A1 and A2, then the mean velocity is replaced by the relative velocity  $\overline{\Delta v} = \overline{v_1 - v_2} = \sqrt{2v^2}$
- So,  $\tau = \frac{1}{n\sigma\sqrt{2v^2}}$

## Problem

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At atmospheric pressure  $P = 10^5$  Pa, the number density of the molecules in the atmosphere  $n \sim 3 \times 10^{19} \text{ cm}^{-3}$ . The elastic collision cross section  $\sigma \sim 45 \times 10^{-16} \text{ cm}^2$ .

Mean free path  $\Lambda = \frac{1}{n\sigma} \sim 7 \times 10^{-6} \text{ cm} = 70 \text{ nm}$ .

Mean velocity  $\langle v \rangle \sim 475 \text{ m/s}$  at  $T = 300 \text{ K}$ .

Mean flight time between two collisions:  $\tau = \frac{\Lambda}{\sqrt{s\langle v^2 \rangle}} = \Lambda \sqrt{\frac{m}{6KT}} =$

$1.1 \times 10^{-10} \text{ sec}$ .