FPRAS for Total Variation Distance in High Dimensions

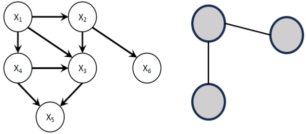
Sutanu Gayen

(Based on a Joint Work with
Arnab Bhattacharyya, Kuldeep S. Meel, Dimitrios Myrisiotis, A. Pavan, N. V. Vinodchandran)

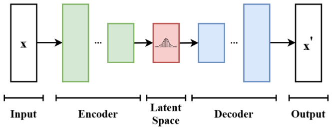
Indian Institute Technology Kanpur

July 27, 2023

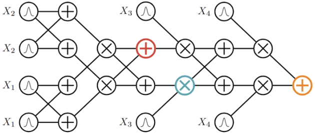
High-Dimensional Statistical Models



Probabilistic graphical models



Deep generative models (GANs, VAEs, normalizing flows, etc)



Probabilistic Circuits

Distance Estimation/Testing

Central Question

Given two models $\mathcal{M}_1$ and $\mathcal{M}_2$ decide whether their distributions are close or far with respect to a certain distance function.

- Additive testing: $\text{dist}(\mathcal{M}_1, \mathcal{M}_2) \leq \varepsilon$ or $> 2\varepsilon$.
  - Equivalent to additive estimation: $(\text{dist}(\mathcal{M}_1, \mathcal{M}_2) \pm \varepsilon)$
- Multiplicative testing: $\text{dist}(\mathcal{M}_1, \mathcal{M}_2) \leq \delta$ or $> (1 + \varepsilon)\delta$ for some δ such as $1/2$.
  - Equivalent to multiplicative estimation: $\text{dist}(\mathcal{M}_1, \mathcal{M}_2)(1 \pm \varepsilon)$
- Efficient algorithms: $\text{poly}(\varepsilon^{-1}, size(M_1), size(M_2))$ time, succeeds with $2/3$ probability

Distance Functions

- Several different choices: f-divergences, integral probability metrics, Wasserstein distances
- f-divergences: $\mathbb{E}_{x\sim P}\left(f\left(\frac{Q(x)}{P(x)}\right)\right)$ for some function $f(t)$ such that $f(1) = 0$.

distance	notation	$\mathbf{f(t)}$	formula
total variation	$\mathrm{TV}(P,Q)$	$\frac{1}{2}\lvert t-1\rvert$	$\frac{1}{2}\sum_{x\in\Omega}\lvert P(x)-Q(x)\rvert$
squared Hellinger	$\mathrm{H}^2(P,Q)$	$\frac{1}{2}(\sqrt{t}-1)^2$	$\frac{1}{2}\sum_{x\in\Omega}\left(\sqrt{P(x)}-\sqrt{Q(x)}\right)^2$
Kullback-Leibler	$\mathrm{KL}(P,Q)$	$\log t$	$\sum_{x\in\Omega}P(x)\log\frac{Q(x)}{P(x)}$
chi-squared	$\chi^2(P,Q)$	$(t-1)^2$	$\sum_{x\in\Omega}\frac{(P(x)-Q(x))^2}{P(x)}$

Total Variation Distance: Properties

$$\begin{aligned}
\mathrm{TV}(P,Q) &= \frac{1}{2}\sum_{x\in\Omega}|P(x)-Q(x)| \\
&= \max_{S\subseteq\Omega}\left[P(S)-Q(S)\right] \\
&= \max_{f:\Omega\to[0,1]}\left(\mathbb{E}_{x\sim P}[f(x)]-\mathbb{E}_{x\sim Q}[f(x)]\right)
\end{aligned}$$

- Only f-divergence which is also an Integral Probability Metric (satisfies triangle inequality)
- Max. difference between the probabilities of P and Q for any event
- $1-2*$ (min. error for distinguishing P and Q by a single sample)
- Minimum probability that $X\neq Y$ among all couplings (X,Y) between P and Q

Prior Work

- Goldreich, Sahai, and Vadhan (1999, 2003) showed that the TV distance is hard to additively approximate for distributions samplable by Boolean circuits.
- Canonne and Rubinfeld (2014) showed how to additively approximate the TV distance for models with efficient inference and sampling.

$$\mathrm{TV}(P,Q) = \sum_{x\in\Omega : P(x)>Q(x)} (P(x)-Q(x)) = \mathbb{E}_{x\sim P}\left[1_{P(x)>Q(x)}(P(x)-Q(x))\right]$$

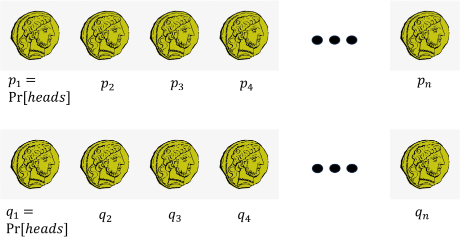
TV Distance of High-dimensional Models

$$\mathrm{TV}(P, Q) = \frac{1}{2} \sum_{x \in \Omega} |P(x) - Q(x)|$$

- A naive computation takes $O(\Omega)$ time, intractable over $\{0, 1\}^n$
- Surprisingly, the complexity of TV distance computation between high-dimensional probabilistic models has not been studied before our work.
- Also, multiplicative approximation algorithms for TV distance has not been studied before.

Notations and Preliminaries

Binary Product Distributions



- Joint distribution of n independent coin flips, $\Omega = \{0, 1\}^n$.
  - $P = P_1 \otimes P_2 \otimes \ldots P_n$.
- Inference and sampling is trivial.
  - $P[x_1, \ldots, x_n] = P_1(x_1) \ldots P_n(x_n)$ where $P_i = \mathrm{Bern}(p_i)$
- Non-binary product distributions, $\Omega = [k]^n$.

Notations and Preliminaries

Approximation algorithms

- FPTAS: fully polynomial time multiplicative approximation algorithm for computing $\mathrm{TV}(P, Q)$
- FPRAS: fully polynomial time *randomized* multiplicative approximation algorithm for computing $\mathrm{TV}(P, Q)$

Recent Development

- Hardness of Computing the TV Distance between Product Distributions and FPTAS for Special Cases.
  [Bhattacharyya, Gayen, Meel, Myrisiotis, Pavan, Vinodchandran; IJCAI 2023, arXiv:2206.07209]
- FPRAS for Computing the TV Distance between Product Distributions.
  [Feng, Guo, Jerrum; SIAM SOSA 2023, TheoretiCS 2023, arXiv:2208.00740]
- Hardness and FPRAS for Computing the TV Distance between Bayesian Networks.
  [Bhattacharyya, Gayen, Meel, Myrisiotis, Pavan, Vinodchandran; 2023+]

Outline

Hardness of Computing the TV Distance between Product Distributions and FPTAS for Special Cases

Our Contribution

Hardness

It is #P-hard in general to exactly compute the TV distance between two binary product distributions P and Q.

FPTAS for special cases

We give an FPTAS for the TV distance between an arbitrary binary product distribution P and the uniform distribution $Q = U$.

Hardness

Hardness

It is #P-hard in general to exactly compute the TV distance between two binary product distributions P and Q.

#SubsetProd

Given positive integers $a_1, \ldots, a_n$ and T, find

$$\left|\left\{S \subseteq [n] : \prod_{i \in S} a_i = T\right\}\right|.$$

(Known to be #P-hard)

#PmfEquals

Given a binary product distribution P with biases $p_1, \ldots, p_n$ and a $0 \leq v \leq 1$, find

$$\left|\{x \in \{0,1\}^n : P(x) = v\}\right|.$$

$$\#\textsc{SubsetProd} \leq \#\textsc{PmfEquals} \leq \mathrm{TV}$$

#SubsetProd $\leq$ #PmfEquals

- Given an instance of #SubsetProd: $a_1, \ldots, a_n$ and T, define an instance of #PmfEquals as follows:

$$p_i = \frac{a_i}{a_i + 1} \qquad \text{and} \qquad v = T \cdot \prod_i (1 - p_i)$$

- Then,

$$\prod_{i \in S} a_i = T \iff P(1_S) = v$$

#PmfEquals $\leq$ TV

- Given $p_1, \ldots, p_n$ and v, find $|\{x \in \{0,1\}^n : P(x) = v\}|$
  - assume: $v < 2^{-n}$ (other case: $v \geq 2^{-n}$)
- Define distributions $\widehat{P}$ and $\widehat{Q}$ on $(n+1)$ bits as follows:
  - $\widehat{p}_i = p_i$ for $i \in [n]$, $\widehat{p}_{n+1} = 1$
  - $\widehat{q}_i = \frac{1}{2}$ for $i \in [n]$, $\widehat{q}_{n+1} = v \cdot 2^n$ (other case: $1/(v \cdot 2^n)$)
- Define distributions P' and Q' on $(n+2)$ bits as follows:
  - $p'_i = p_i$ for $i \in [n]$, $p'_{n+1} = 1$, $p'_{n+2} = \frac{1}{2} + \beta$
  - $q'_i = \frac{1}{2}$ for $i \in [n]$, $q'_{n+1} = v \cdot 2^n$ (other case: $1/(v \cdot 2^n)$), $q'_{n+2} = \frac{1}{2} - \beta$
  - β is small depending on the granularity of precision

Claim

$\mathrm{TV}\left(P', Q'\right) = \mathrm{TV}(\widehat{P}, \widehat{Q}) + |\{x \in \{0,1\}^n : P(x) = v\}| \cdot 2\beta v$

Approximation Algorithms for TV(P, Q)

- Zero vs non-zero testing is easy!
- Factor n-approximation is easy!
  - $\mathrm{TV}(P_i, Q_i) \leq \mathrm{TV}(P, Q) \leq \sum_i \mathrm{TV}(P_i, Q_i)$

FPTAS for Distance to Uniformity for Binary Product Distributions

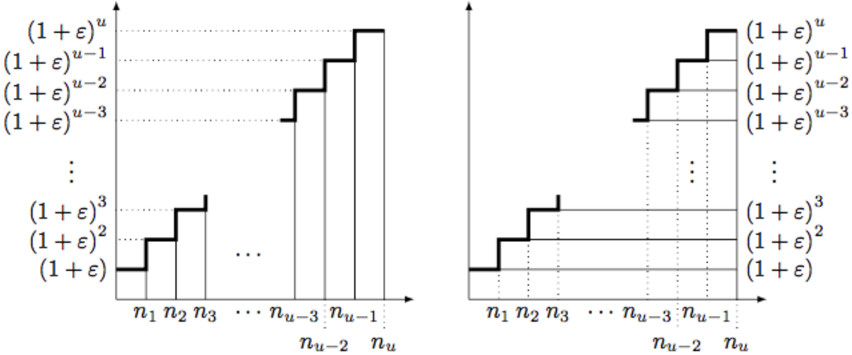
We give an FPTAS that returns $(1 \pm \varepsilon)\mathrm{TV}(P, U)$ where U is the uniform distribution over $\{0,1\}^n$. w.l.o.g. $\frac{1}{2} < p_i < 1$ for every i.

$$
\begin{aligned}
\mathrm{TV}(P,U) &= \sum_{x \in \{0,1\}^n} \max\left(0, P(x) - 1/2^n\right) \\
&= \sum_{S \subseteq [n]} \max\left(0, \prod_{i \in S} p_i \prod_{i \notin S} (1 - p_i) - 1/2^n\right) \\
&= \prod_{i \in [n]} (1 - p_i) \sum_{S \subseteq [n]} \underbrace{\max\left(0, \prod_{i \in S} \frac{p_i}{1 - p_i} - \prod_{i \in [n]} \frac{1}{2(1 - p_i)}\right)}_{Y_S}
\end{aligned}
$$

Approximating $\sum_{S\subseteq[n]} Y_S$

- $Y_S > 0$ lies in some range $[m, M]$ for each S (depending on precision)
- We create $u = \log_{1+\varepsilon}\frac{M}{m}$ levels: $[m(1+\varepsilon)^j, m(1+\varepsilon)^{j+1}]$ depending on the contribution of Y_S.
- Let n_j be the *count of sets* $S \subseteq [n]$ which contributes in the range $[m, m(1+\varepsilon)^j]$
  - $(n_{j+1} - n_j)$ sets contribute in the range $[m(1+\varepsilon)^j, m(1+\varepsilon)^{j+1}]$
  - $\sum_j (n_{j+1} - n_j) m(1+\varepsilon)^j$ is a $(1+\varepsilon)$-factor approximation of $\sum_{S\subseteq[n]} Y_S$

Reorganization trick



$$\sum_j (n_{j+1} - n_j) m (1+\varepsilon)^j = \sum_j (n_u - n_j) \left((1+\varepsilon)^{j+1} - (1+\varepsilon)^j \right)$$

It suffices to approximate $(n_u - n_j)$!

Approximating $(n_u - n_j)$

- $n_u = 2^n$, $n_j = \#$ sets with $Y_S \leq m(1+\varepsilon)^j$.
- Therefore, $(n_u - n_j) = \#$ sets with $Y_S > m(1+\varepsilon)^j > 0$

$$\left| \left\{ S \subseteq [n] : \prod_{i \in S} \frac{p_i}{1-p_i} > \underbrace{m(1+\varepsilon)^j + \prod_{i \in [n]} \frac{1}{2(1-p_i)}}_{A} \right\} \right|$$

Reduction to #KnapSack

- Define weights $w_i = \log \frac{p_i}{1-p_i} > 0$ for every $i \in [n]$.
- $\left\{S : \prod_{i \in S} \frac{p_i}{1-p_i} > A\right\} = \left\{S : \sum_{i \in S} w_i > \log A\right\}$
- $\left|\left\{S \subseteq [n] : \sum_{i \in S} w_i > \log A\right\}\right| = \underbrace{\left|\left\{T \subseteq [n] : \sum_{j \in T} w_j \leq B\right\}\right|}_{\#\text{KnapSack}}$
  - $T = [n] \setminus S$, $B = \sum_{i \in [n]} w_i - \log A$

[Gopalan, Klivans, Meka] [Stefanovic, Vempala, Vigoda]

Use existing FPTAS for #KnapSack.

Extensions for Binary Product Distributions

- FPTAS when Q has constantly many different biases
- FPRAS when $p_i \geq \frac{1}{2}, q_i \leq p_i$ for every i

FPRAS for Computing the TV Distance between Arbitrary Product Distributions

[Feng, Guo, Jerrum]

Gives an FPRAS for computing the TV distance between any two product distributions P and Q.
(could be non-binary but we focus on binary for simplicity).

Coupling interpretation of $\mathrm{TV}(P, Q)$

A coupling between P and Q is a joint distribution (X, Y) such that $X \sim P$ and $Y \sim Q$.

$$\mathrm{TV}(P, Q) = \min_{\text{couplings } (X,Y) \text{ between } P,Q} \Pr[X \neq Y]$$

Optimal Coupling O Given $\mathrm{TV}(P,Q)$

- $\mathrm{TV}(P,Q) = 1 - \sum_{w\in\Omega} \min\left(P(w),Q(w)\right) = \Pr_O(X \neq Y)$
- Make sure, $\Pr_O[X=Y] = \sum_{w\in\Omega} \min\left(P(w),Q(w)\right)$
  - define $\Pr_O[X=Y=w] = \min\left(P(w),Q(w)\right)$

Optimal Coupling O

$$\begin{aligned}
\Pr[X=x,Y=y] &= \min\left(P(w),Q(w)\right) && \text{(if } X=Y=w\text{)}\\
&= 0 && \text{(if } P(x)<Q(x) \text{ or } Q(y)<P(y)\text{)}\\
&= \frac{\left(P(x)-Q(x)\right)\left(Q(y)-P(y)\right)}{\mathrm{TV}(P,Q)} && \text{(otherwise)}
\end{aligned}$$

Optimal Coupling for Product Distributions

- Computing $\sum_{w\in\Omega} \min\left(P(w), Q(w)\right)$ is hard for product distributions
- Is the coordinate wise optimal coupling C also optimal overall?
  - Let (X_i, Y_i) be the optimal coupling for (P_i, Q_i). Is the coupling $C = \left((X_1, \ldots, X_n), (Y_1, \ldots, Y_n)\right)$ optimal?
  - If so, computing $\Pr_C(X = Y)$ is easy! Since $(X_i, Y_i) \perp (X_j, Y_j)$.
- No! Let us see an example.

Globally Optimal (O) $\neq$ Coordinate-wise Optimal (C)

$$(X_1, X_2) \sim P = \text{Bern}\left(\frac{1}{2} + \delta\right) \otimes \text{Bern}\left(\frac{1}{2} - \delta\right)$$

$$(Y_1, Y_2) \sim Q = \text{Bern}\left(\frac{1}{2}\right) \otimes \text{Bern}\left(\frac{1}{2}\right)$$

$$\begin{aligned} C_1 : \Pr[X_1 = 0, Y_1 = 0] &= \frac{1}{2} - \delta \\ \Pr[X_1 = 1, Y_1 = 1] &= \frac{1}{2} \\ \Pr[X_1 = 0, Y_1 = 1] &= 0 \\ \Pr[X_1 = 1, Y_1 = 0] &= \delta \end{aligned}$$

$$\begin{aligned} C_2 : \Pr[X_2 = 0, Y_2 = 0] &= \frac{1}{2} \\ \Pr[X_2 = 1, Y_2 = 1] &= \frac{1}{2} - \delta \\ \Pr[X_2 = 0, Y_2 = 1] &= \delta \\ \Pr[X_2 = 1, Y_2 = 0] &= 0 \end{aligned}$$

Coordinate-wise Optimal Coupling ($C = C_1 \otimes C_2$)

$\mathbf{X_1}$	$\mathbf{X_2}$	$\mathbf{Y_1}$	$\mathbf{Y_2}$	Pr
0	0	0	0	$\frac{1}{2}\left(\frac{1}{2}-\delta\right)$
0	0	0	1	$\delta\left(\frac{1}{2}-\delta\right)$
0	0	1	0	0
0	0	1	1	$\left(\frac{1}{2}-\delta\right)^2$
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0

$\mathbf{X_1}$	$\mathbf{X_2}$	$\mathbf{Y_1}$	$\mathbf{Y_2}$	Pr
1	0	0	0	$\frac{\delta}{2}$
1	0	0	1	δ^2
1	0	1	0	0
1	0	1	1	$\delta\left(\frac{1}{2}-\delta\right)$
1	1	0	0	$\frac{1}{4}$
1	1	0	1	$\frac{\delta}{2}$
1	1	1	0	0
1	1	1	1	$\frac{1}{2}\left(\frac{1}{2}-\delta\right)$

Overall Optimal Copling (O)

$\mathbf{X_1}$	$\mathbf{X_2}$	$\mathbf{Y_1}$	$\mathbf{Y_2}$	Pr
0	0	0	0	$\frac{1}{4} - \delta^2$
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	$\left(\frac{1}{2} - \delta\right)^2$
0	1	1	0	0
0	1	1	1	0

$\mathbf{X_1}$	$\mathbf{X_2}$	$\mathbf{Y_1}$	$\mathbf{Y_2}$	Pr
1	0	0	0	δ^2
1	0	0	1	$\delta(1-\delta)$
1	0	1	0	$\frac{1}{4}$
1	0	1	1	δ^2
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	$\frac{1}{4} - \delta^2$

- We will multiplicatively approximate the ratio: $\frac{\Pr_O[X \neq Y]}{\Pr_C[X \neq Y]}$.
- $\Pr_C\left[X \neq Y\right]$ can be exactly computed.

$$\begin{aligned}
\Pr_C\left[X \neq Y\right] &= 1 - \Pr_C\left[X = Y\right] \\
&= 1 - \prod_{i \in [n]} \Pr_{C_i}\left[X_i = Y_i\right] \\
&= 1 - \prod_{i \in [n]} (1 - \mathrm{TV}(P_i, Q_i))
\end{aligned}$$

Estimator

- Let Π be the distribution of $X \sim C \mid X \neq Y$. i.e. $\Pi(w) = \Pr_C[X = w \mid X \neq Y]$
- Let
$$f(w) := \frac{\Pr_O[X \neq Y \wedge X = w]}{\Pr_C[X \neq Y \wedge X = w]}$$

Claim

$$\mathbb{E}_{w \sim \Pi}[f(w)] = \frac{\Pr_O[X \neq Y]}{\Pr_C[X \neq Y]}$$

Proof of Claim

$$
\begin{aligned}
\mathbb{E}_{w\sim\Pi}\left[f(w)\right] &= \sum_{w\in\Omega} \Pr_C\left[X = w \mid X \neq Y\right] \cdot \frac{\Pr_O\left[X \neq Y \wedge X = w\right]}{\Pr_C\left[X \neq Y \wedge X = w\right]} \\
&= \sum_{w\in\Omega} \frac{\Pr_C\left[X = w \wedge X \neq Y\right]}{\Pr_C\left[X \neq Y\right]} \cdot \frac{\Pr_O\left[X \neq Y \wedge X = w\right]}{\Pr_C\left[X \neq Y \wedge X = w\right]} \\
&= \sum_{w\in\Omega} \frac{\Pr_O\left[X \neq Y \wedge X = w\right]}{\Pr_C\left[X \neq Y\right]} \\
&= \frac{\Pr_O\left[X \neq Y\right]}{\Pr_C\left[X \neq Y\right]}
\end{aligned}
$$

Properties of the Estimator

- Sampling $w \sim \Pi$ is efficient
- Computing $f(w)$ is efficient
- $$0 \leq f(w) \leq 1$$
- $$\frac{1}{n} \leq \mathbb{E}_{w\sim\Pi}\left[f(w)\right] \leq 1$$

$$\Pi(w) = \Pr_C\left[X = w \mid X \neq Y\right]$$

$$f(w) := \frac{\Pr_O\left[X \neq Y \wedge X = w\right]}{\Pr_C\left[X \neq Y \wedge X = w\right]}$$

$$\mathbb{E}_{w\sim\Pi}\left[f(w)\right] = \frac{\Pr_O\left[X \neq Y\right]}{\Pr_C\left[X \neq Y\right]}$$

FPRAS using monte-carlo sampling.

Computing the TV Distance between Bayesian Networks

Bayesian Networks

- A joint distribution over $X_1, \ldots, X_n$ that is a product of conditional probabilities (as opposed to marginal probabilities as in a product distribution)
- Defined with respect to a DAG G over $[n]$
- Notations:
  - $\Pi(i)$= parents of node i
  - $nde(i)$= non-desecendants of node i
  - $X_S = \{X_i\}_{i \in S}$
  - $\max_i |\Pi(i)|$ is called the in-degree of the Bayes net

Bayesian Networks Factorization

$$\Pr[X_1, \ldots, X_n] = \prod_{i \in [n]} \Pr[X_i \mid X_{\Pi(i)}]$$

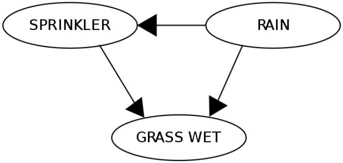
- Any node X_i is independent of its non-desendants conditioned on its parents

$$X_i \perp X_{nde(i)} \mid X_{\Pi(i)}$$

$$\implies \Pr[X_i, X_{nde(i)} \mid X_{\Pi(i)}] = \Pr[X_i \mid X_{\Pi(i)}]\Pr[X_i \mid X_{nde(i)}]$$

Example

	SPRINKLER	
RAIN	T	F
F	0.4	0.6
T	0.01	0.99



RAIN	
T	F
0.2	0.8

		GRASS WET	
SPRINKLER	RAIN	T	F
F	F	0.0	1.0
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.99	0.01

$$\Pr[R, S, G] = \Pr[R] \Pr[S \mid R] \Pr[G \mid S, R]$$

TV Approximation for Bayes Nets

Let $P[X_1, \ldots, X_n]$ and $Q[Y_1, \ldots, Y_n]$ be two Bayes nets over the same DAG G over $[n]$. Return:

$$d \in (1 \pm \varepsilon)\mathrm{TV}(P, Q)$$

Our Results

- Deciding $\mathrm{TV}(P,Q) = 0$ or not is NP-hard for Bayes nets of indegree 2
- We give an FPRAS for $\mathrm{TV}(P,Q)$ for Bayes nets of indegree 1 (tree distributions)
- More generally, FPRAS whenever inference is feasible (computing $\Pr[X_i] = 1$ is feasible e.g. fixed treewidth)

Proof Sketch: Hardness

- Given a sat formula, create two Bayes nets P and $Q = U$ such that $\mathrm{TV}(P, Q)$ counts the number of satisfying assignments

- The Bayes net mimicks the formula computation. Inputs are n random bits.

A Coarse Multiplicative Approximation for Trees

$$\begin{aligned}
&\mathrm{TV}(P,Q)\\
&\le \sum_{i\in[n]} \sum_{a\in 0,1^{|\Pi(i)|}} \Pr_P[X_{\Pi(i)} = a]\mathrm{TV}(P[X_i \mid X_{\Pi_i} = a], Q[Y_i \mid Y_{\Pi_i} = a])\\
&\le 2\sum_{i\in[n]} \mathrm{TV}(P[X_i, X_{\Pi(i)}], Q[Y_i, Y_{\Pi(i)}])
\end{aligned}$$

$$\begin{aligned}
&TV(P,Q)\\
&\ge \mathrm{TV}(P[X_i, X_{\Pi(i)}], Q[Y_i, Y_{\Pi(i)}])
\end{aligned}$$

Therefore, $\max_i \mathrm{TV}(P[X_i, X_{\Pi(i)}], Q[Y_i, Y_{\Pi(i)}])$ is a $2n$-factor approximation.

Proof Sketch: FPRAS

- We give an estimator for $\frac{\Pr_O[X \neq Y]}{\Pr_C[X \neq Y]}$. Infer: $\Pr_C[X \neq Y]$
- Except now, C is not the product coupling
  - If we couple each factor individually, it need not be a valid coupling overall!
- C is a *partial coupling*, a joint distribution over (X, Y):
  - corresponding factors are still coupled:

$$\begin{aligned}&\Pr[X_i = Y_i = w | X_{\Pi(i)} = a, Y_{\Pi(i)} = b] \\ &= \min\{\Pr\left[X_i = w \mid X_{\Pi(i)} = a\right], \Pr\left[Y_i = w \mid Y_{\Pi(i)} = b\right]\}\end{aligned}$$

  - Only $X \sim P$.

The 4 Required properties of [Feng, Guo, Jerrum] still goes through!

Thank you!

Questions?