Indian Institute of Technology Jodhpur

CS112 Discrete Mathematics

Assignment 1

The Invariance Principle

1. $2n$ ambassadors are invited to a banquet. Every ambassador has at most $n-1$ enemies. Prove that the ambassadors can be seated around a round table, so that nobody sits next to an enemy.*Hint: Let H be the number of neighboring hostile couples. We must find an algorithm which reduces H*

2. There are 3675 white, 322 black, and 4787 red chips on a table. In one step, you may choose two chips of different colors and replace them by a chip of the third color. If just one chip will remain at the end, its color will not depend on the evolution of the game. What color it will be?

3. There is a positive integer in each square of a rectangular table. In each move, you may double each number in a row or subtract 1 from each number of a column. Prove that you can reach a table of zeros by a sequence of these permitted moves.

Counting

4. A certain company has 30 female employees, including 3 in management ranks and 150 male employees including 12 in management rank. A committee of 3 women and 3 men is to be chosen. How many ways are there to choose the committee if

    - It includes at least 1 person of management rank of each gender.
    - It includes at least 1 person of management rank.

5. Show that
$$\binom{n}{0} + \binom{n+1}{1} \cdots \binom{n+r}{r} = \binom{n+r+1}{r}$$

6. How many odd numbers between 1000 and 9999 have distinct digits?