CS314 Algorithms Design and Analysis, August 2016
Problems on NP-completeness
(Chapter 34 in CLRS 3rd edition)

Due Date: November 28, 2016
Usual Instructions: The solution you hand in must be your own even if you discussed with others. If you do discuss with others or use other resources, please acknowledge. If you copy or let someone copy, then your answer sheet and further submissions (including tests) will NOT be evaluated.

1. 34.2-3 (If the Decision version of Ham cycle is in P, then we can find the cycle in polynomial time)

2. 34.4-7 (2CNF Sat in polynomial time)

3. 34.5-1 (Subgraph isomorphism is NP-complete)

4. Consider the feedback vertex set problem (FVS) in undirected graphs where you are given an undirected graph G, and the question is whether there exists a subset S of at most k vertices such that $G \setminus S$ is a forest? I.e. S covers all cycles of the graph. Show that this problem is NP-complete. (Hint: reduce from Vertex Cover by converting edges into cycles, but be careful to prove rigorously.)

5. Consider the following Hitting Set problem whose input consists of a pair $(\mathcal{U}, \mathcal{F})$ where $\mathcal{U}$ is a finite set of elements and $\mathcal{F}$ is a family of subsets of U. The question is whether there is a subset $\mathcal{U}'$ of $\mathcal{U}$ on at most k elements, such that $\mathcal{U}' \cap \mathcal{F} \neq \emptyset$ for every $F \in \mathcal{F}$. Such a set $\mathcal{U}'$ is called a hitting set.

   (a) Show that the problem is in NP.

   (b) Consider the following reduction from FVS of the previous problem. Given a graph $G = (V, E)$, the universe $\mathcal{U} = \mathcal{V}$, the vertex set. And $\mathcal{F}$ contains subsets of vertices of all cycles in G. I.e. find a cycle C in G, and take the vertex set of C as a set in $\mathcal{F}$.

   Show that G has a feedback vertex set of size at most k if and only if the reduced $(\mathcal{U}, \mathcal{F})$ has a hitting set of size at most k.

   (c) Despite the claim in the previous subproblems, we haven't proved the problem NP-complete, why?

   (d) Give a reduction from Vertex Cover to prove the problem NP-complete.