Classified Matchings under one sided preferences

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IIT Madras

Recent Trends in Algorithms

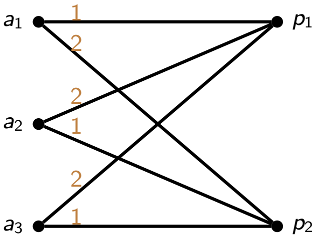
NISER, Bhubaneshwar
Feb 07, 2019

joint work with Prajakta Nimbhorkar (CMI) and Nada Pulath (IIT-M)

Problem setup

Input:

- A set of applicants A.
- A set of posts P.
- Applicants have preferences over a subset in P.



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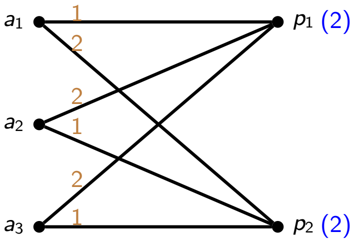

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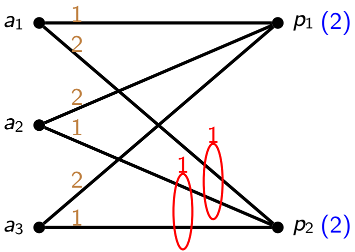

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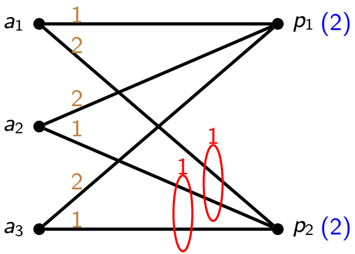

Classes are subsets on the neighborhood.

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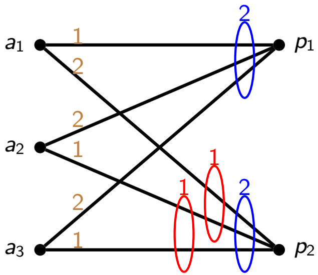

The neighborhood is a trivial class!

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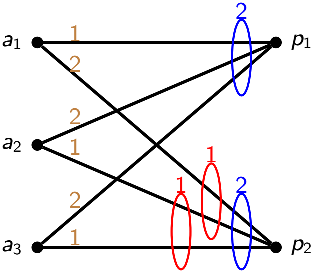

Goal: Match applicants to posts optimally.

Why classifications?

Some natural constraints that can be modelled:

Allotting courses to students

- Course - may not want many students from the same Dept.

Allotting tasks to employees

- Task - wasteful to have many employees with the same skills.

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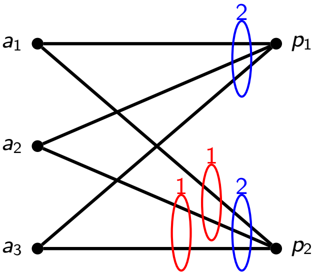

Goal: Compute a maximum cardinality matching.

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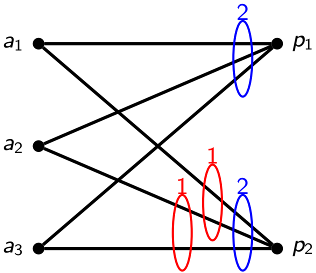

Goal: Compute a maximum cardinality matching.

- Arbitrary classes then problem is NP-Hard.

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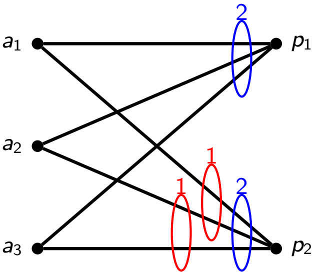


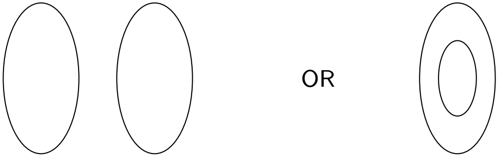
Goal: Compute a maximum cardinality matching.

- Arbitrary classes then problem is NP-Hard.
- Consider special classification families.

Laminar classification

Huang (2010); 2-sided pref.

$\boxed{\text{Laminar classification} \iff \text{any pair of classes is non-intersecting}}$

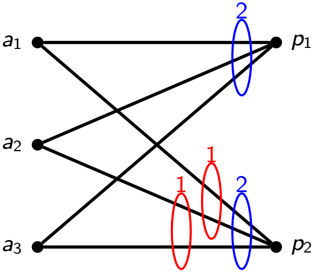


- Example: Countries , States , Districts , Cities
- Special case: Partition

Problem setup

Input:

- A set of applicants A
- A set of posts P
- ~~Applicants have preferences over a subset in~~ ~~P~~
- Posts have quotas
- Posts have laminar classes and quotas



Goal: Compute a maximum cardinality matching.

$\boxed{\text{Maximum matchings under laminar classifications}}$

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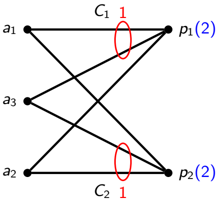
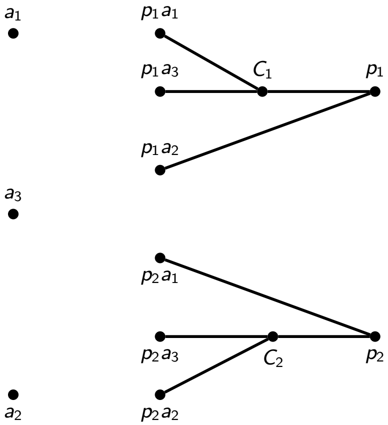
$\Downarrow$

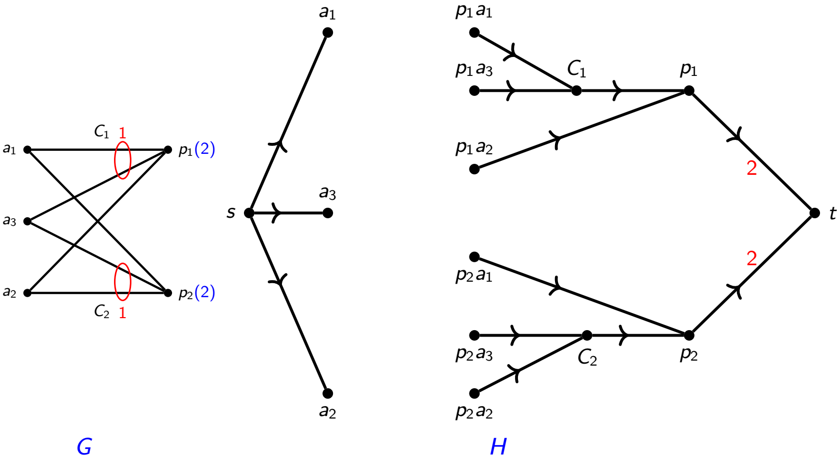
$\boxed{\text{Maximum flow in a flow network}}$

Classification tree - property of laminar classification



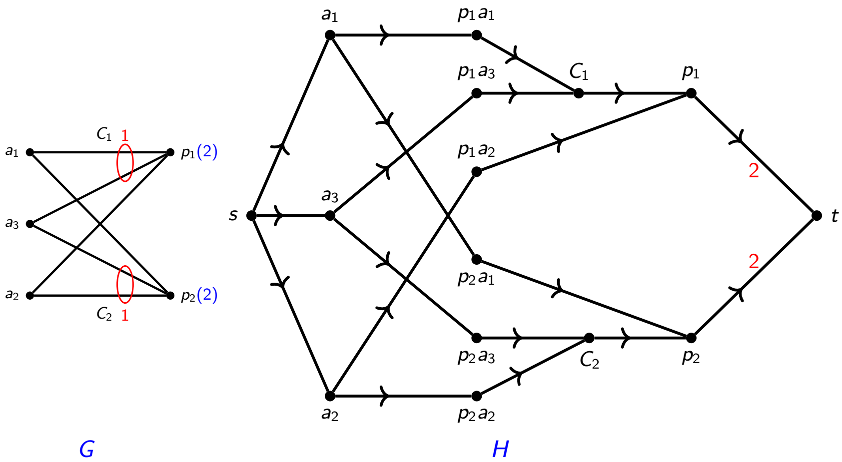


Feasible matchings to feasible flows



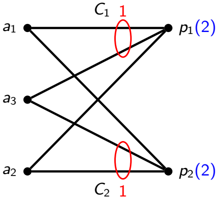
Add source s and sink t

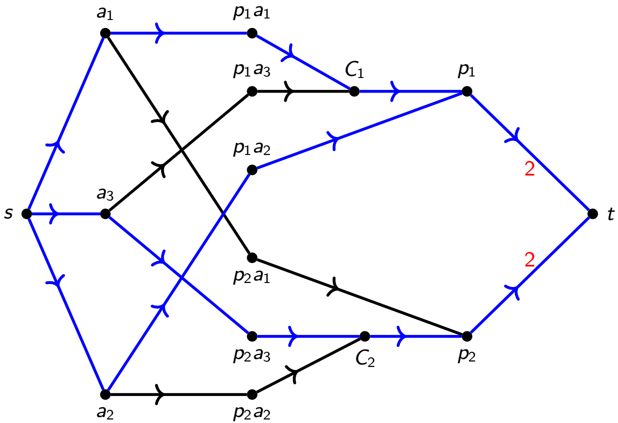
Feasible matchings to feasible flows



Add (a, pa) edges

Feasible matchings to feasible flows

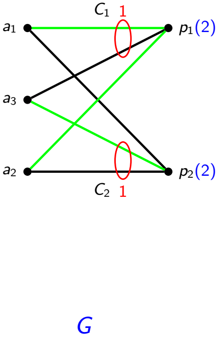


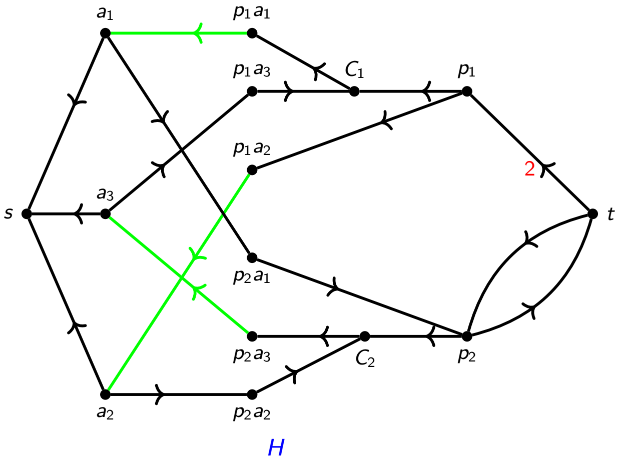


G H

$\boxed{\text{Compute a max-flow in } H}$

Feasible matchings to feasible flows





$\boxed{\text{Maximum matching in } G \iff \text{Maximum flow in } H}$

Back to our problem

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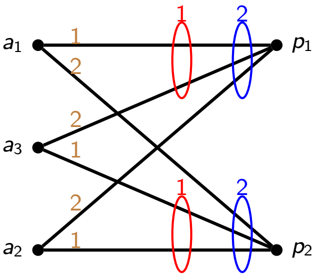

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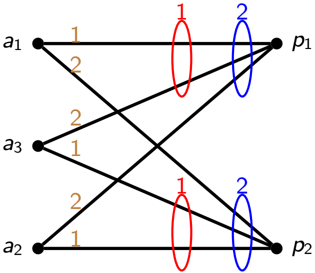

Goal: Match applicants to posts optimally.

Popularity: majority does not want to deviate.

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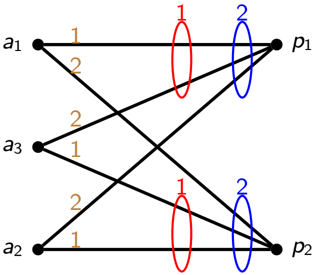

Goal: Match applicants to posts optimally.

Rank-maximality: max. number to rank-1, subject to this max. number to rank-2, ...

$\boxed{\text{Popularity in the one-to-one setting}}$

Popularity: definition

Gärdenfors (1975), Abraham et al.(2005)

Compare matchings M and M' using applicant votes.

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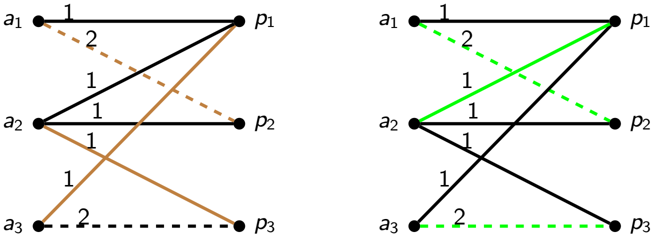

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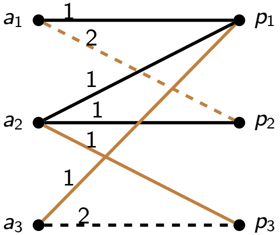

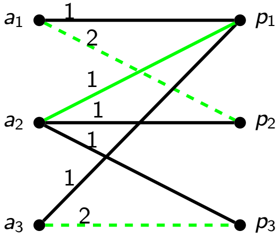

	M	M'
a_1	-	-
a_2	-	-
a_3	✓	-

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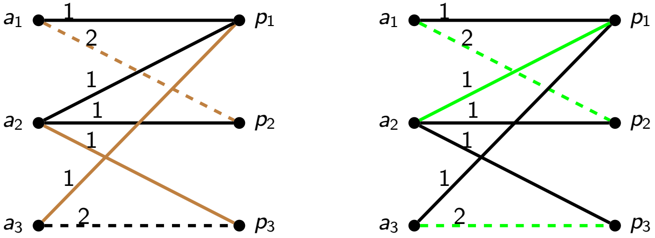

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M beats M' $\implies$ M' is not popular

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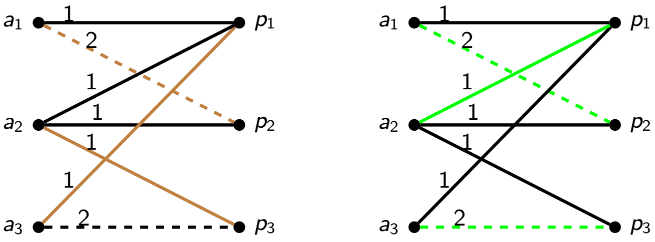

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A matching M is popular if no matching beats it.

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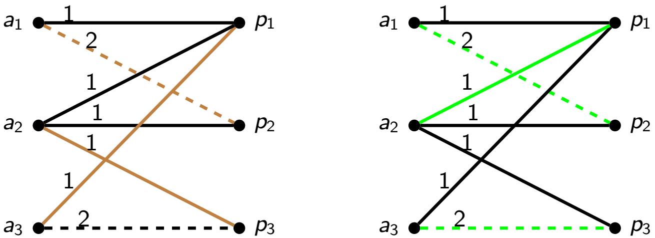

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Popular matchings: characterization

Abraham et al. (2005)

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- If such a matching does not exist, no popular matching exists.

Computing popular matchings Abraham et al. (2005); Manlove and Sng (2006)

Overall idea: Reduction to two maximum matching computations.

Computing popular matchings

Abraham et al. (2005); Manlove and Sng (2006)

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1. Construct G_1: every a adds $(a, f(a))$ edges.

   G_1 is graph on rank-1 edges.

Computing popular matchings

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Steps 3 & 4: Dulmage Mendelsohn Decomposition

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Partition of vertices into three sets w.r.t. a maximum matching.

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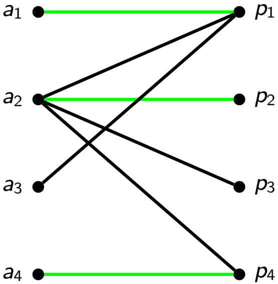

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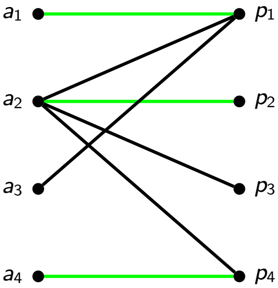

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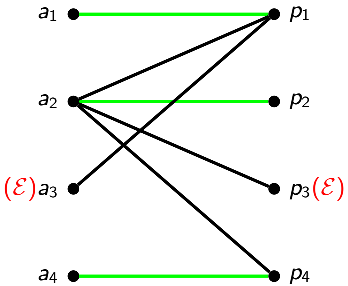

M is a maximum matching

- $\mathcal{E}$: reachable from unmatched vertex via even length alt. path.

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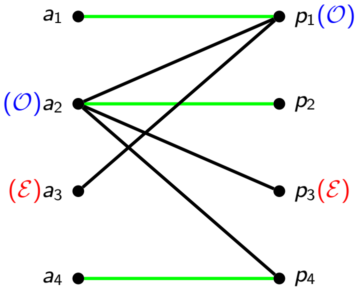

M is a maximum matching

- $\mathcal{E}$: reachable from unmatched vertex via even length alt. path.
- $\mathcal{O}$: reachable from unmatched vertex via odd length alt. path.

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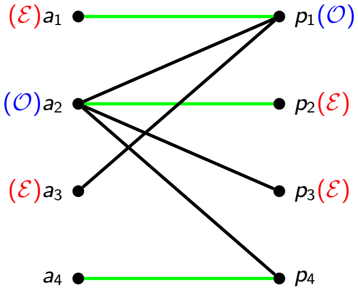

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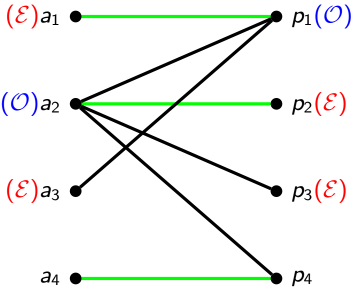

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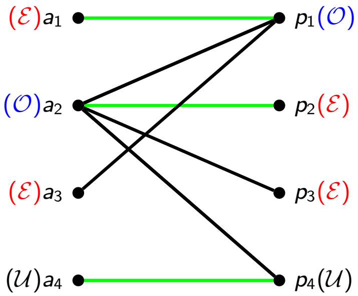

M is a maximum matching

- $\mathcal{E}$: reachable from unmatched vertex via even length alt. path.
- $\mathcal{O}$: reachable from unmatched vertex via odd length alt. path.
- $\mathcal{U}$: **unreachable** from unmatched vertex via length alt. path.

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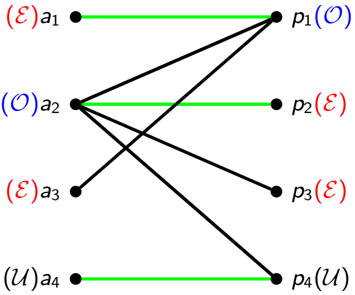

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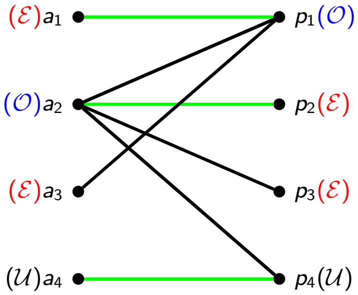

$\boxed{\mathcal{E}, \mathcal{O} \text{ and } \mathcal{U} \text{ invariant of } M}$

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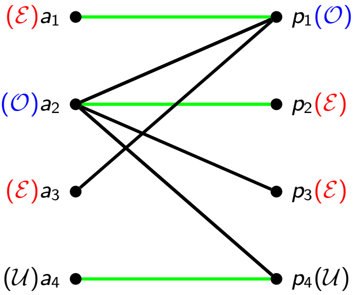

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For any maximum matching:

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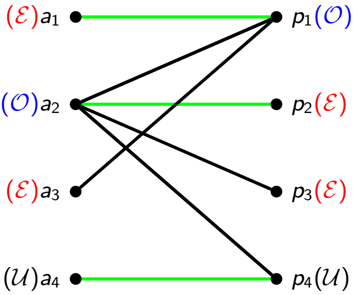

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For any maximum matching:

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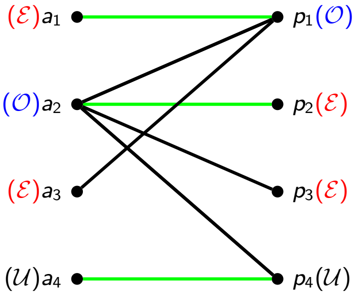

$\boxed{\mathcal{E},\ \mathcal{O} \text{ and } \mathcal{U} \text{ invariant of } M}$

For any maximum matching:

- Every $\mathcal{O}$ and $\mathcal{U}$ vertex is matched.
- No $\mathcal{OO}$,$\mathcal{OU}$ edges are matched.

Computing popular matchings

Abraham et al. (2005); Manlove and Sng (2006)

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3. Delete "unnecessary" rank-1 edges. $\mathcal{OO}$,$\mathcal{OU}$ edges.
4. Even applicants add $(a, s(a))$ edges.
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6. If M matches all applicants, declare popular,
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Computing popular matchings

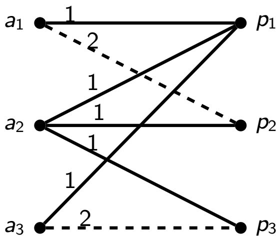
Abraham et al. (2005); Manlove and Sng (2006)

Overall idea: Reduction to two maximum matching computations.

1. Construct G_1: every a adds $(a, f(a))$ edges.
   G_1 is graph on rank-1 edges.
2. Compute a maximum matching M_1 in G_1.
3. Delete "unnecessary" rank-1 edges. $\mathcal{OO}$,$\mathcal{OU}$ edges.
4. Even applicants add $(a, s(a))$ edges.
5. Augment M_1 to compute a maximum matching M.
6. If M matches all applicants, declare popular,
   else no popular matching.

$\boxed{\text{Step 4: } s(a) \text{ – most preferred even post in } G_1.}$

Is deletion necessary?



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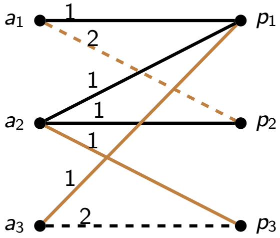

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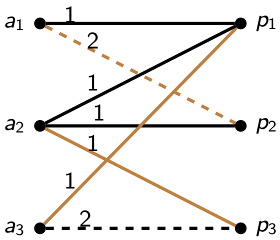

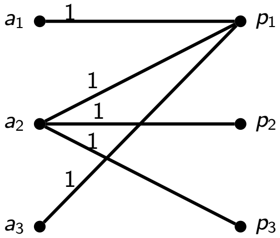

Graph G_1

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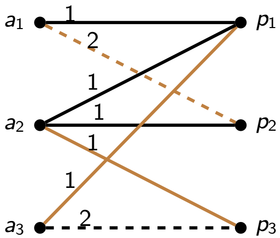
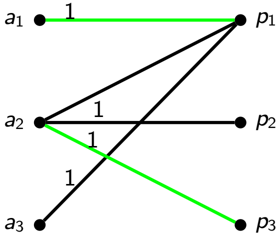
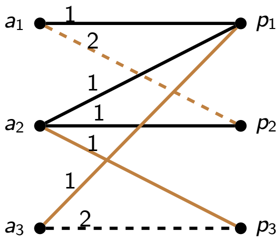
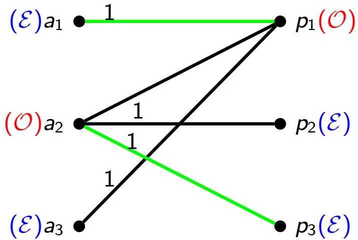




Max. matching M_1 in G_1

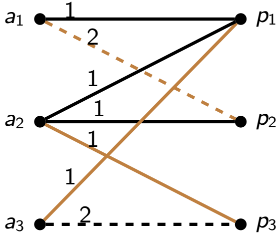
Is deletion necessary?

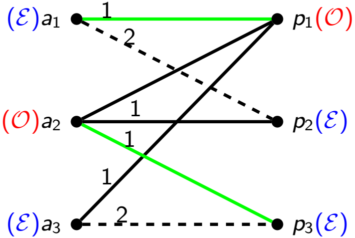




Label vertices

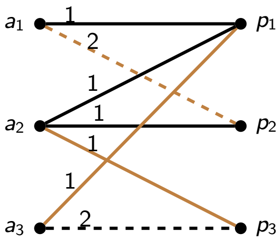
Is deletion necessary?

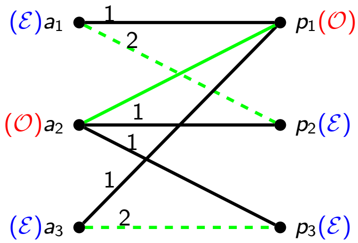




Add $(a, s(a))$ edges

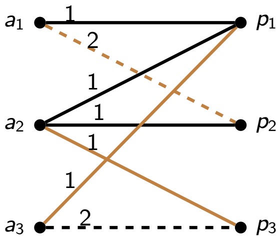
Is deletion necessary?

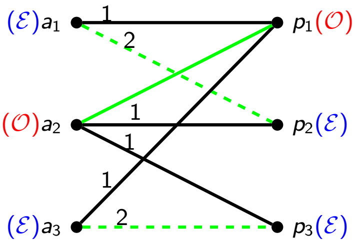




Augment M_1 to get M'

Is deletion necessary?

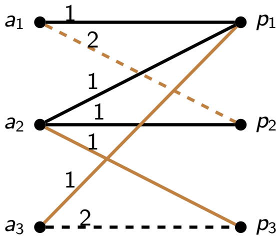


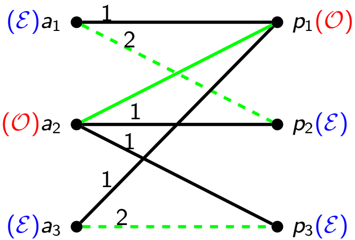


	M	M'
a_1	-	-
a_2	-	-
a_3	✓	-

M' is not popular

Is deletion necessary?





Deletion of $\mathcal{OO}$, $\mathcal{OU}$ edges is crucial!

Back to our problem

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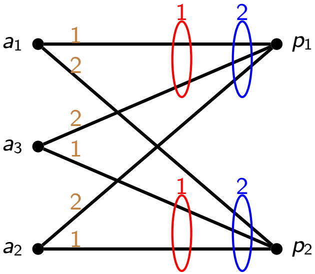
Input:

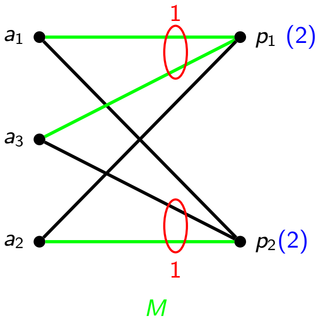
- A set of applicants A.
- A set of posts P.
- Applicants have preferences over a subset in P.
- Posts have quotas.
- Posts have laminar classes.



Goal: Compute a popular matching of applicants to posts (if one exists).

$\boxed{\text{Laminar classified popular matchings}}$

Classified matchings: challenges



- Deal with capacitated matchings.

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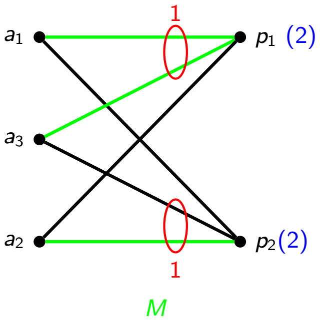

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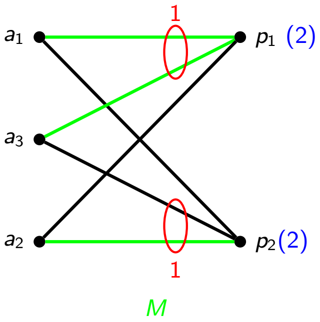

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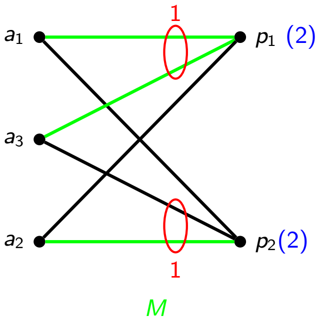

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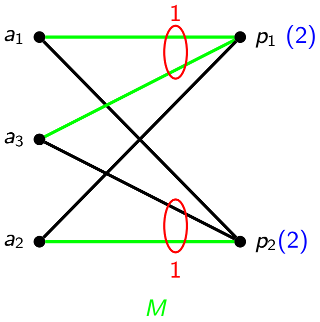

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Use max-flow and min-cut properties!

Properties of max-flow

Let H be any flow network and f be a max-flow in H.

$$\begin{aligned} S &= \{v \mid \text{ is reachable from } s \text{ in } H_f\} \\ T &= \{v \mid v \text{ can reach } t \text{ in } H_f\} \\ U &= \{v \mid v \notin T \cup S\} \end{aligned}$$

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Known Facts:

- Forward edges $(S, T \cup U)$: saturated in **every** max-flow.
- Reverse edges $(T \cup U, S)$: *zero* flow in **every** max-flow.

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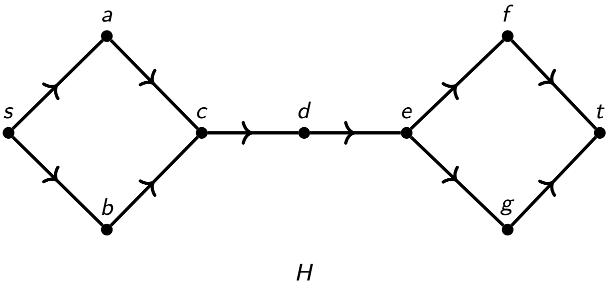

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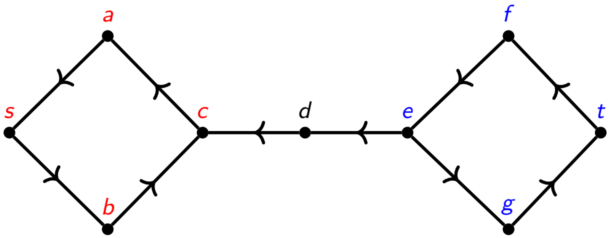

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The sets S, T and U are **invariant** of the max-flow f.

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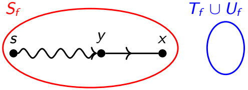

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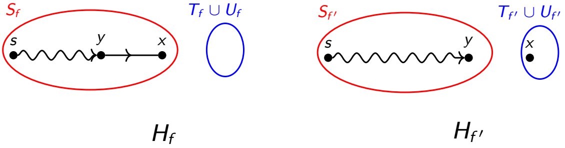

- $(y, x) \in H$
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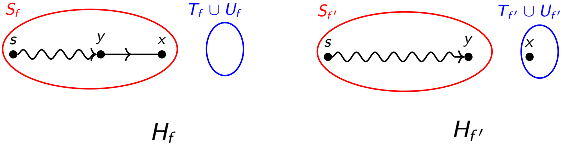

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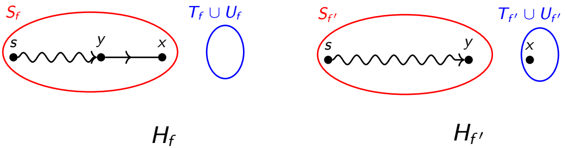

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- An $O(|A| \cdot |E|)$ time algorithm.

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Proof technique:

1. Two promotions at the cost of one demotion.
2. Cut edges were deleted $\implies$ augmentation preserves rank-1 edges.

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Thank you!