Ensuring your *favourite* player wins: Tournament Rigging and Bribery

Sushmita Gupta,

Sanjukta Roy,

Saket Saurabh,

Meirav Zehavi

(Knockout) Tournaments

How can we ensure our *favorite* player/team wins the tournament?

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An example …

FIFA WORLD CUP'18
KNOCKOUT STAGES

Round of 16	Quarterfinals	Semifinals

FT	Match: 49
Uruguay	2
Portugal	1

FT	Match: 50
France	4
Argentina	3

FT	Match: 53
Brazil	2
Mexico	0

FT	Match: 54
Belgium	3
Japan	2

FT - PENS	Match: 51
Spain	1 (3)
Russia	1 (4)

FT - PENS	Match: 52
Croatia	1 (3)
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Sweden	1
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France	2

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Formally we say...

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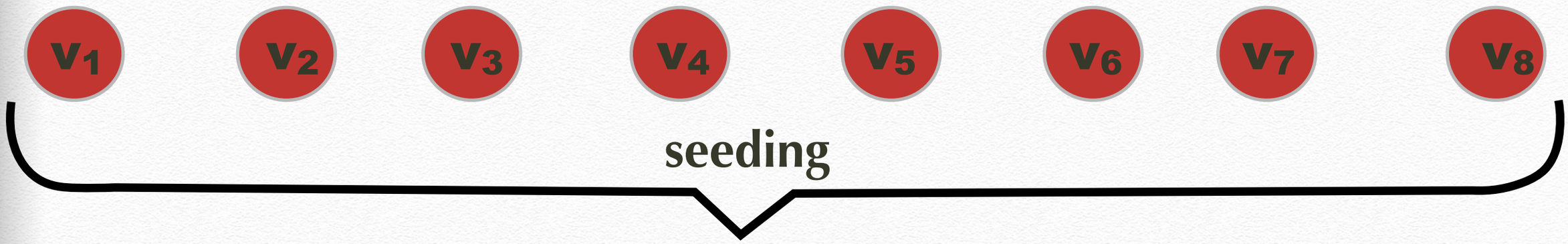

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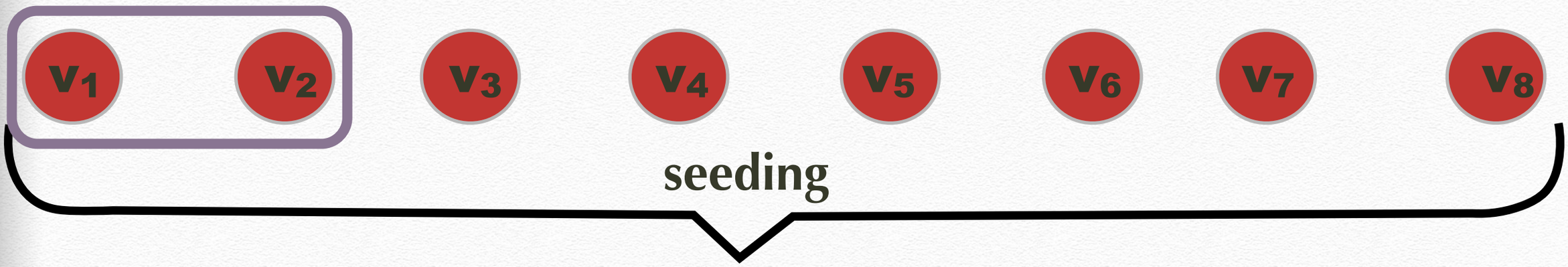

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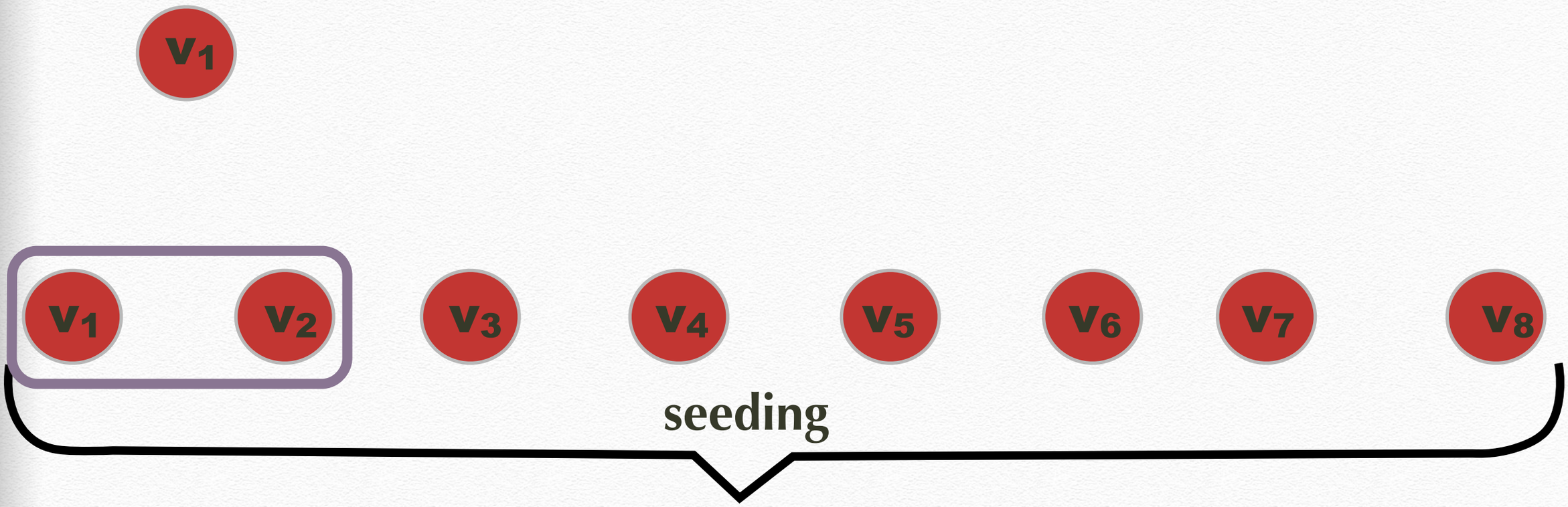

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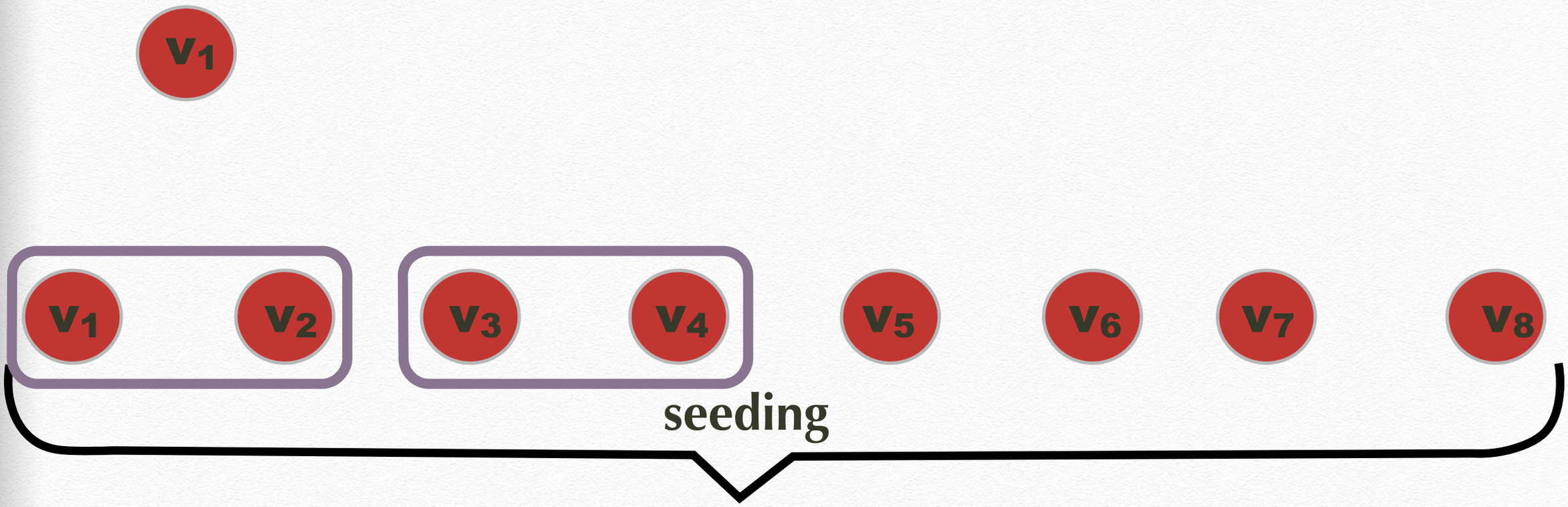

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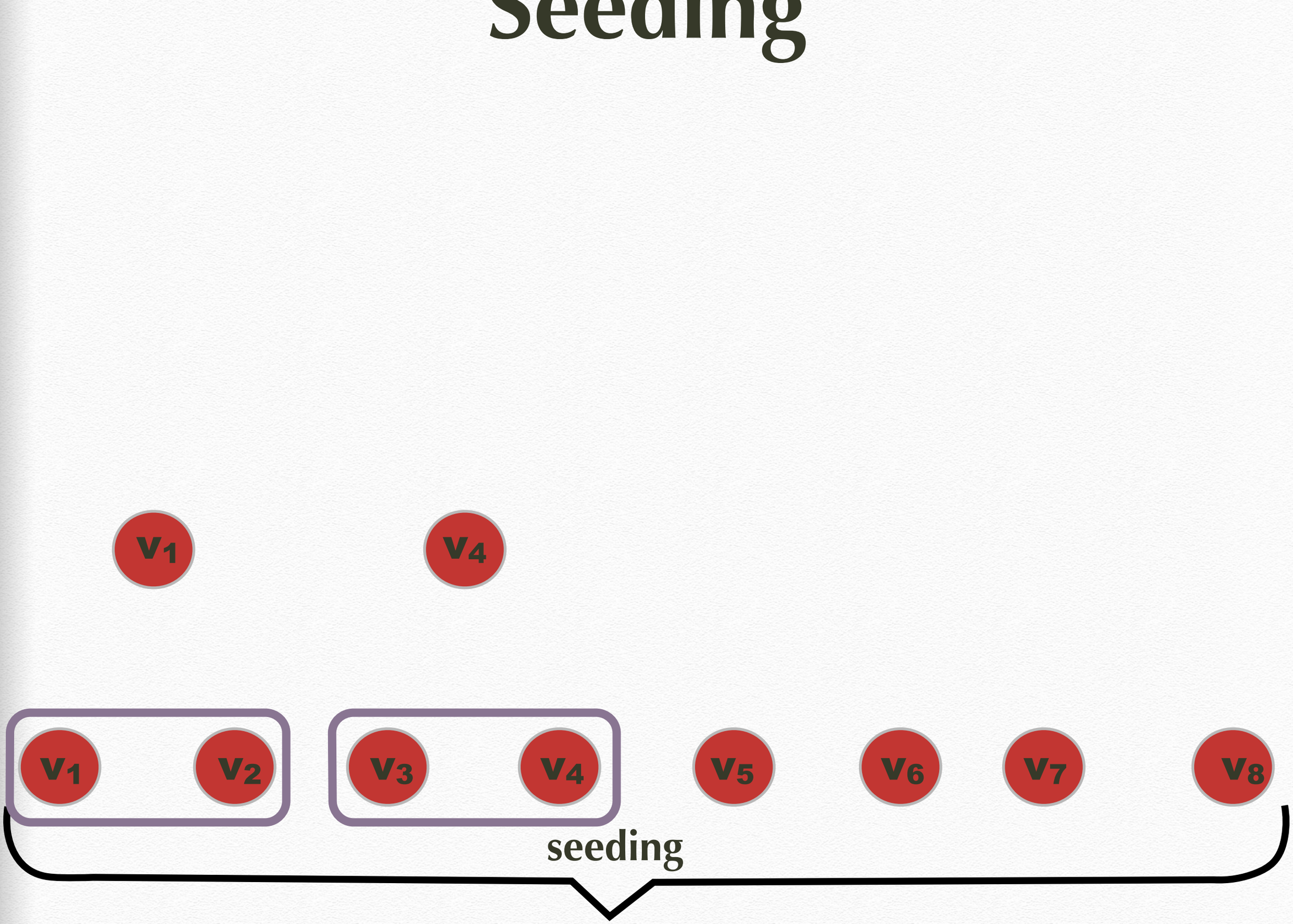

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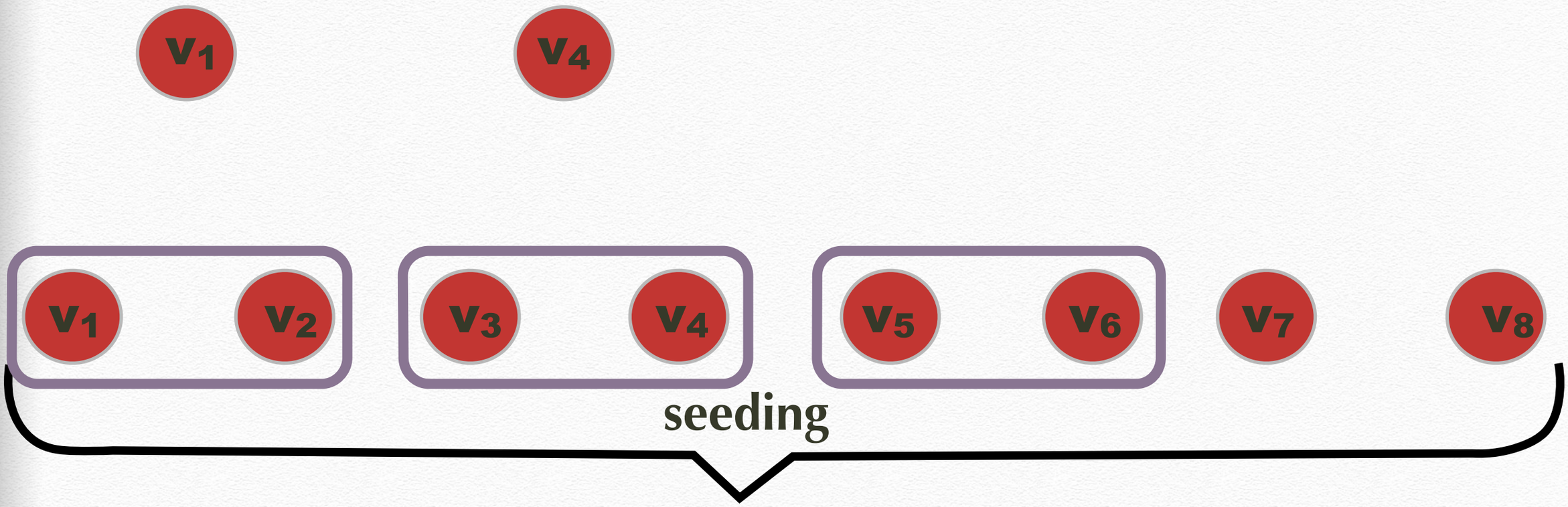

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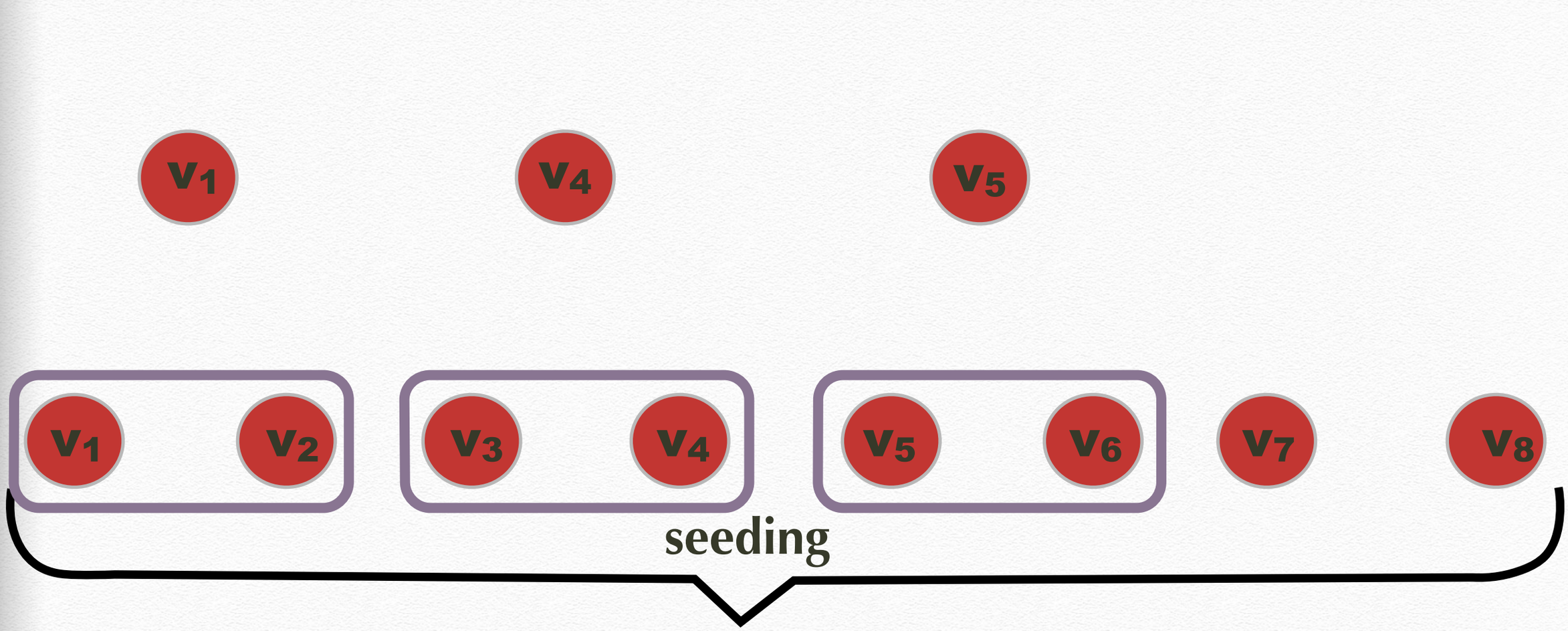

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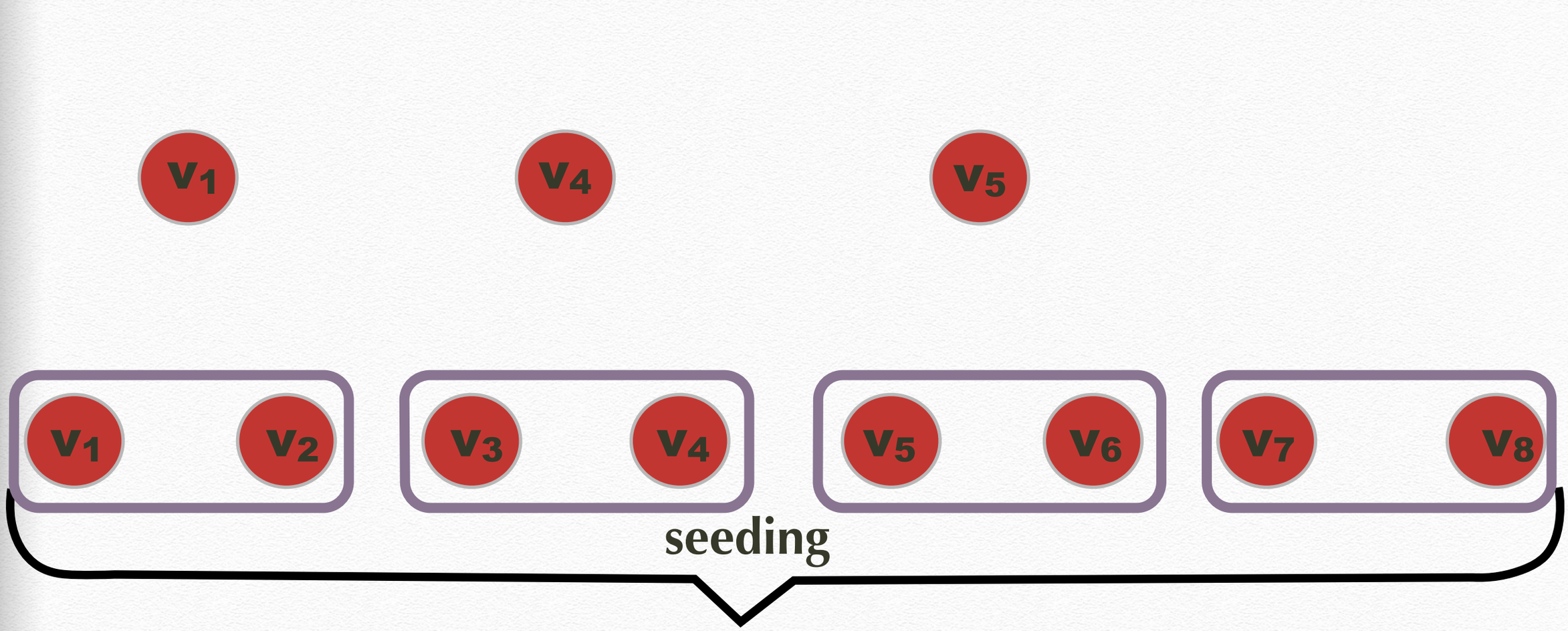

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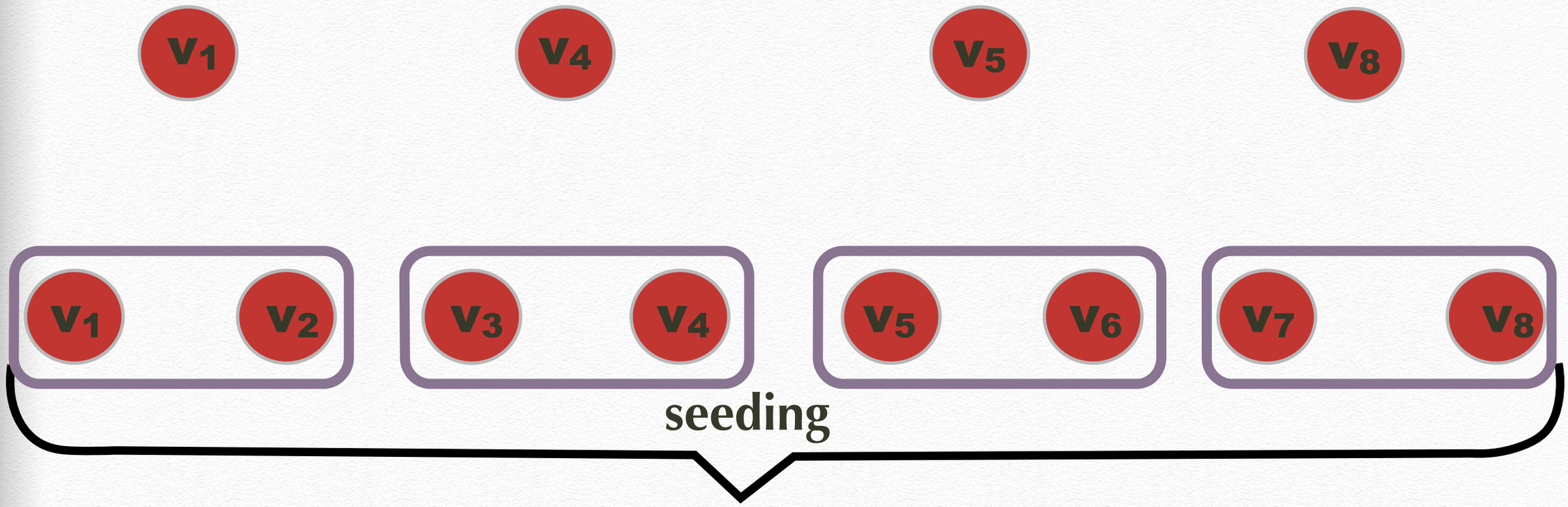

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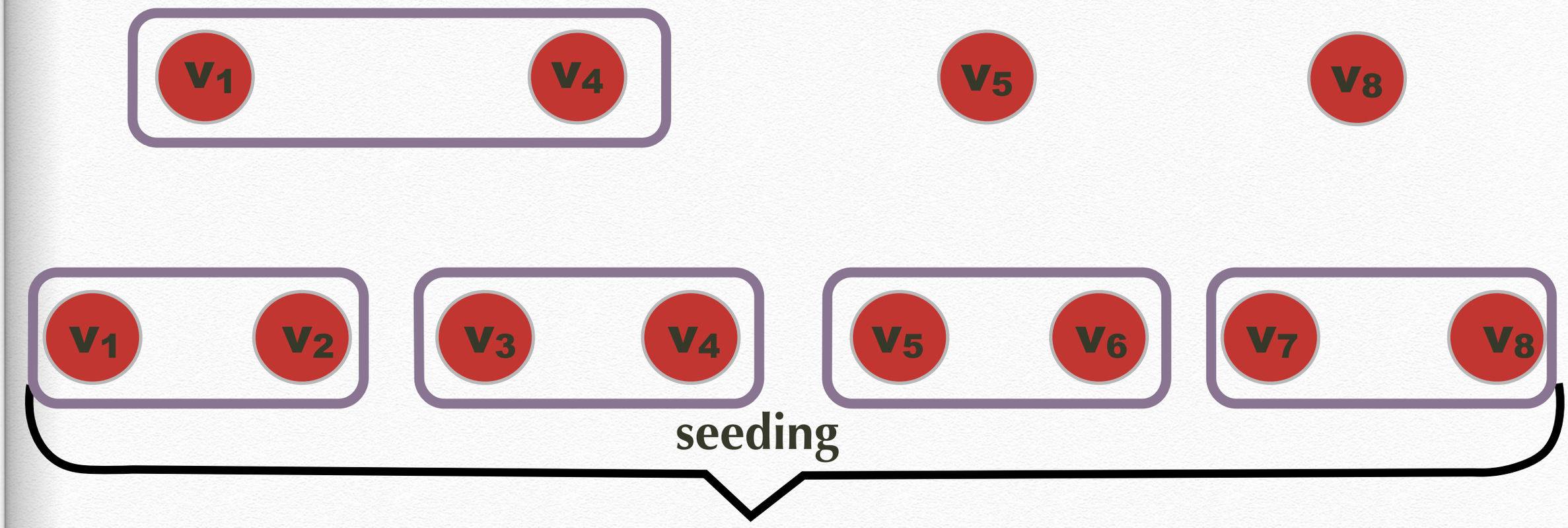

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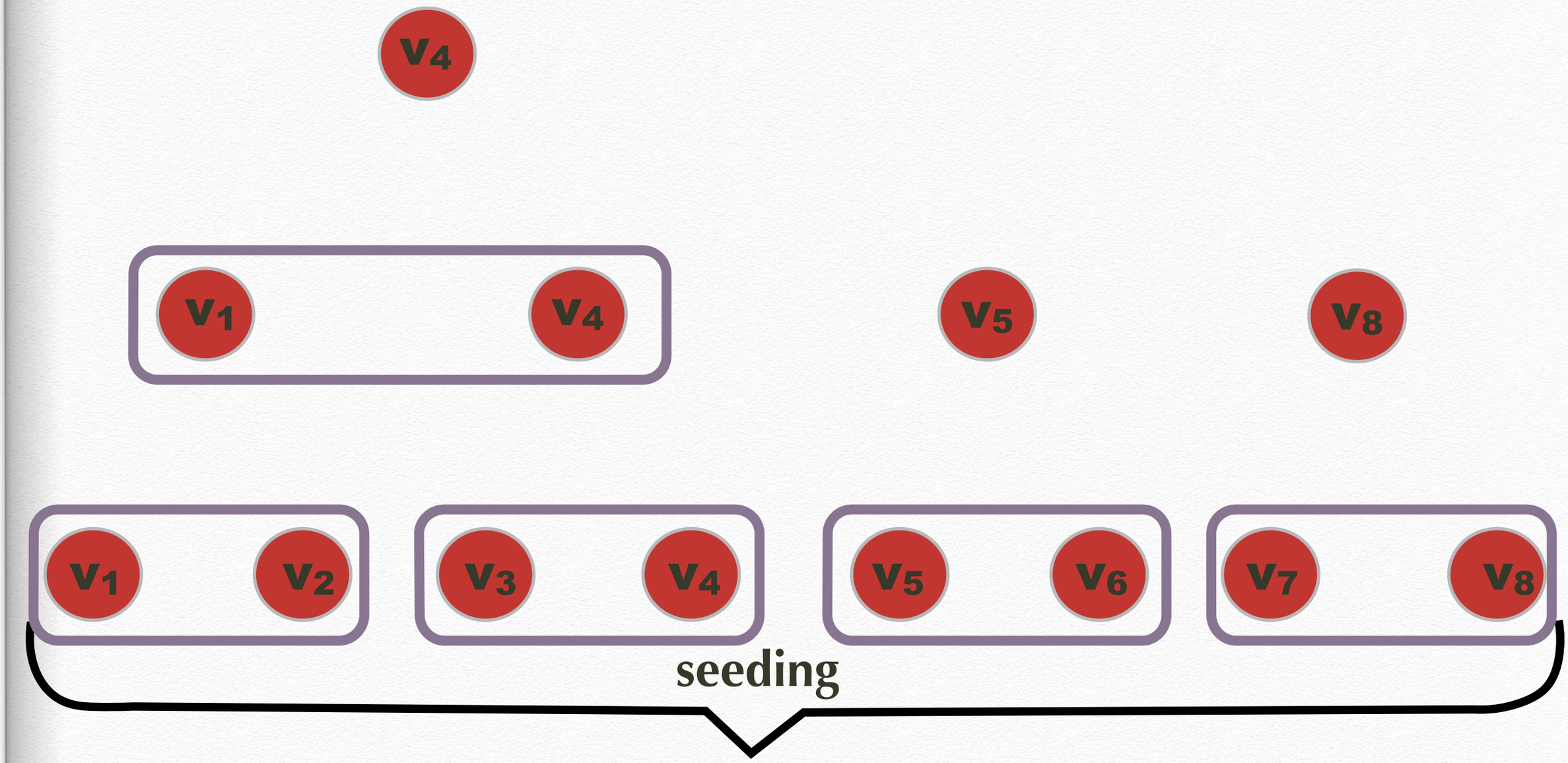

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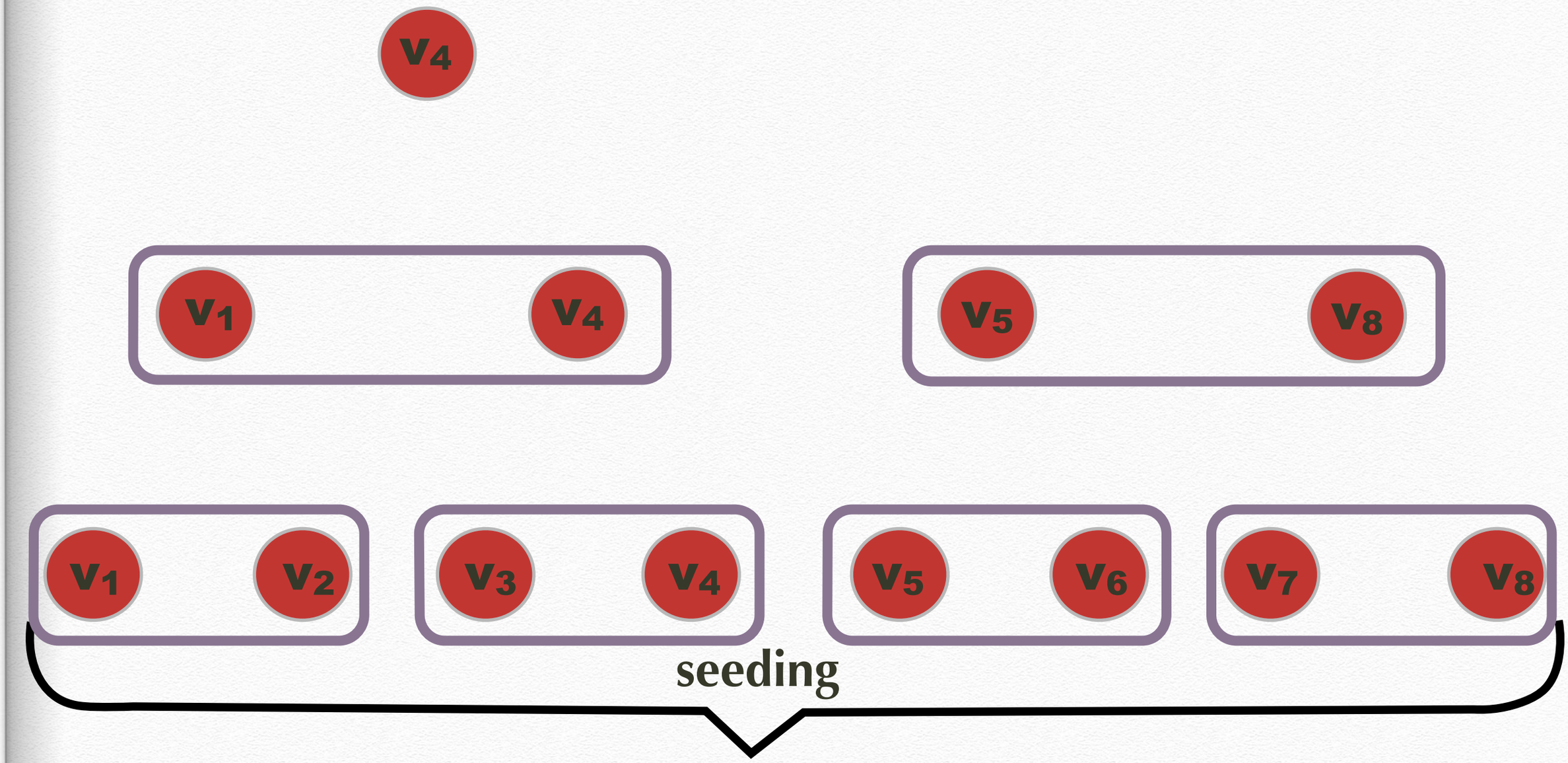

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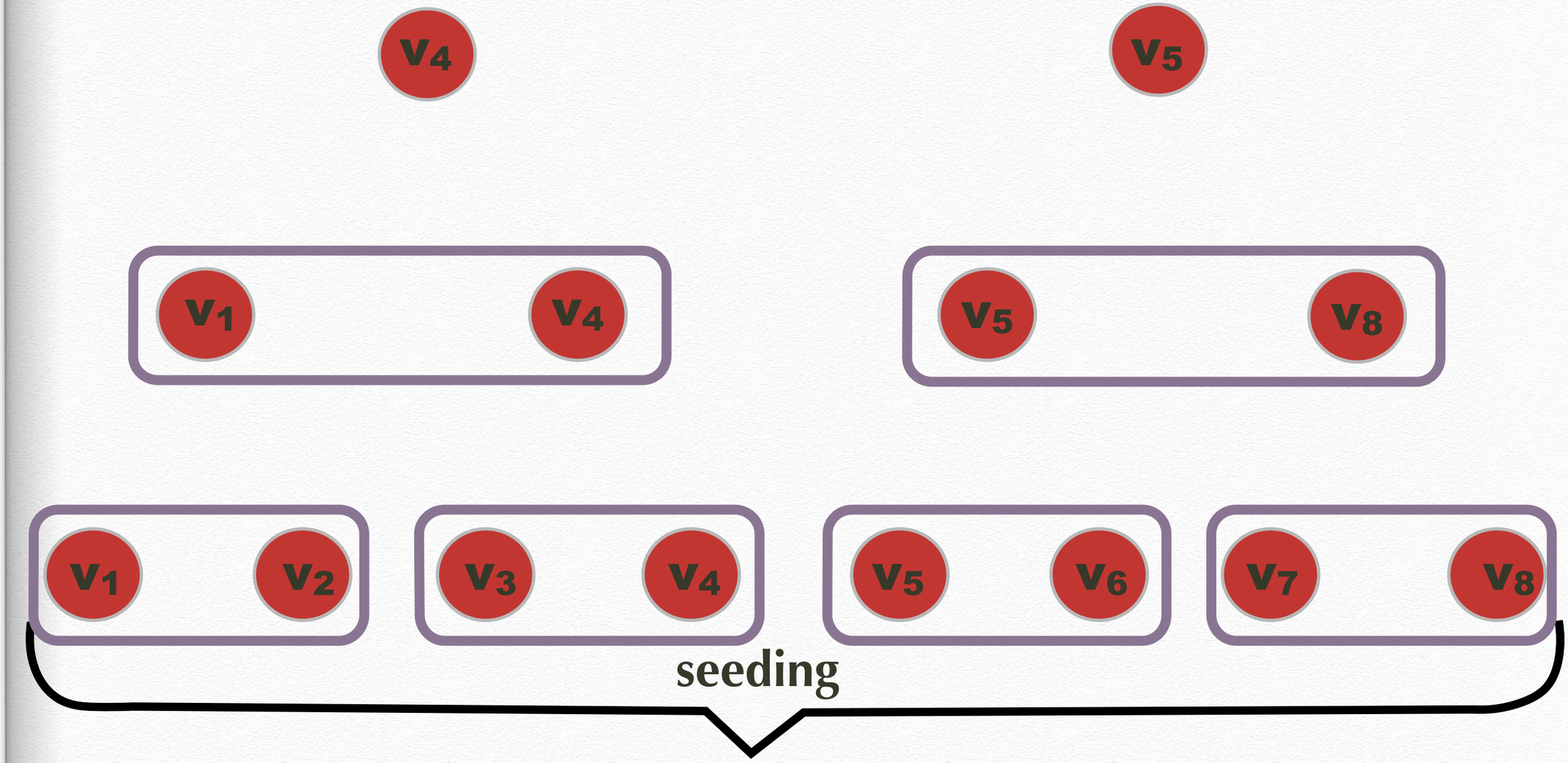

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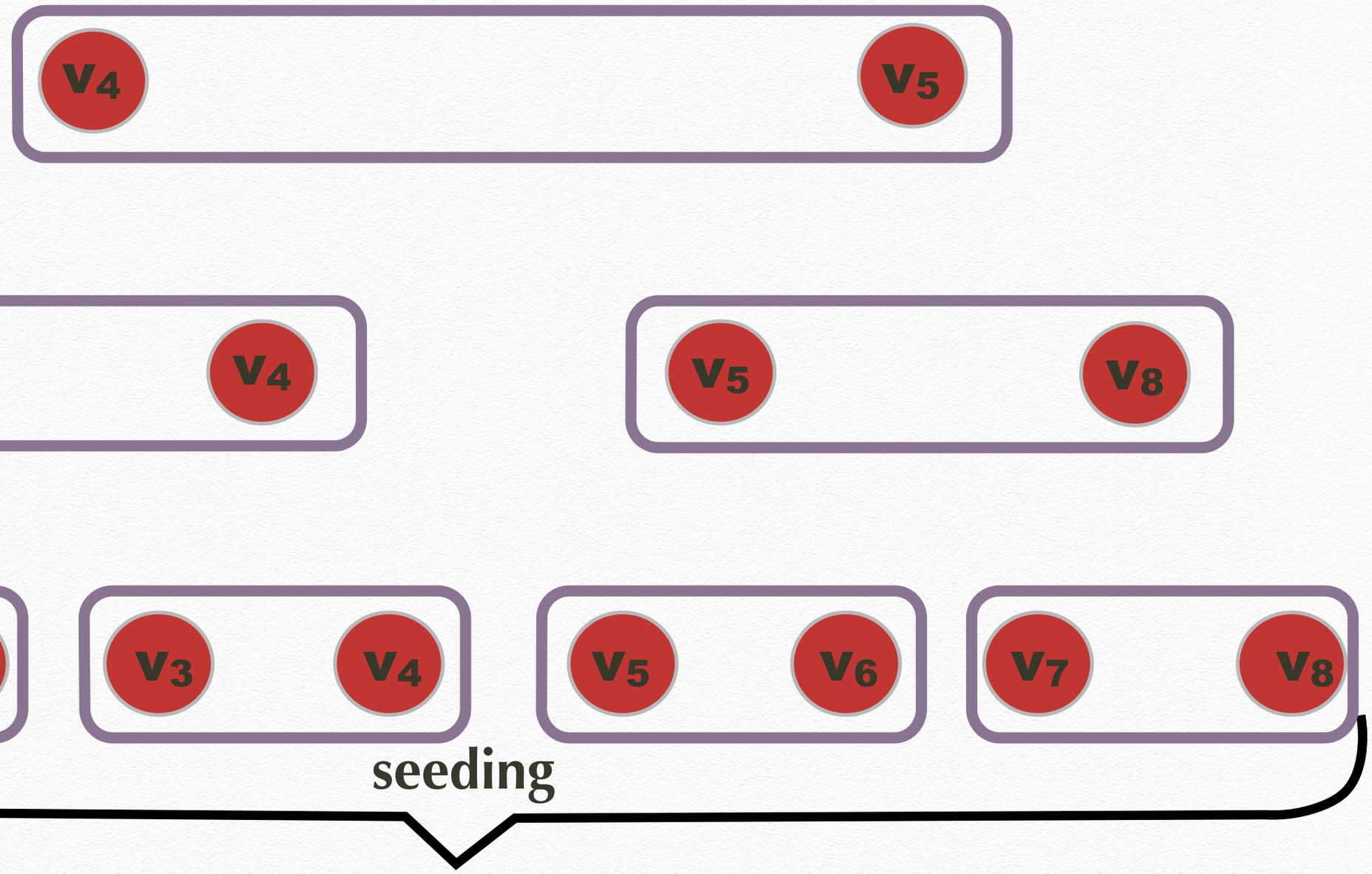

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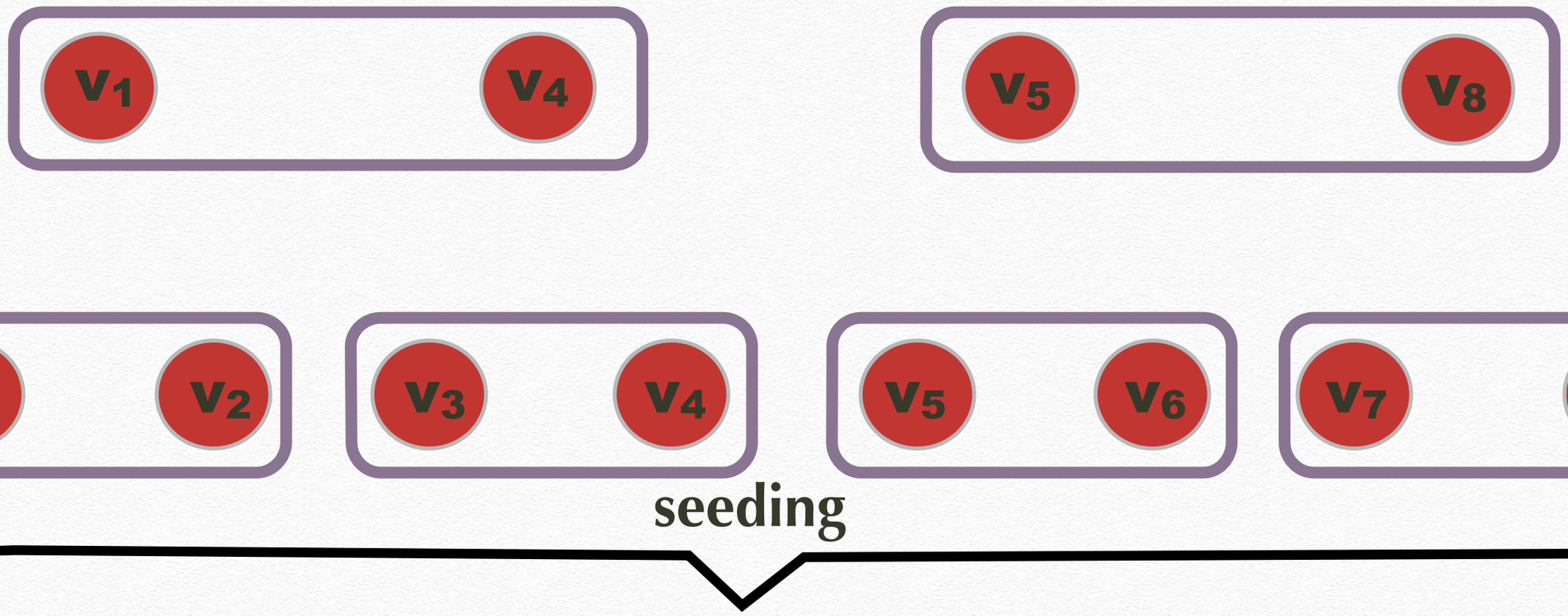

Is there a **seeding** to ensure that *favourite* wins the tournament ?

Our problem: TOURNAMENT FIXING

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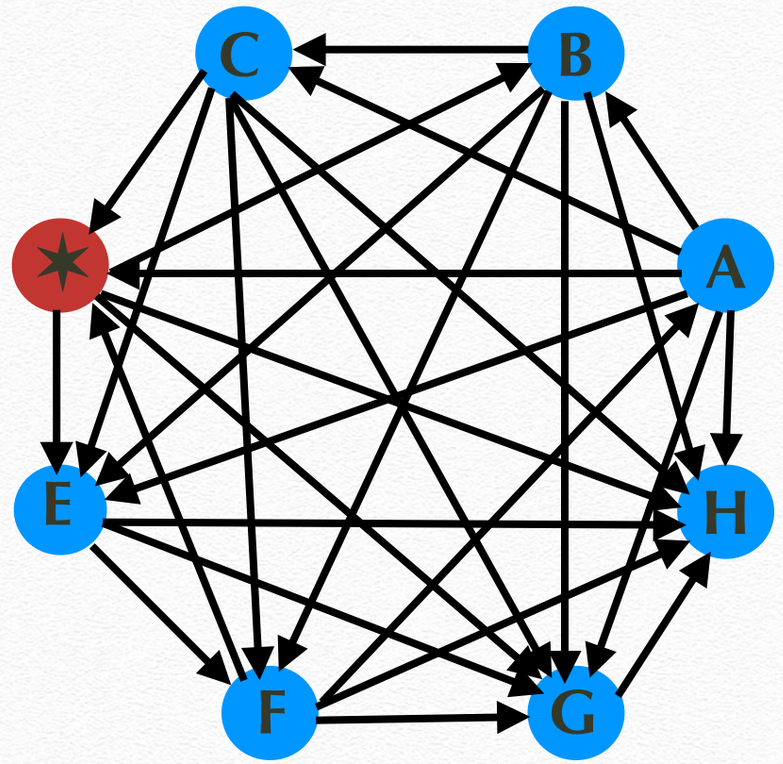
INPUT: Win-lose graph



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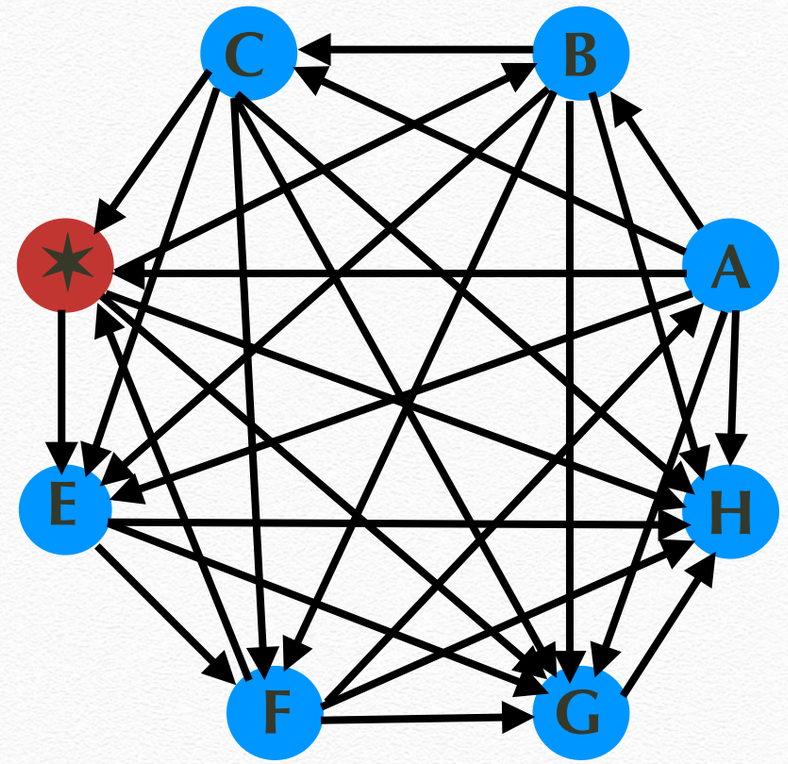

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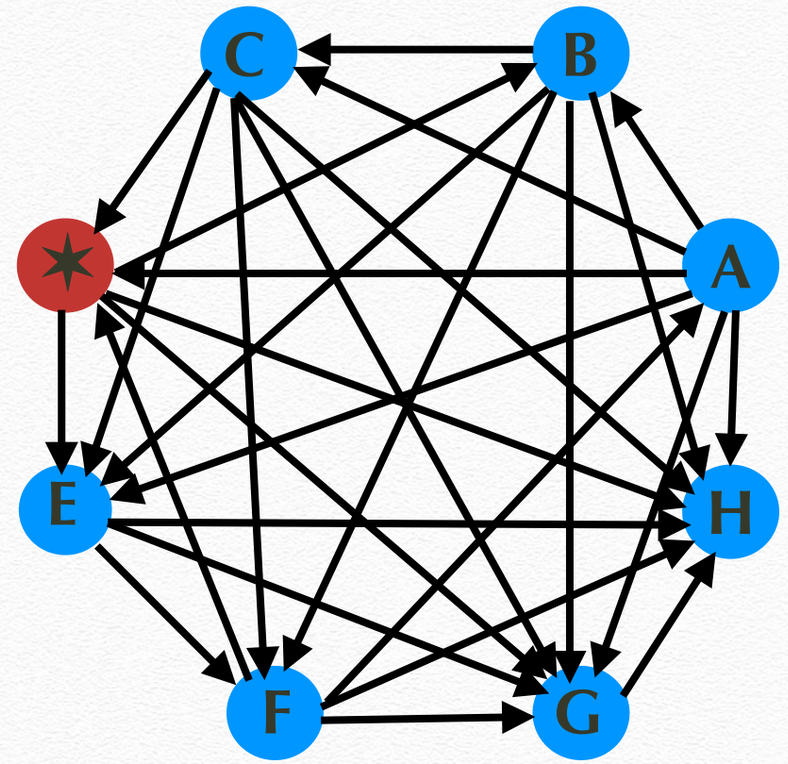

…introduced by Vu,Altman,Shoham
AAMAS'09

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[Aziz et al. AAAI'14]

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....But, how do we define **rank**?

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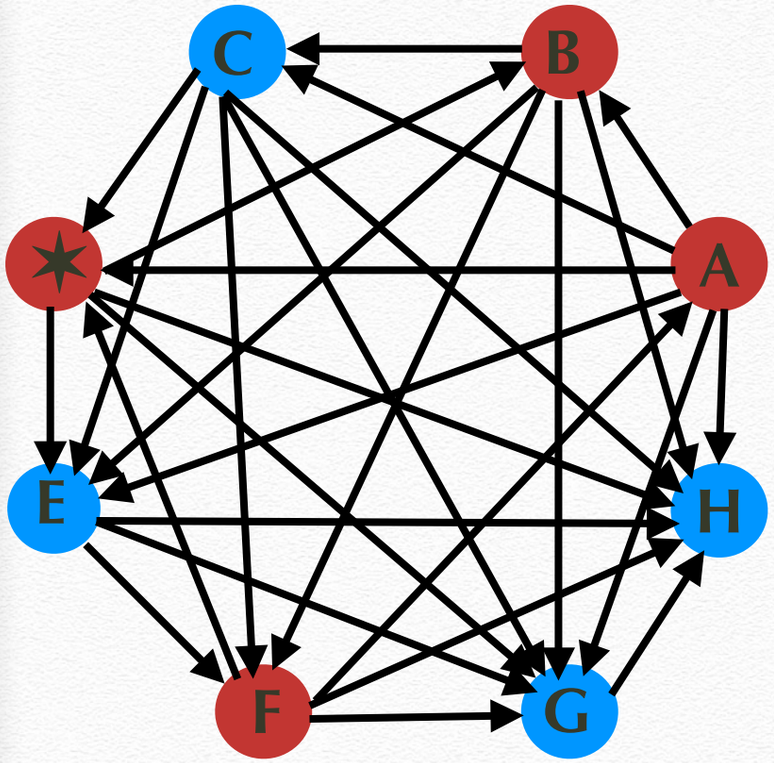

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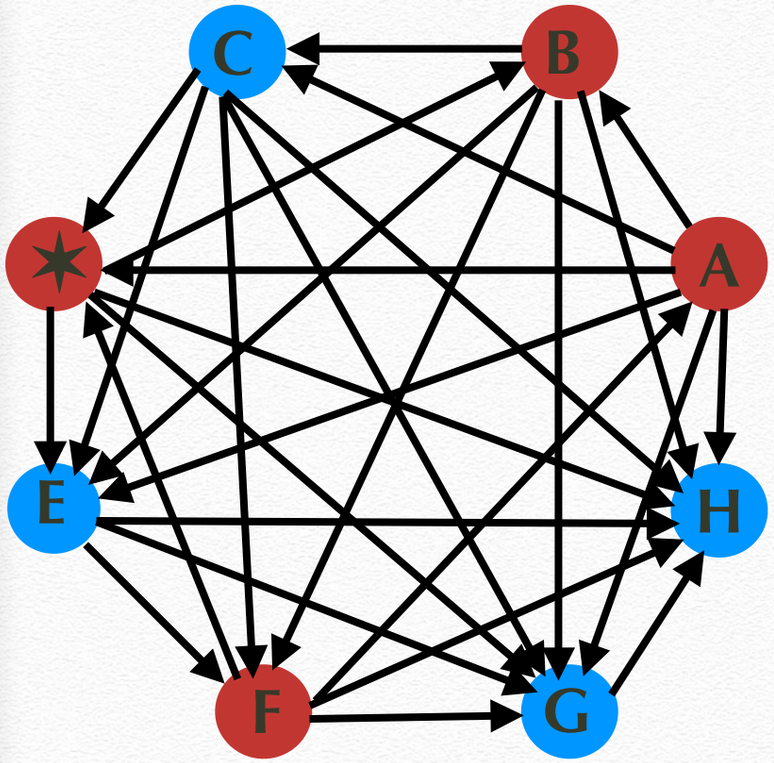

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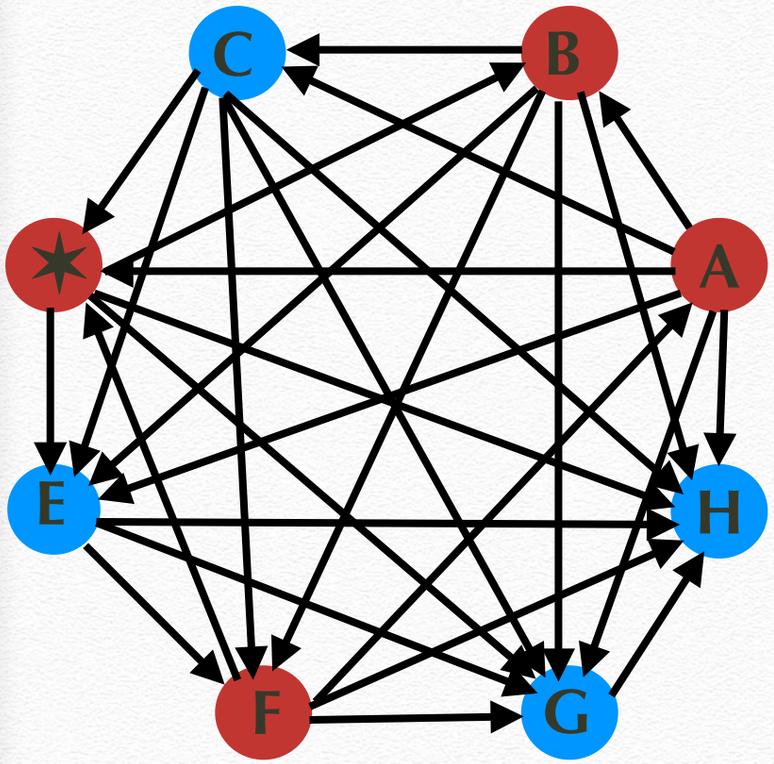

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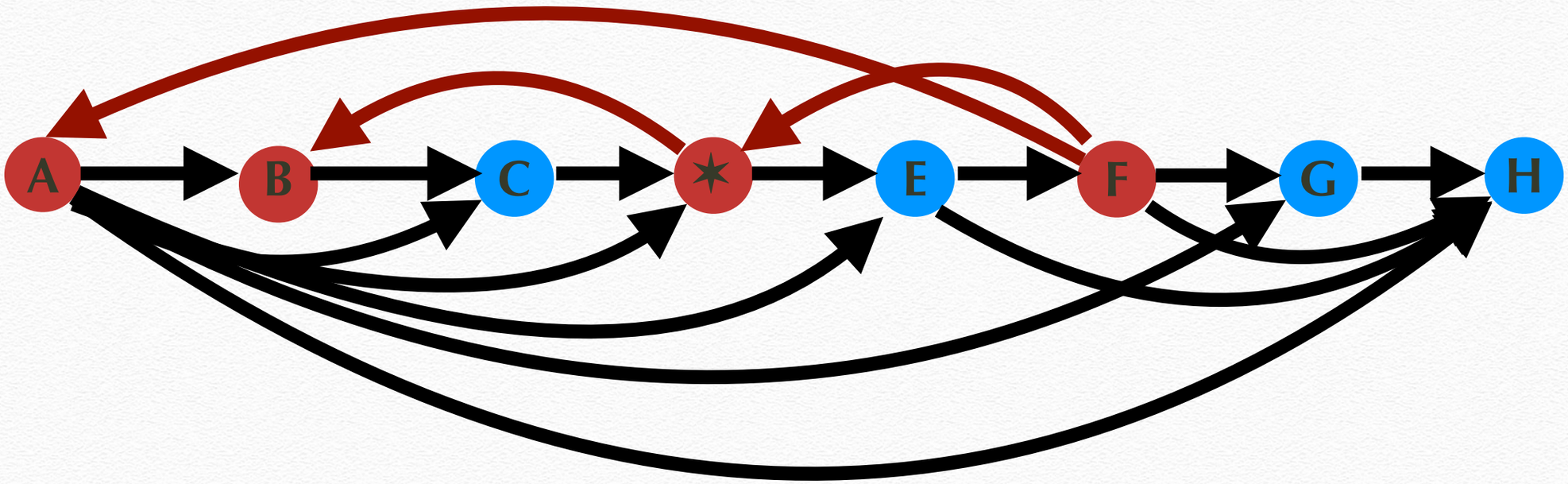

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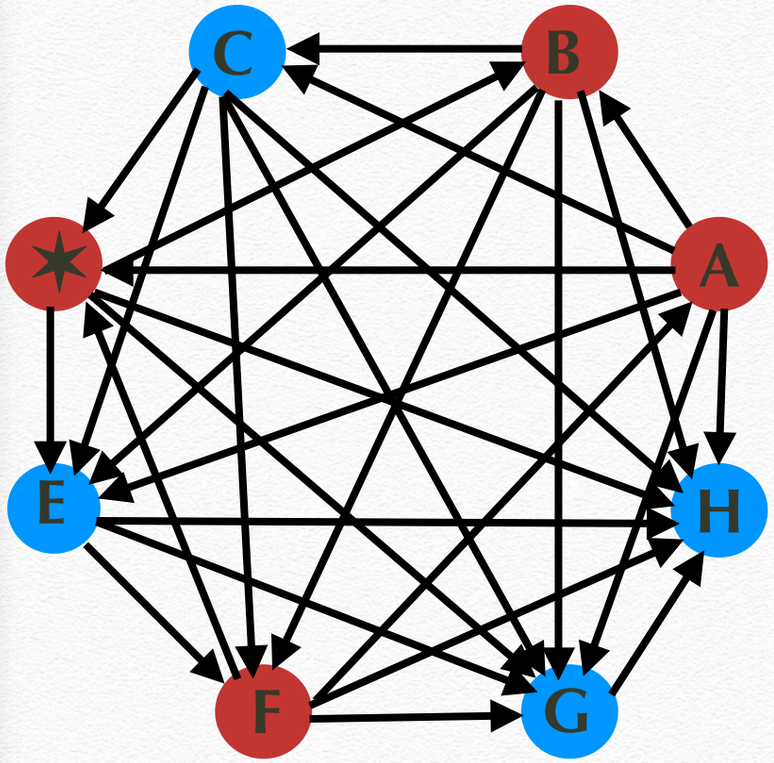

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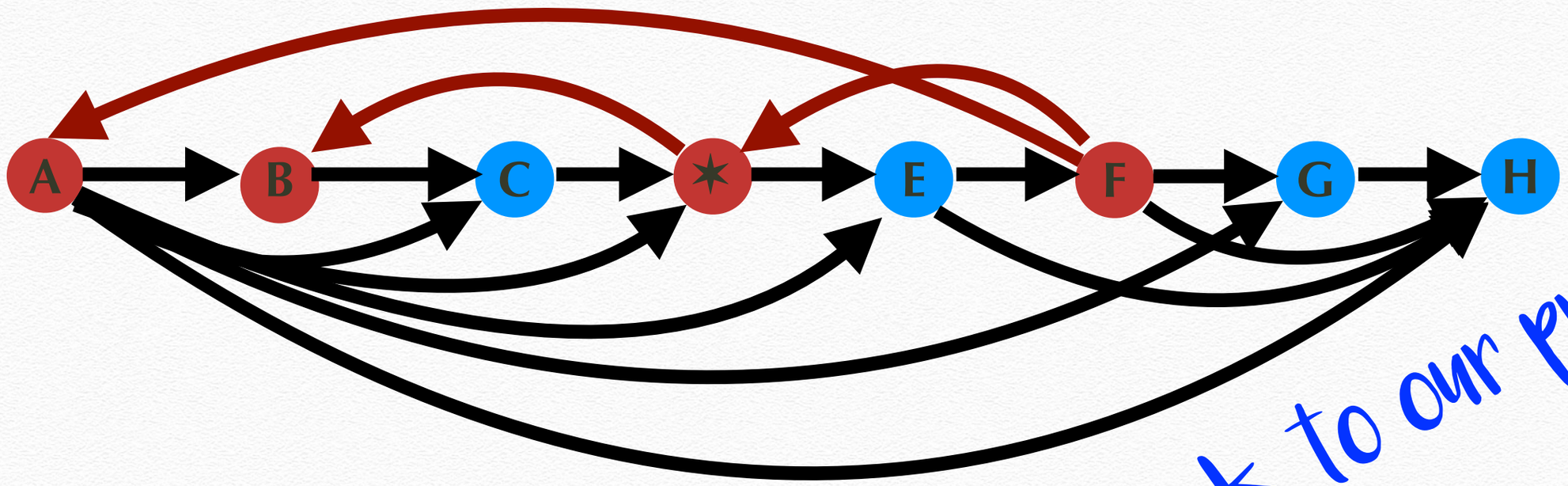

Back to our problem

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(2) $2^{O(k^2 \log k)}poly(n)$ using ILP [Ramanujam and Szeider AAAI'17]

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Gupta,Roy,Saurabh & Zehavi IJCAI'18

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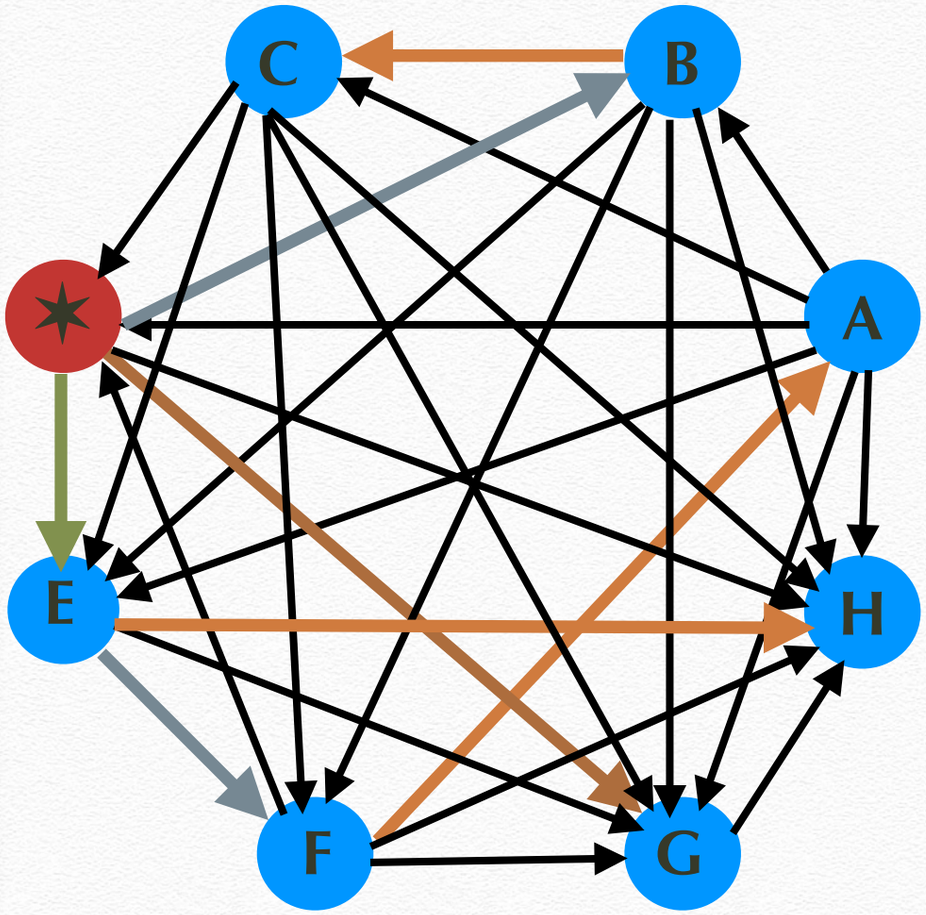
- **Algorithm runs in time $2^{O(k \log k)}poly(n)$**
- **Combinatorial algorithm using a greedy strategy**
- **Reveals structural properties**

Spanning Binomial Arborescences (SBA)



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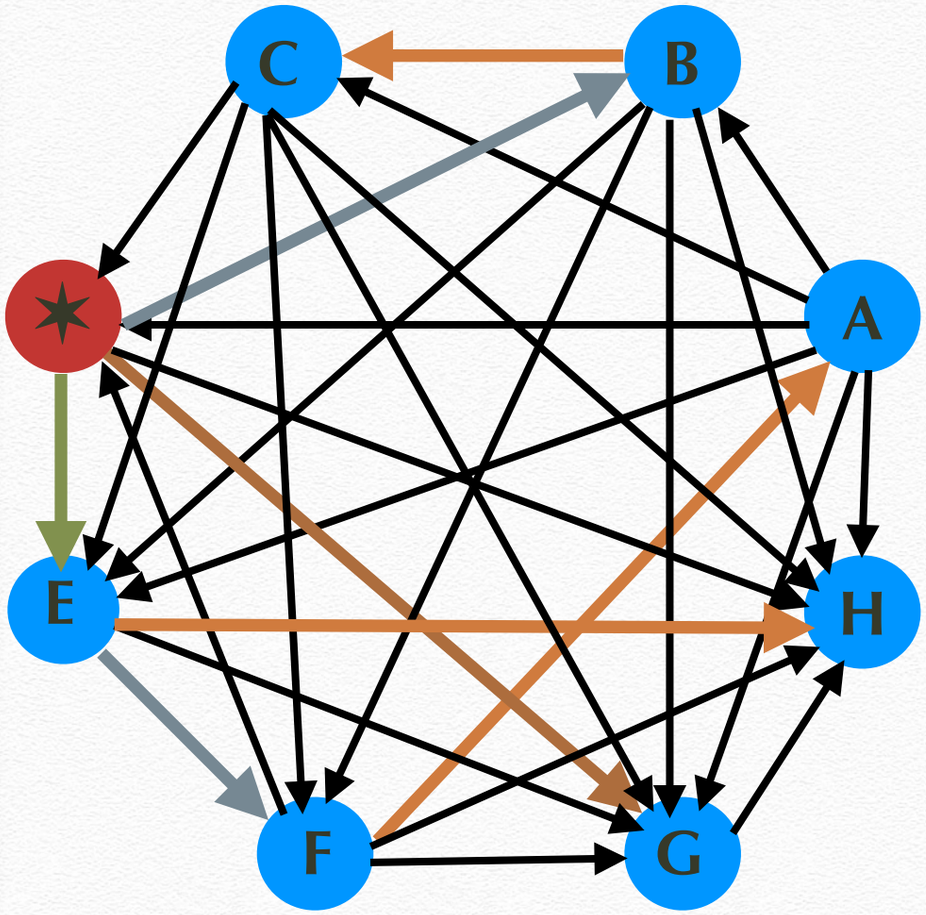

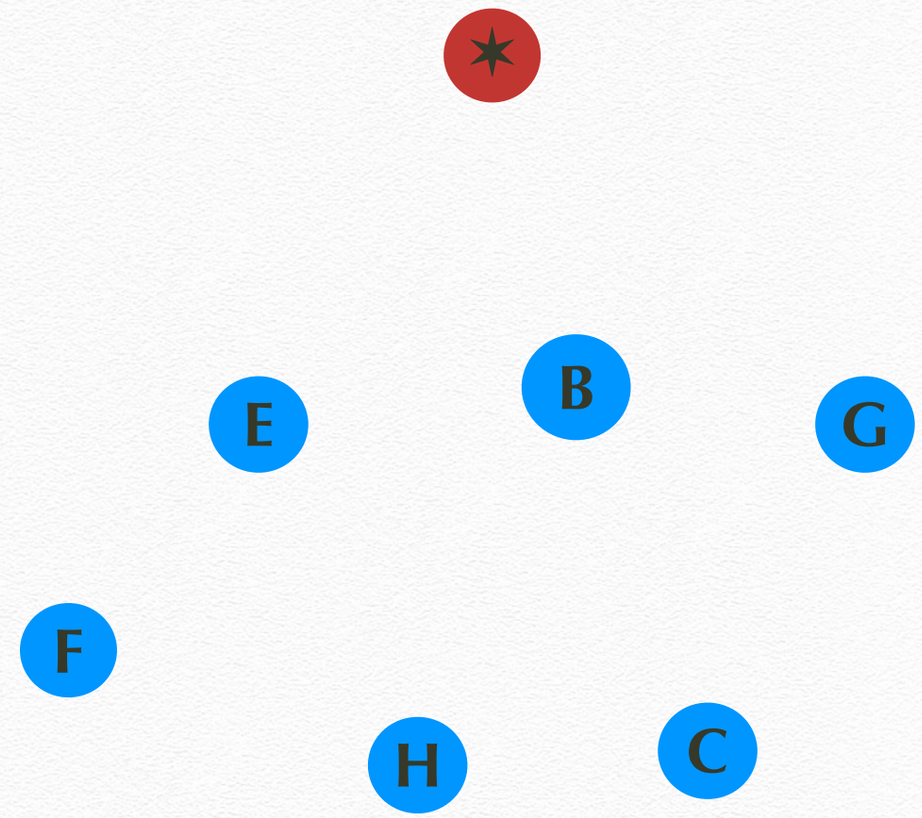

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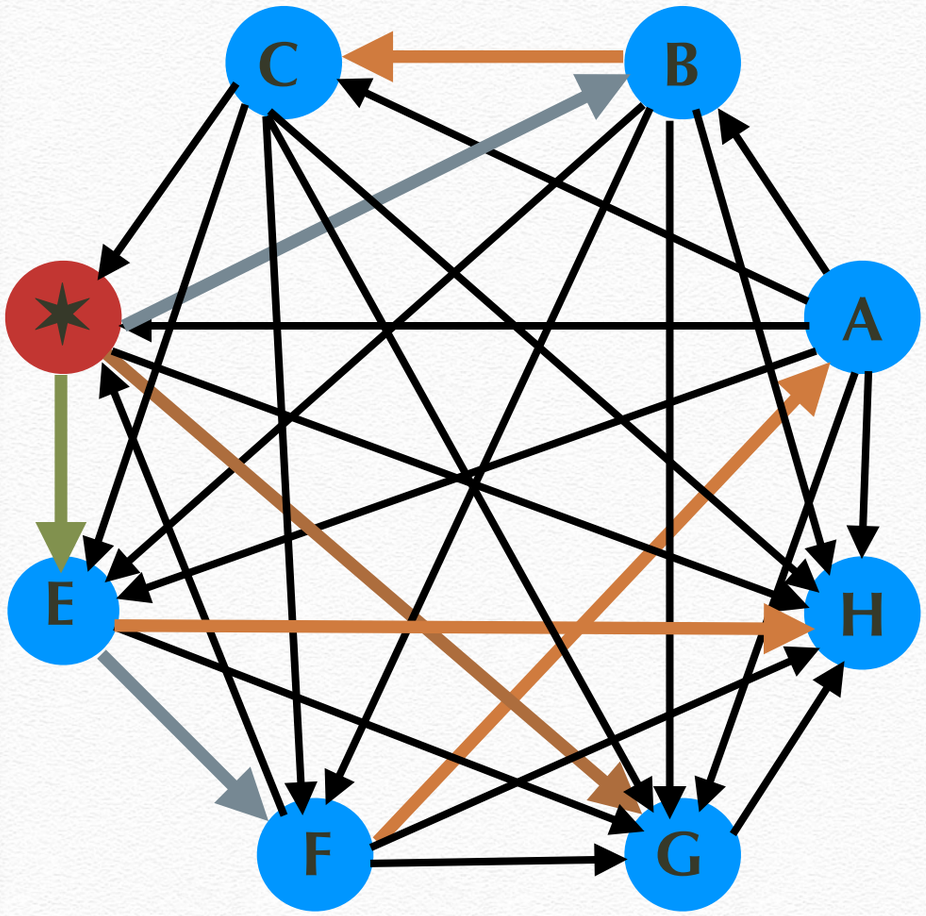

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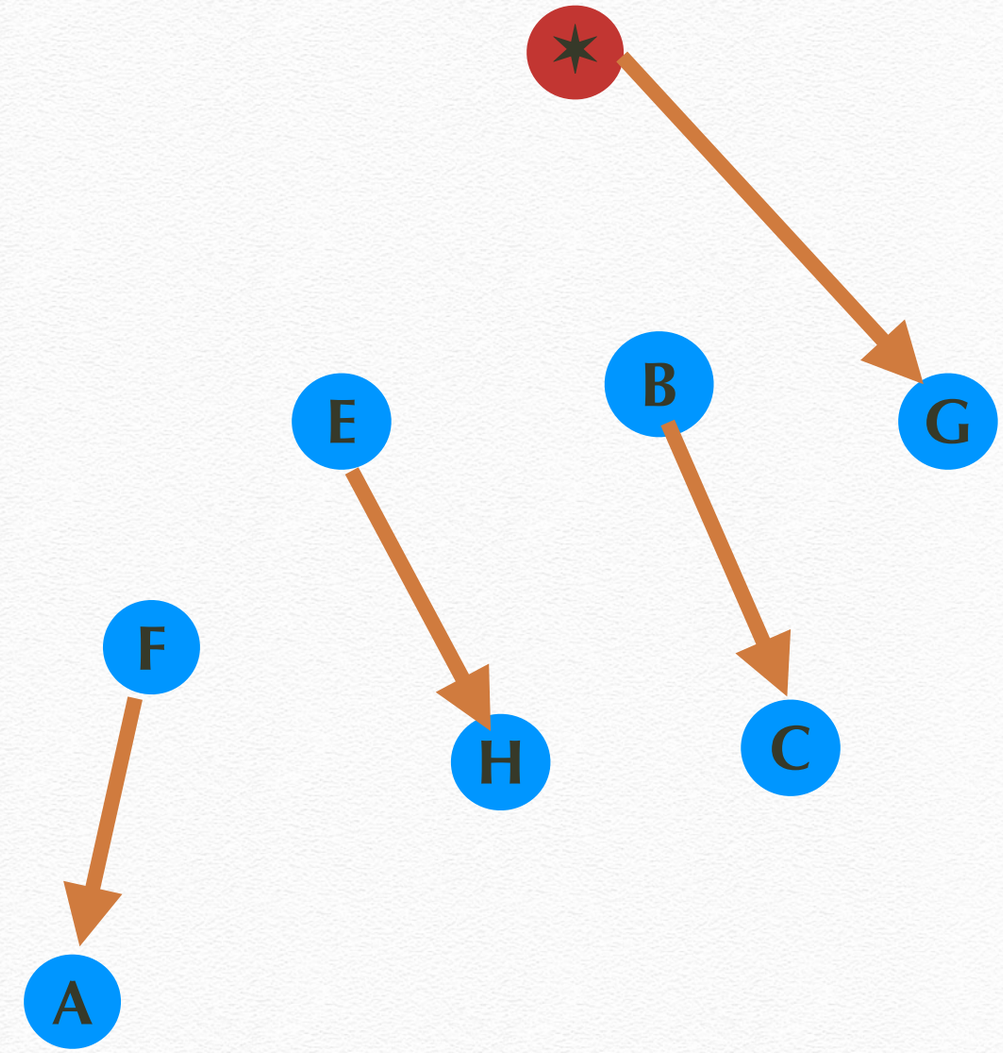

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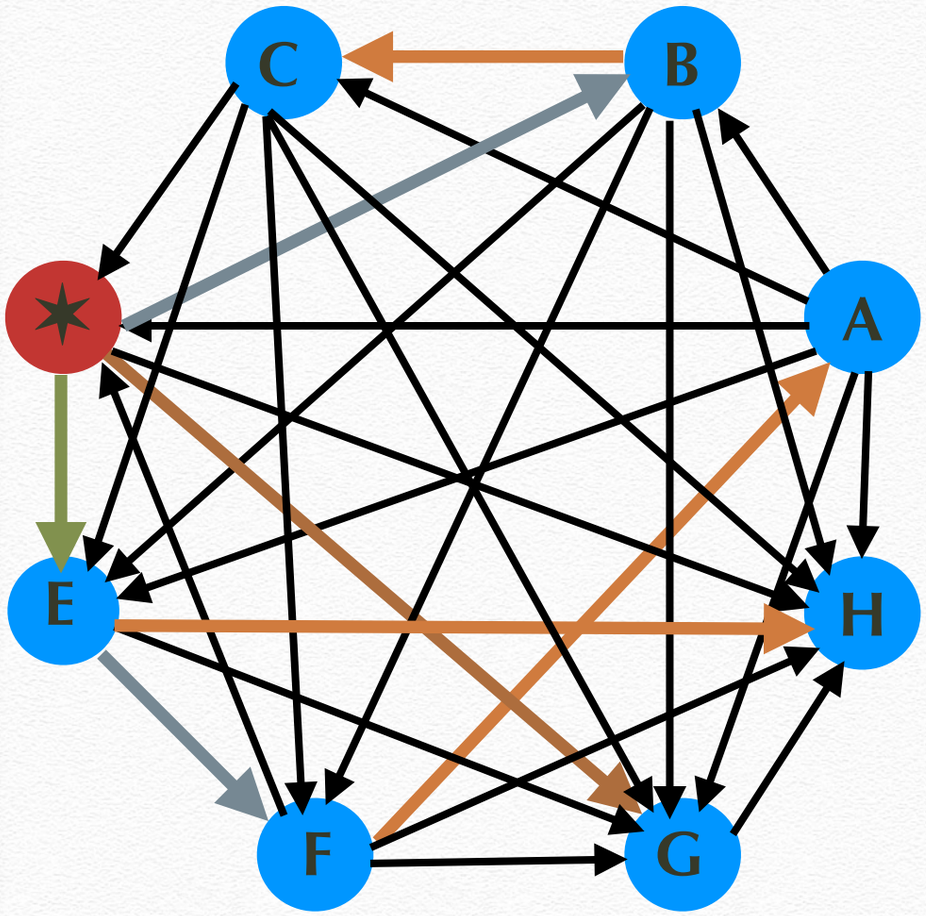

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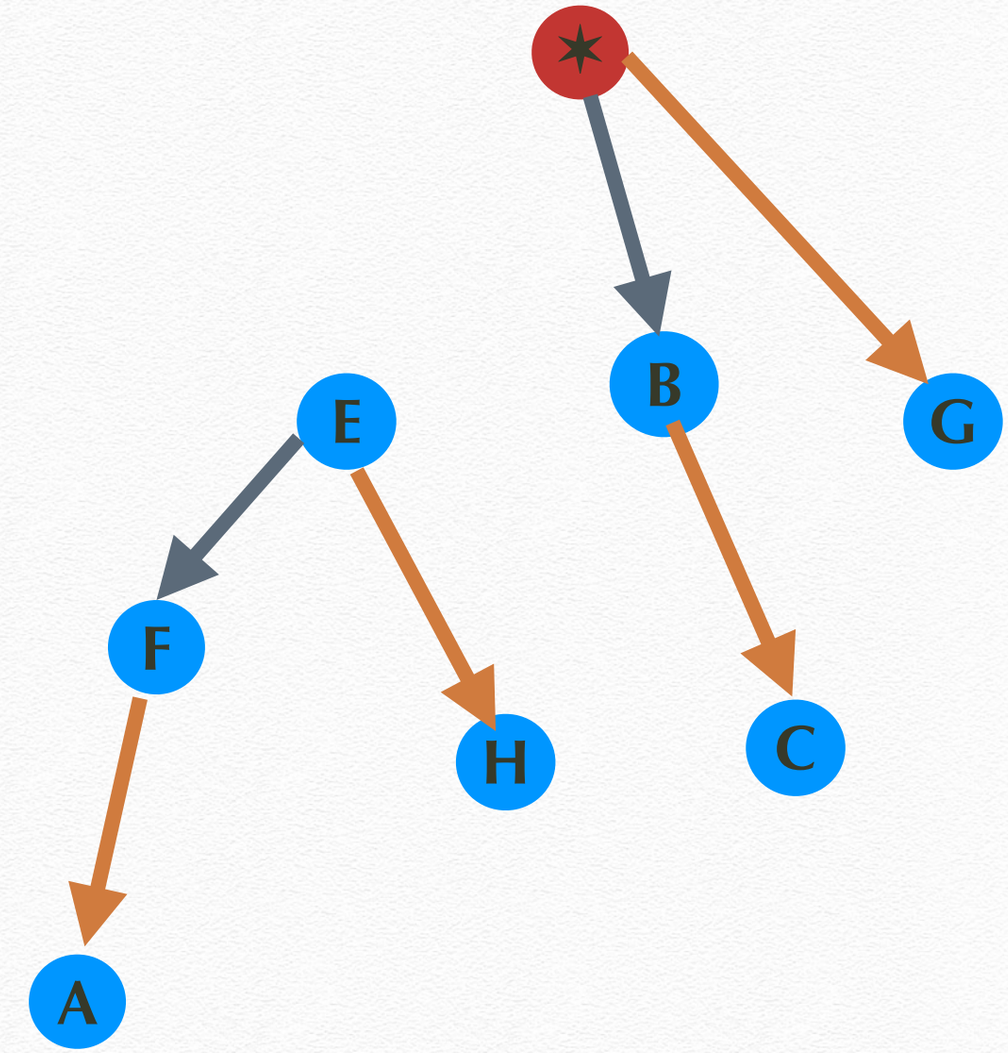

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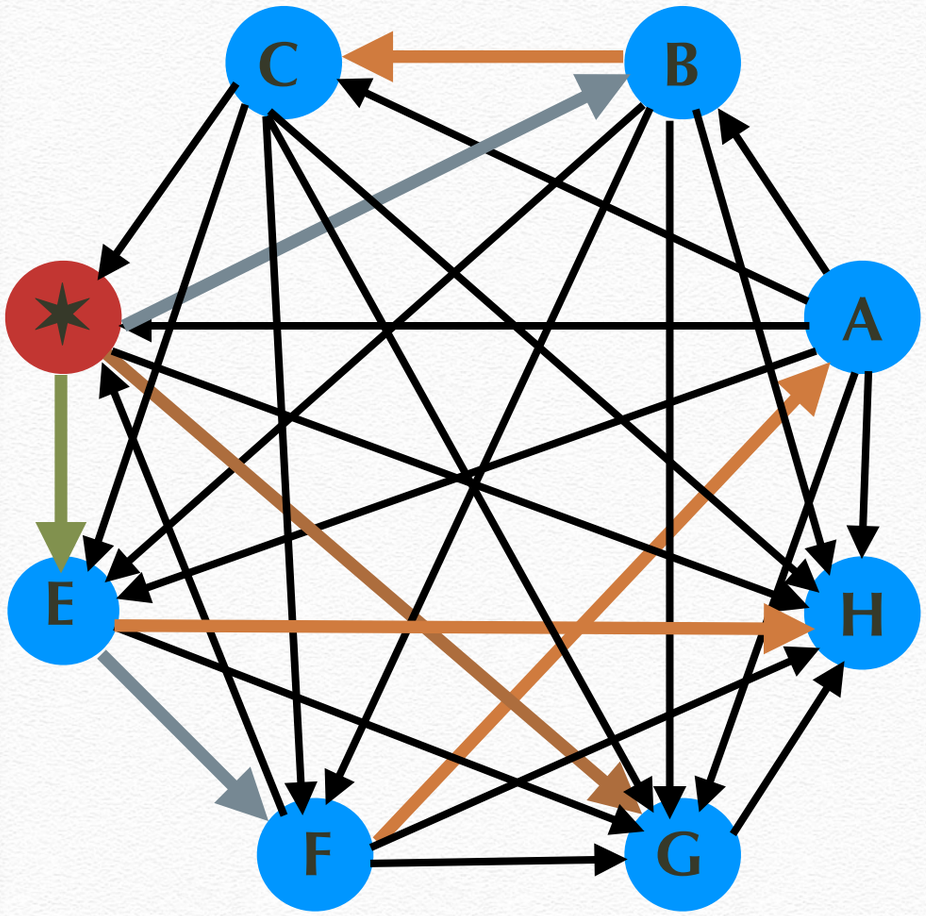
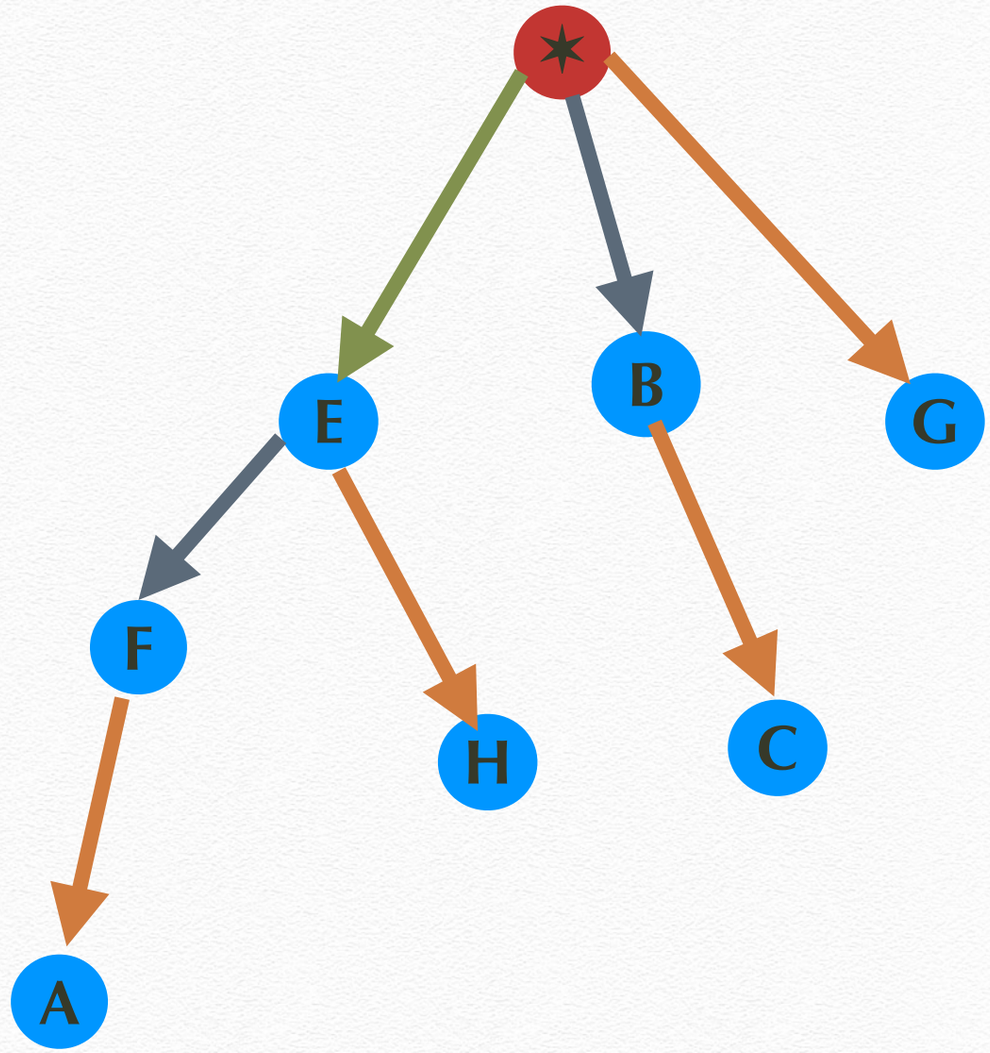


Win-lose graph



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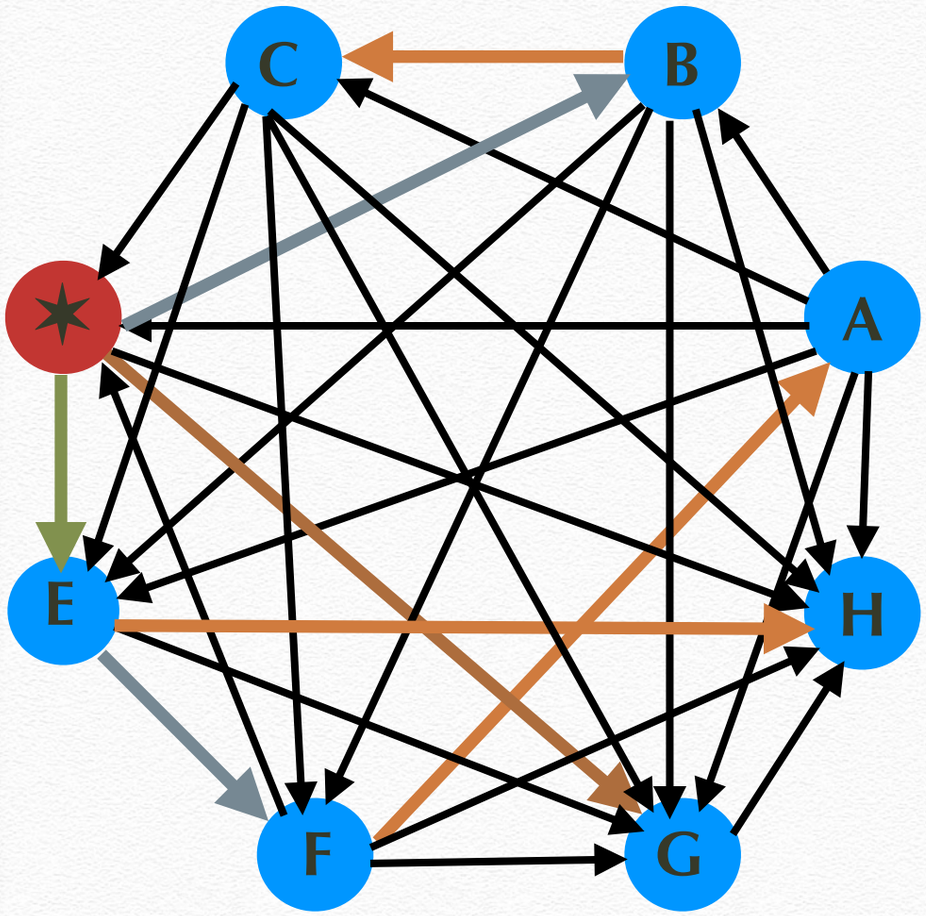

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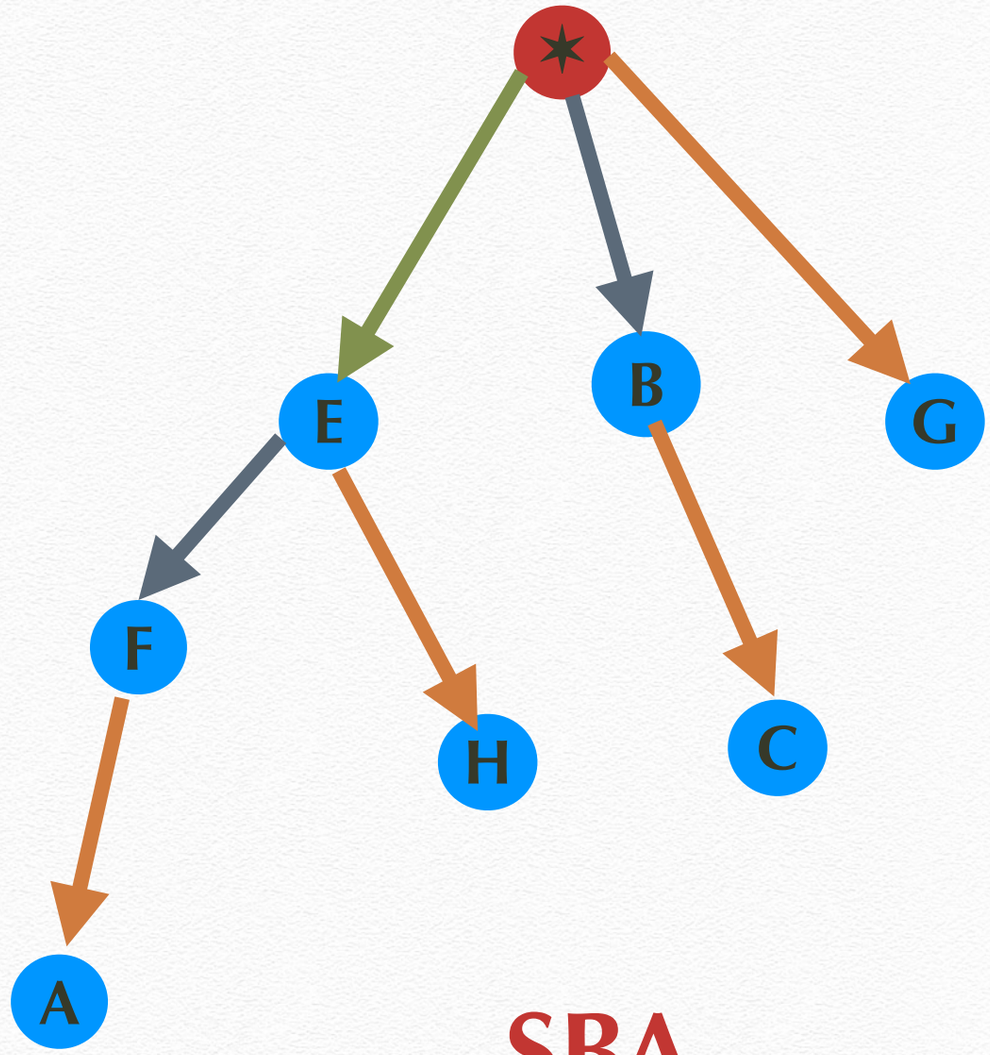

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If **T** ⊆ **D** (a directed graph) and **V(T) = V(D)**, then **T** is **labeled spanning BA (SBA)**

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[Williams AAAI'10]

Tournament Fixing in terms of an SBA

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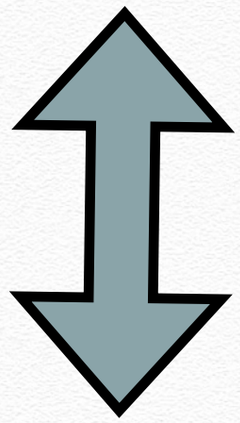

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(i) Fill up the paths & subtrees of the template

III. GREEDY: (ii) If final outcome is an SBA, then done.

(iii) Or else, guess again

What if no favorable seeding exists for *favorite* ?

Can *favorite* win with bribery?

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INPUT: Win-lose graph

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In terms of an SBA ….

Can we ***reverse*** l arcs in the win-lose graph so that there will be an SBA with *favorite* as the root ?

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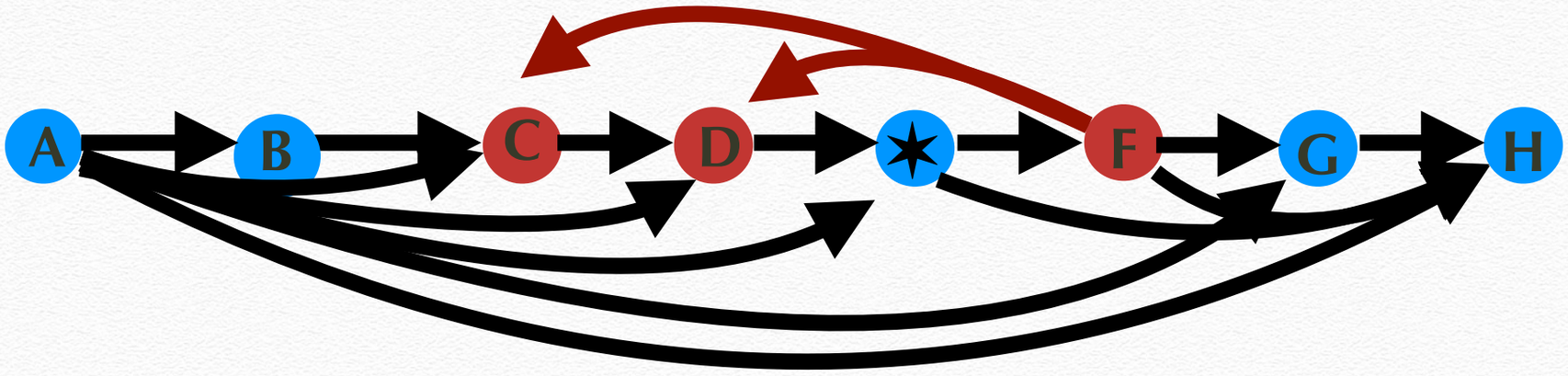
  - Uses our algorithm for TOURNAMENT FIXING

Elite (Players) Club



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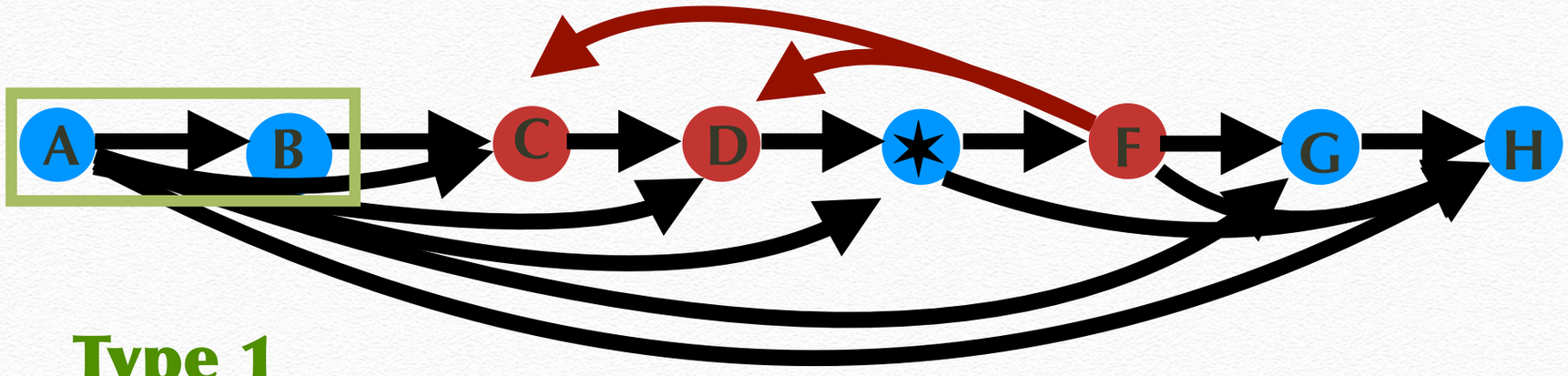

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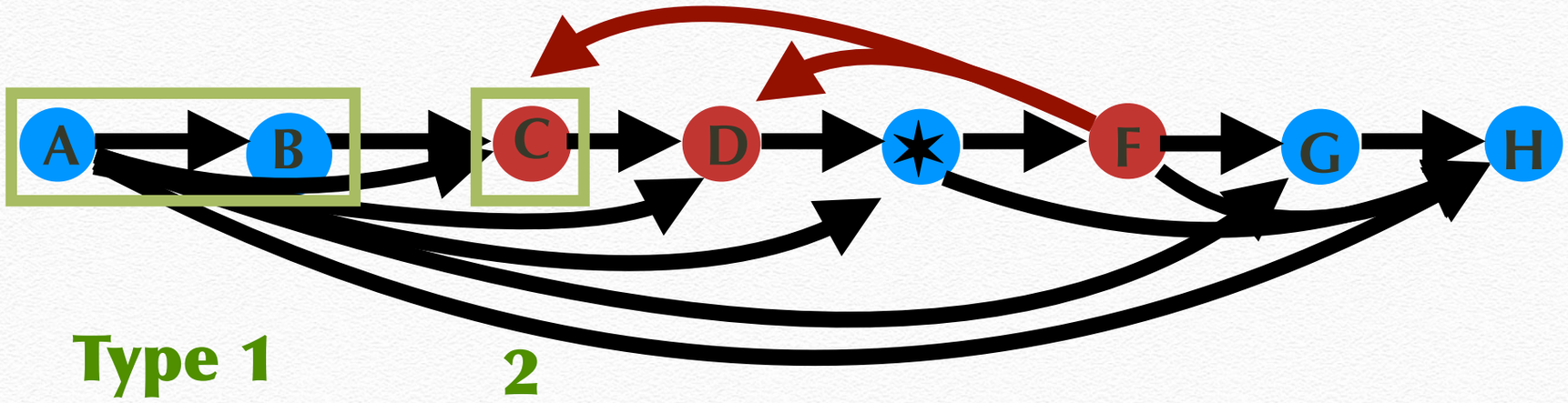
Elite (Players) Club



Type 1 2

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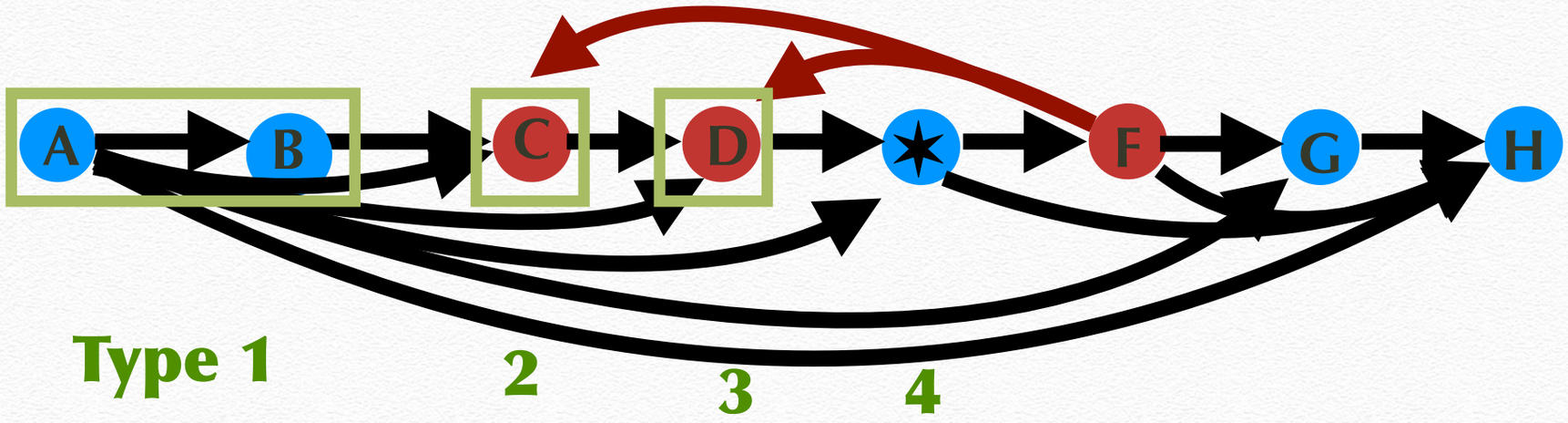

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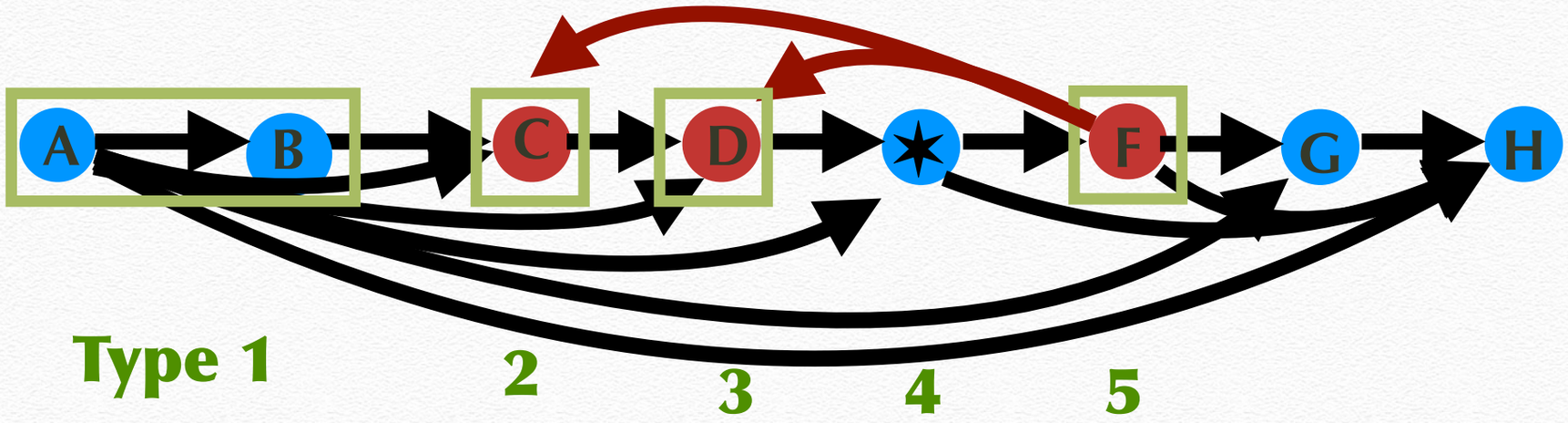

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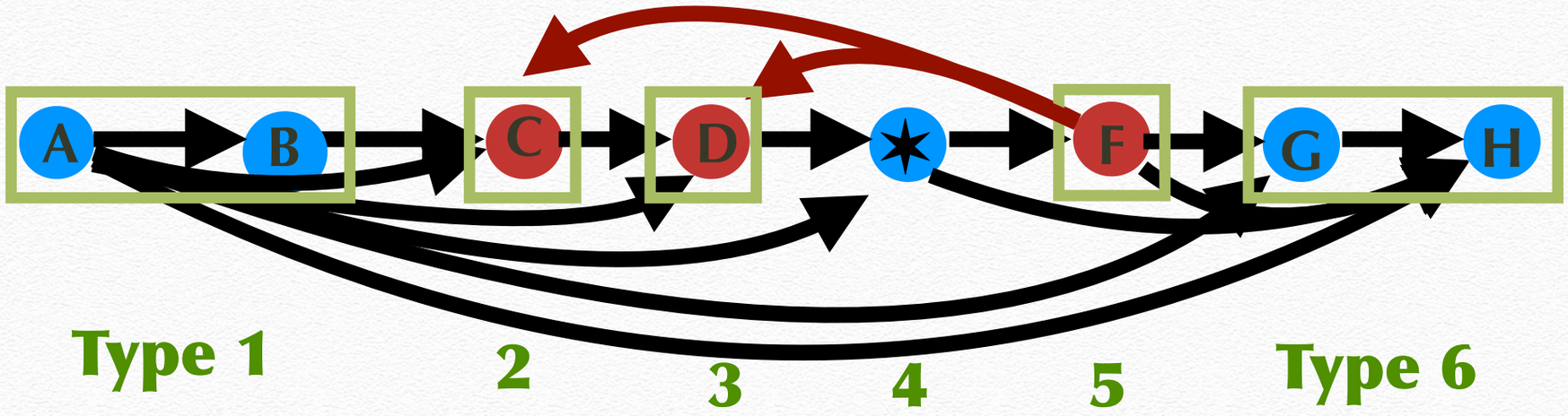

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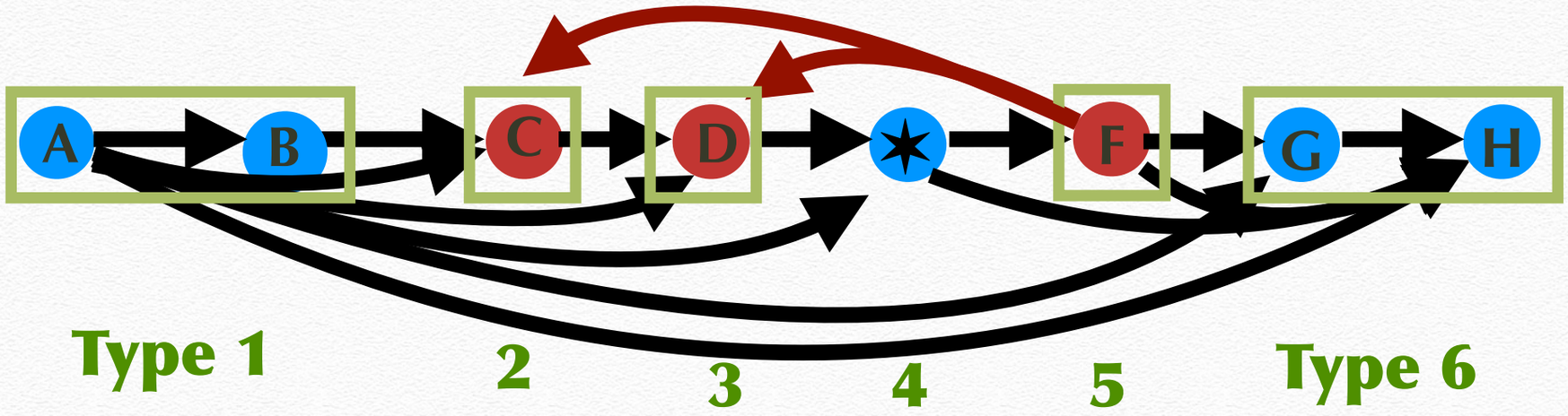

S is an Elite Club if

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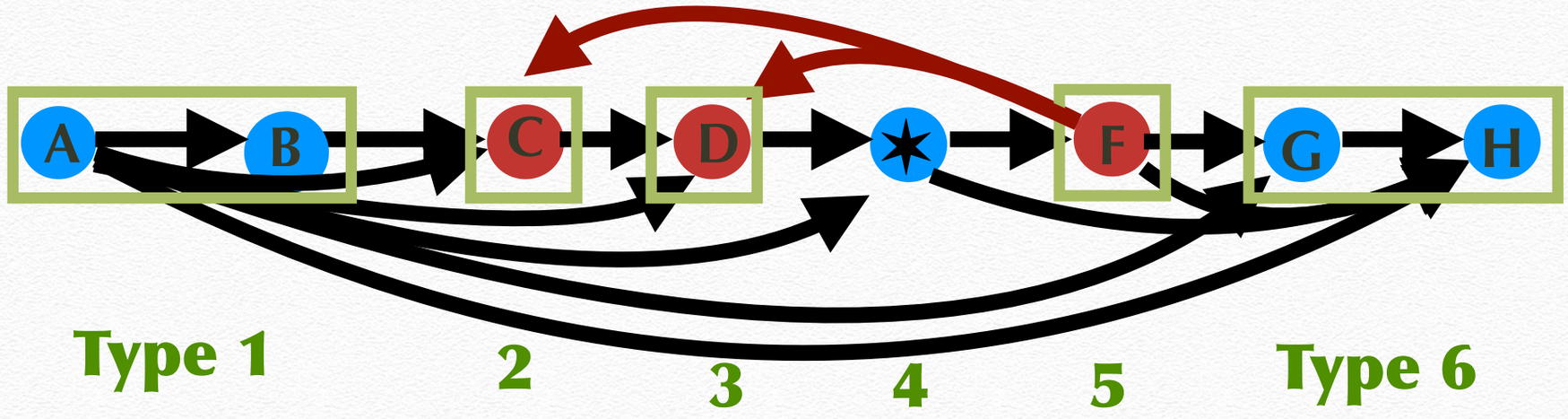

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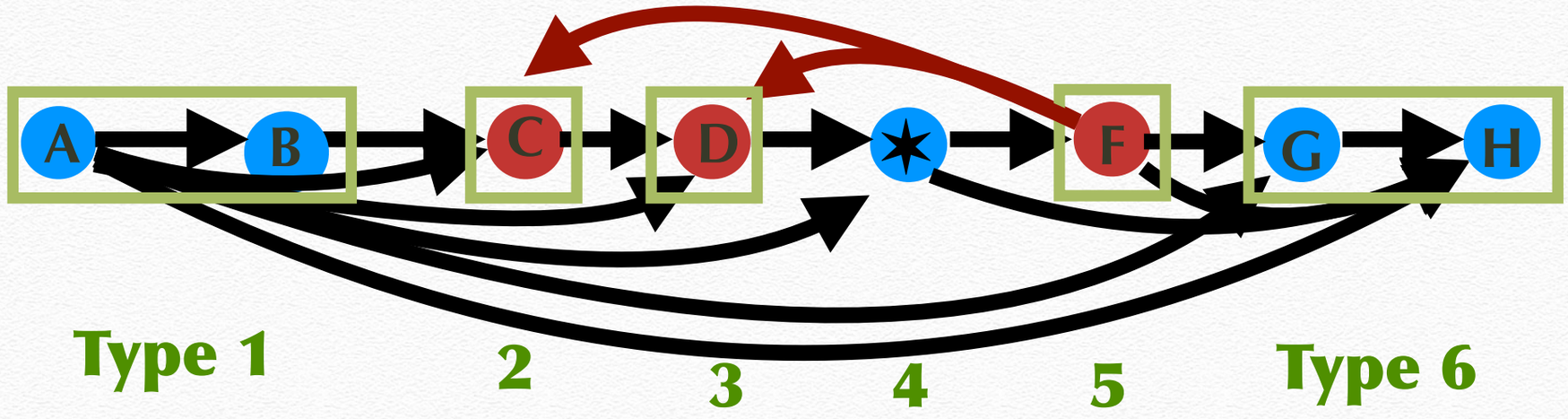
Elite (Players) Club



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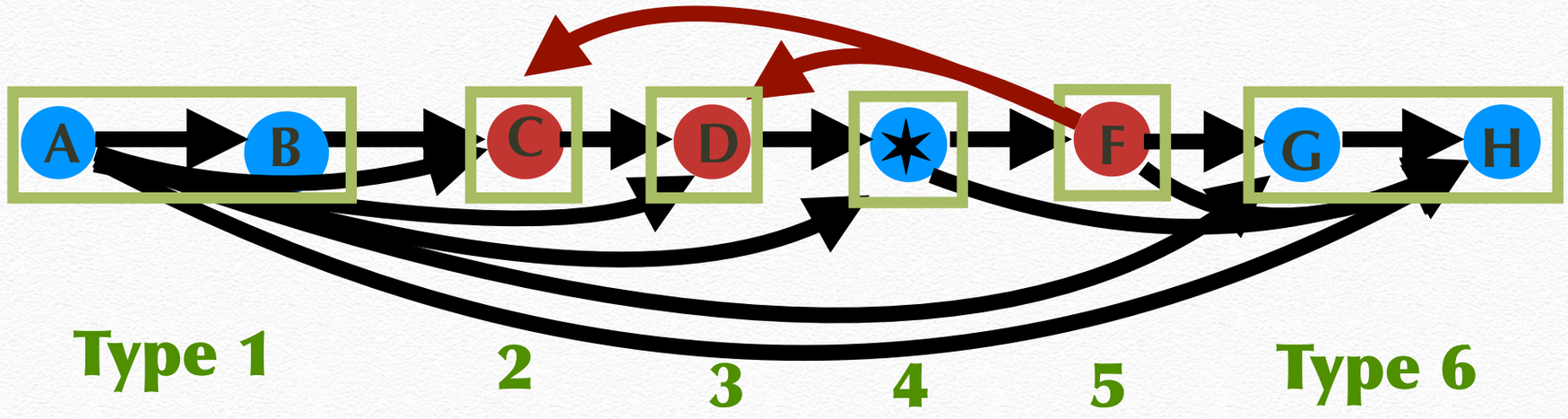

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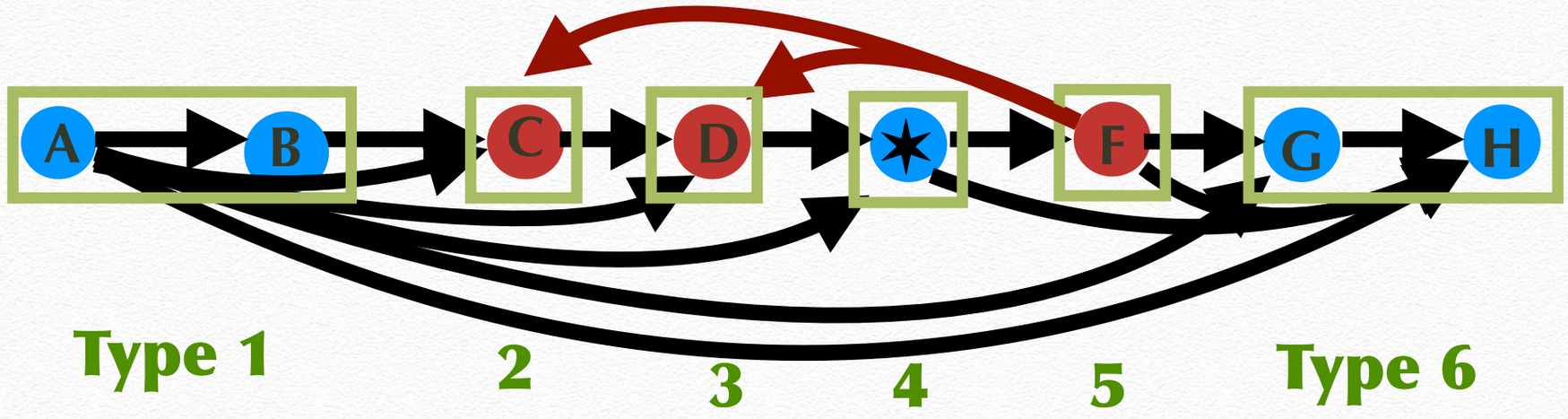

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Elite Club={A,B, C,D }

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Properties used by our algorithm.

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  - Are these problems solvable in time f(k)poly(n), k := FVS in win-lose graph

Thank You!

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- **Win lose graph** is not acyclic
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    - *favorite* wins if it beats $\frac{n}{2^l} - 1$ players

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  - Decision making