CHAMBERLIN-COURANT ON RESTRICTED DOMAINS

Neeldhara Misra

IIT Gandhinagar

RECENT TRENDS IN ALGORITHMS

NATIONAL INSTITUTE OF SCIENCE EDUCATION AND RESEARCH

THE STANDARD VOTING SETUP

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and typical computational problems.

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SINGLE-PEAKED & SINGLE-CROSSING PREFERENCES

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Getting realistic about domain restrictions.

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CONCLUDING REMARKS

THE STANDARD VOTING SETUP

and typical computational problems.

SINGLE-PEAKED & SINGLE-CROSSING PREFERENCES

…better winner determination, greater resilience to manipulation, etc.

ALMOST SPECIAL

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CONCLUDING REMARKS

Red flags and research directions.

THE STANDARD VOTING SETUP

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and typical computational problems.

Candidates/Alternatives

Voters express their preferences over alternatives (here, as rankings).

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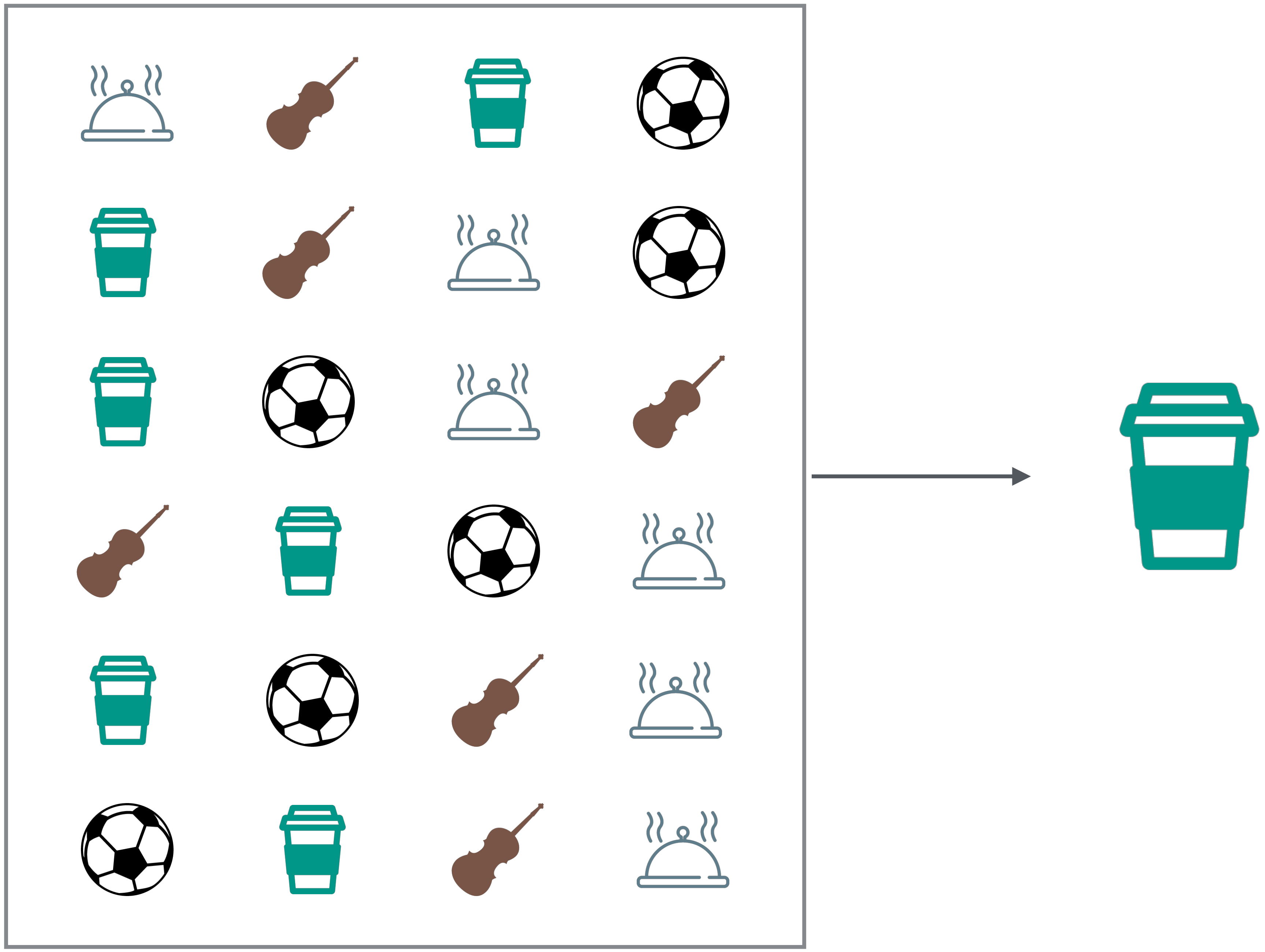
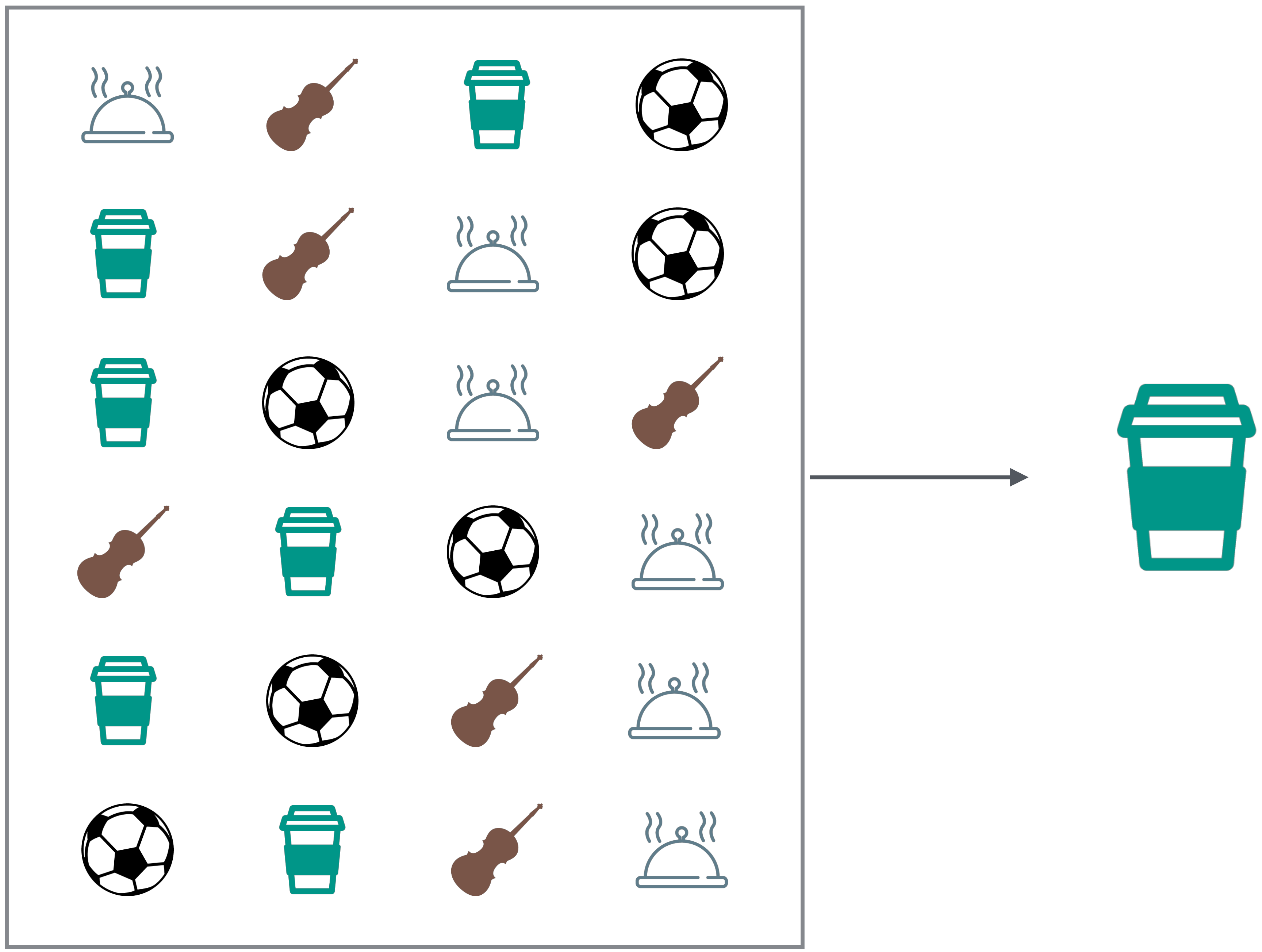
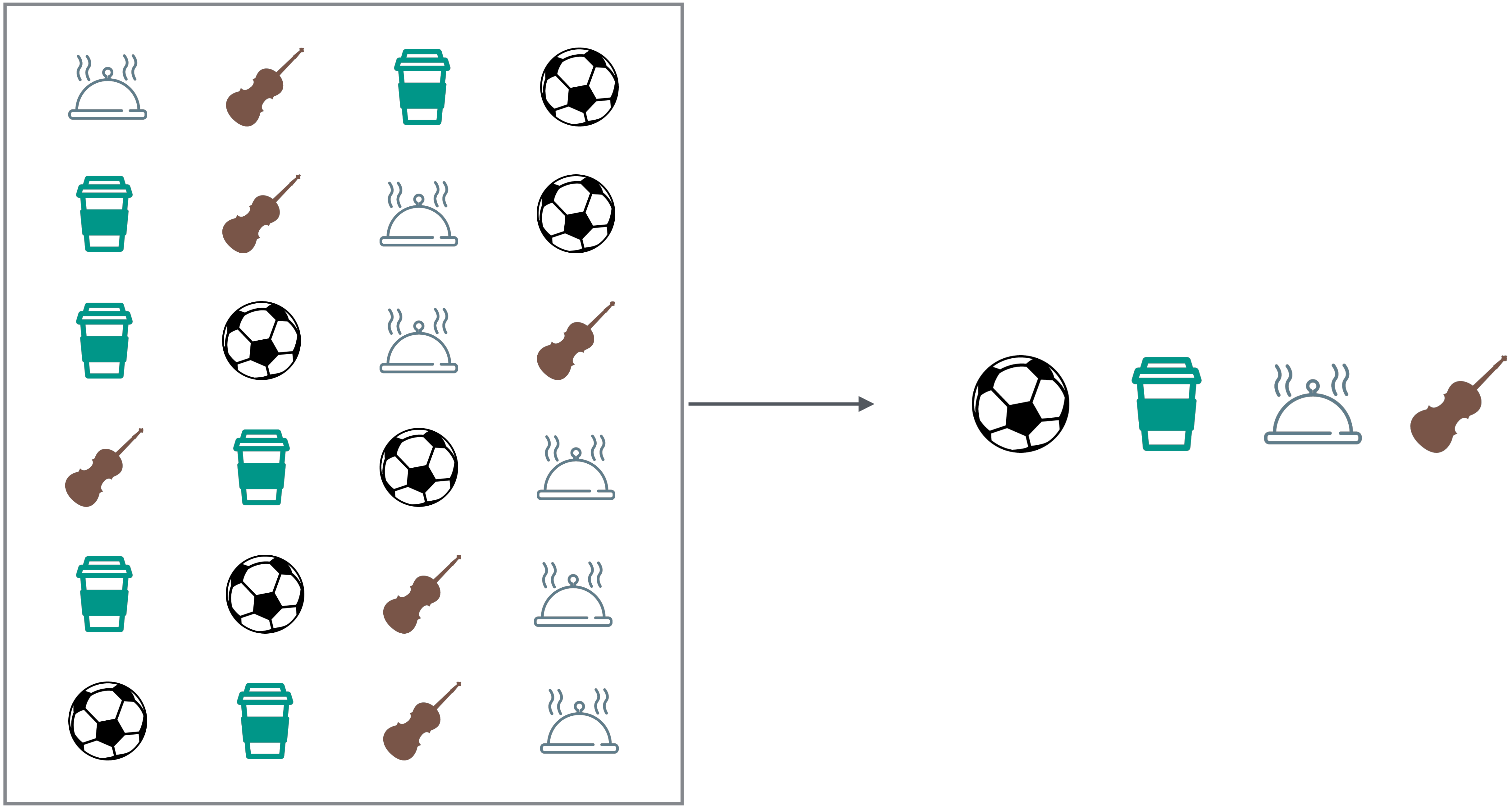
(could also be approval ballots).

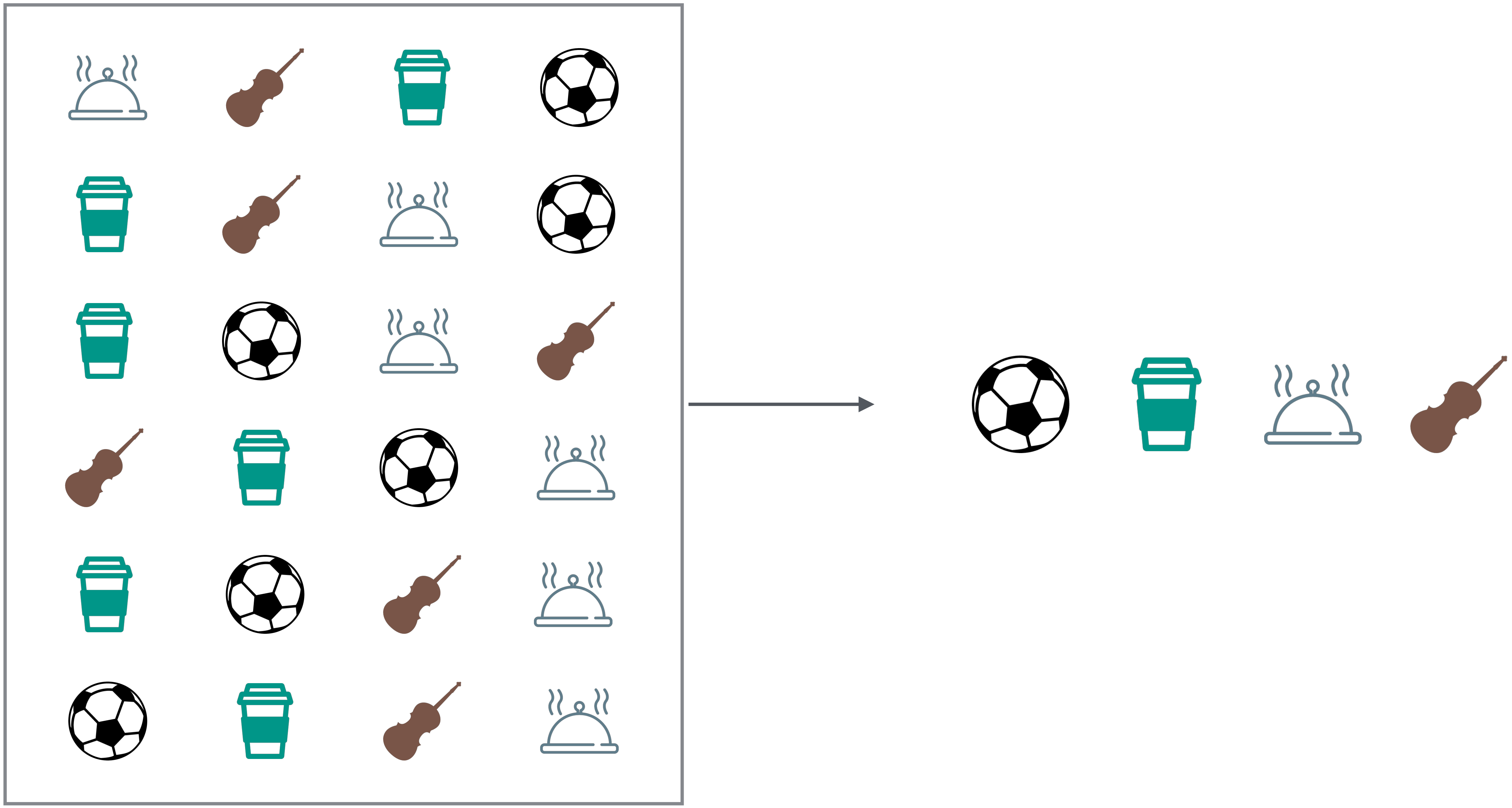


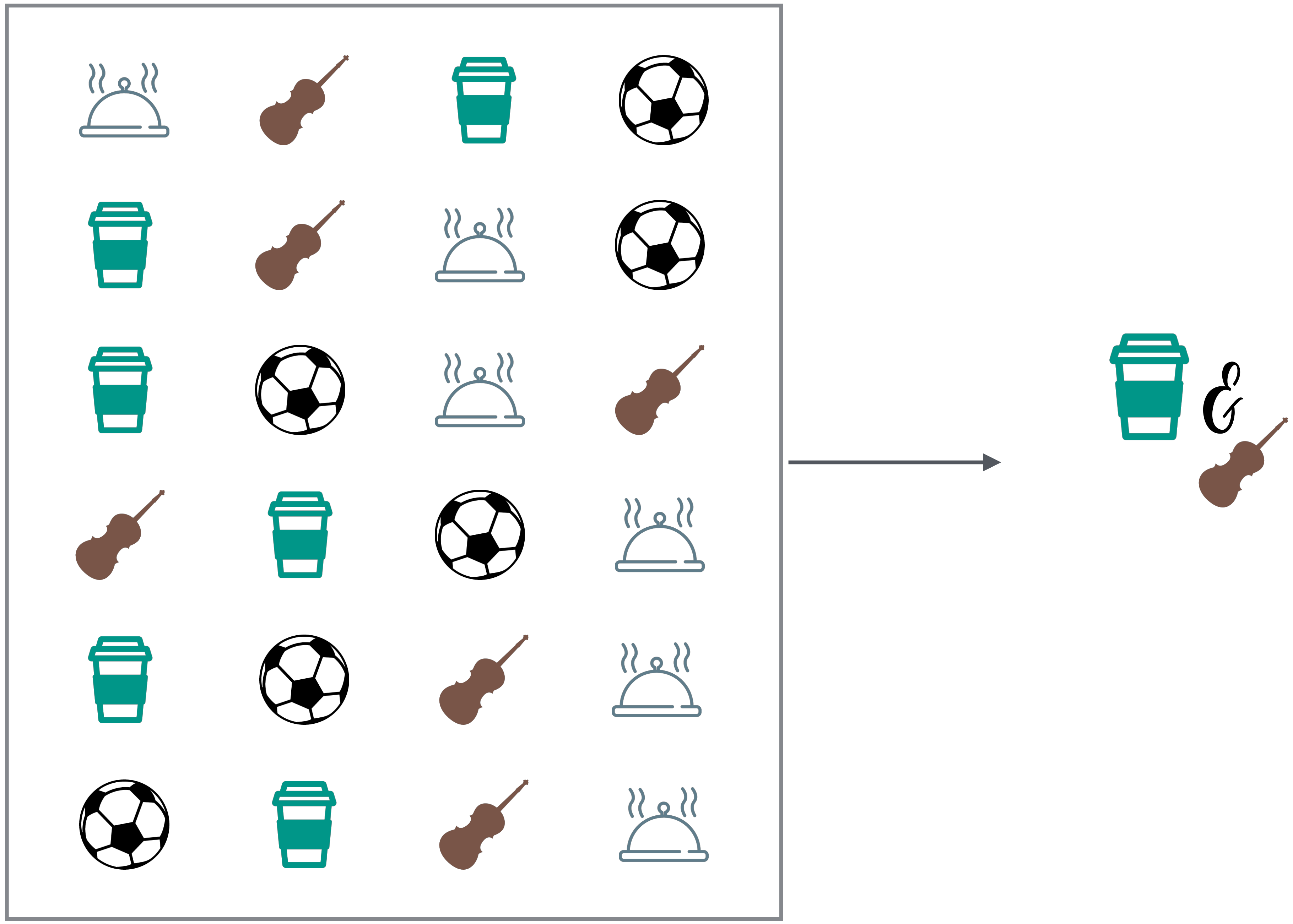
Social Choice Functions



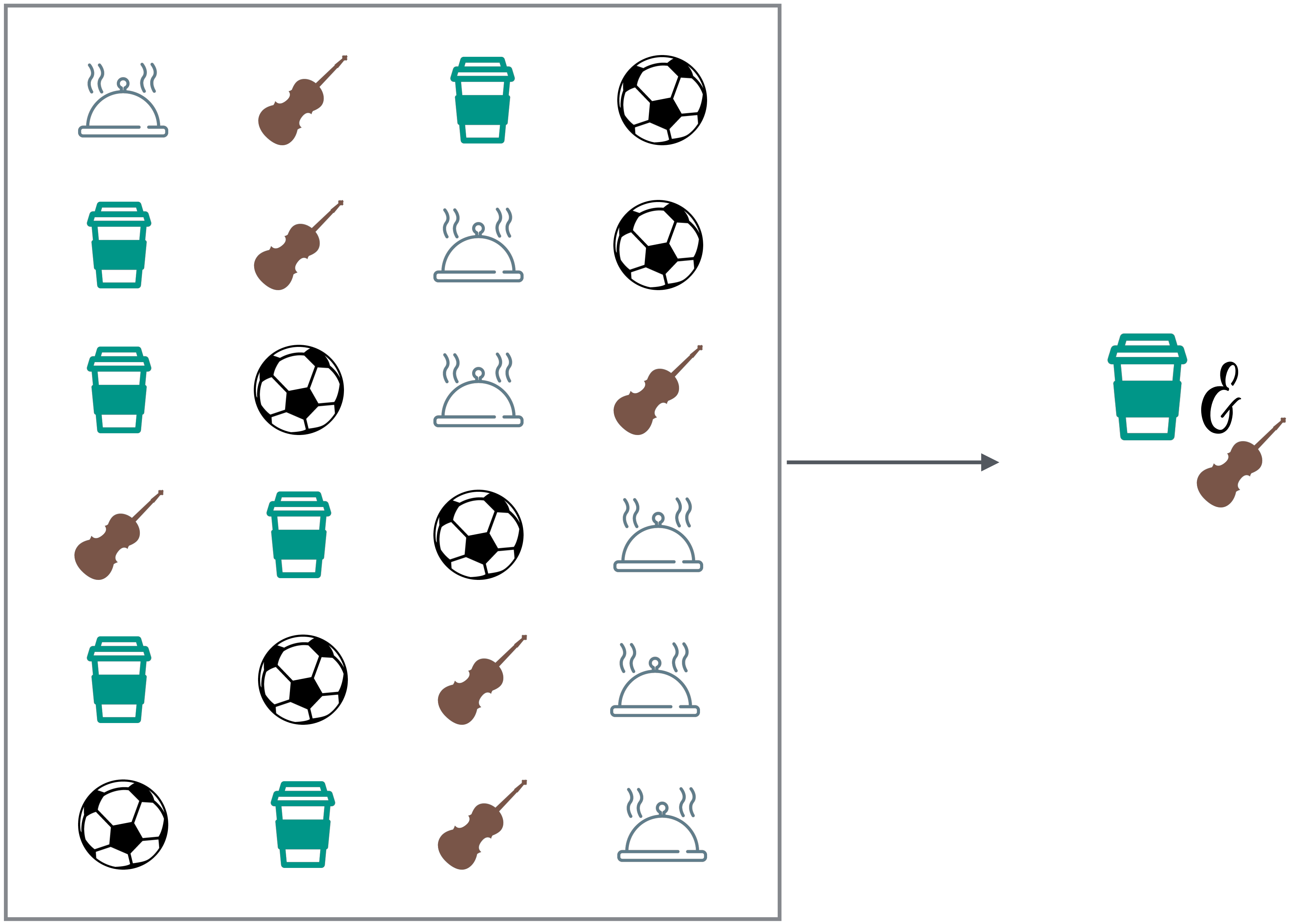


Social Welfare Functions





Multiwinner Voting Rules



Typical problems

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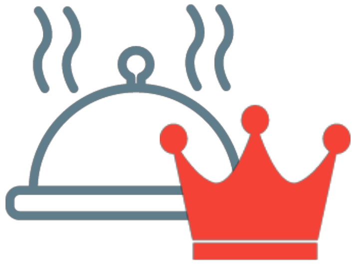
VOTING RULES

Some Examples

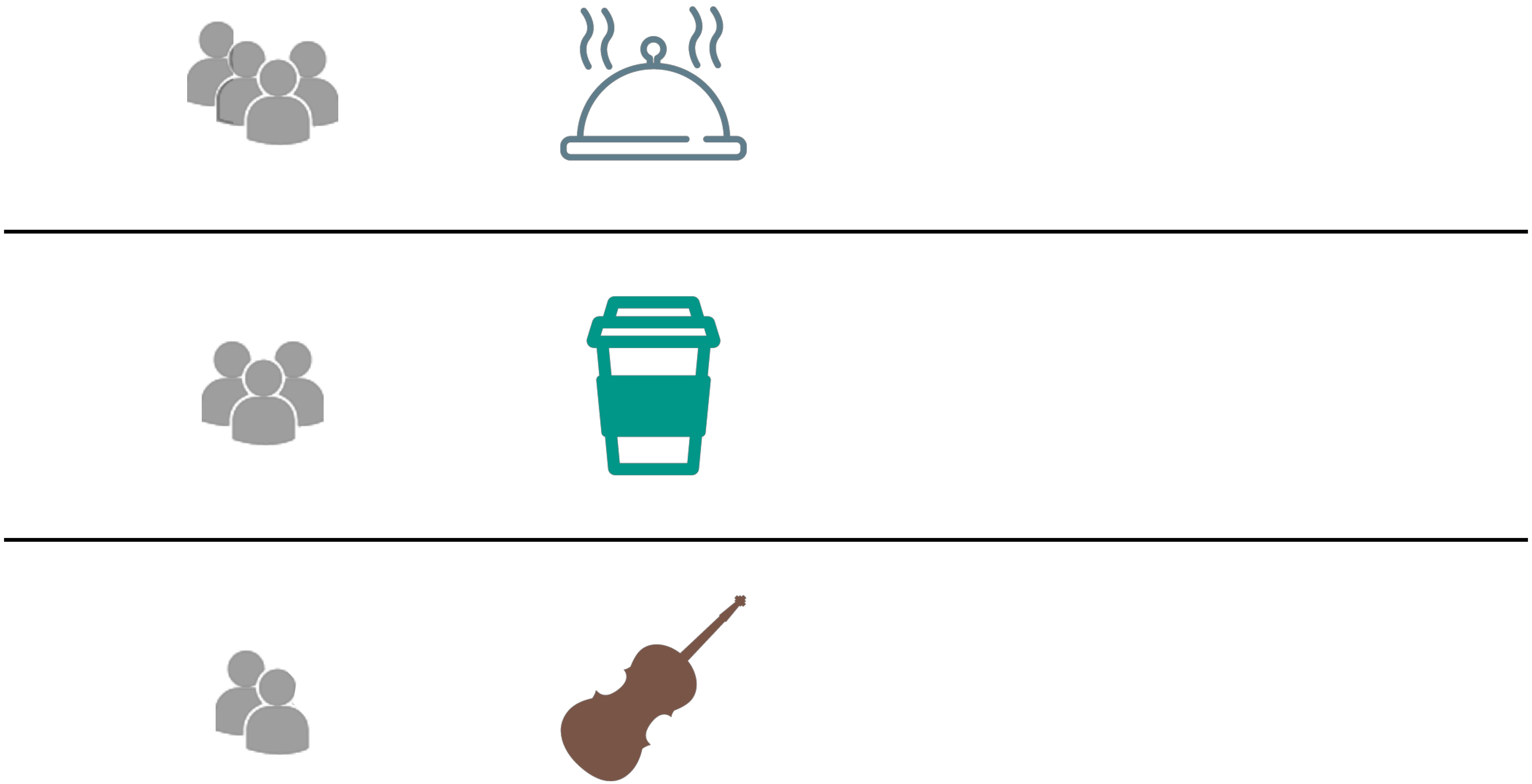
VOTING RULES

Plurality



(Plurality)

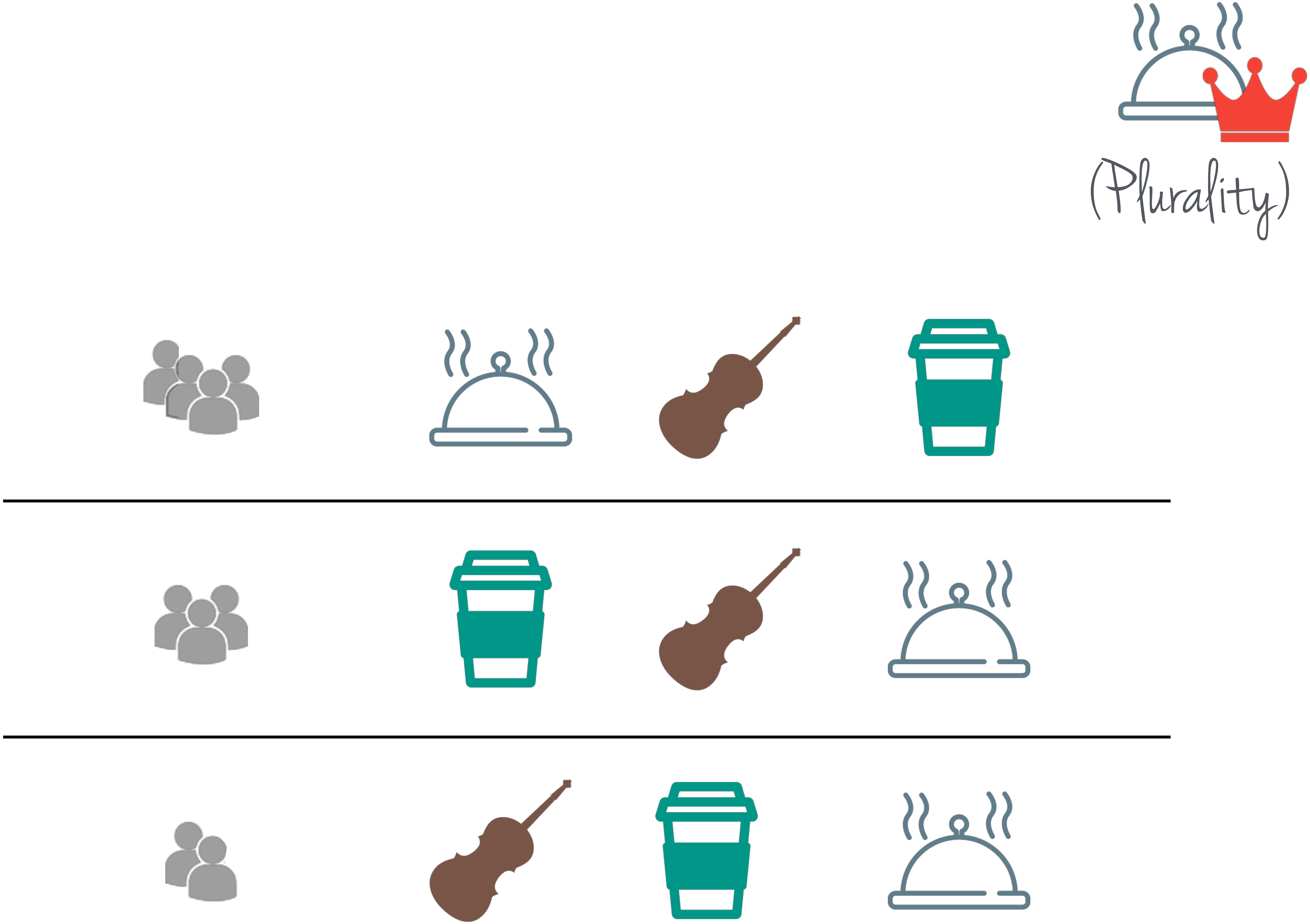



(Plurality)

The plurality winner can also be among the least popular options.

(Plurality)

We say that a voter (or a group of voters) can **manipulate** if they can obtain a more desirable outcome by misreporting their preferences.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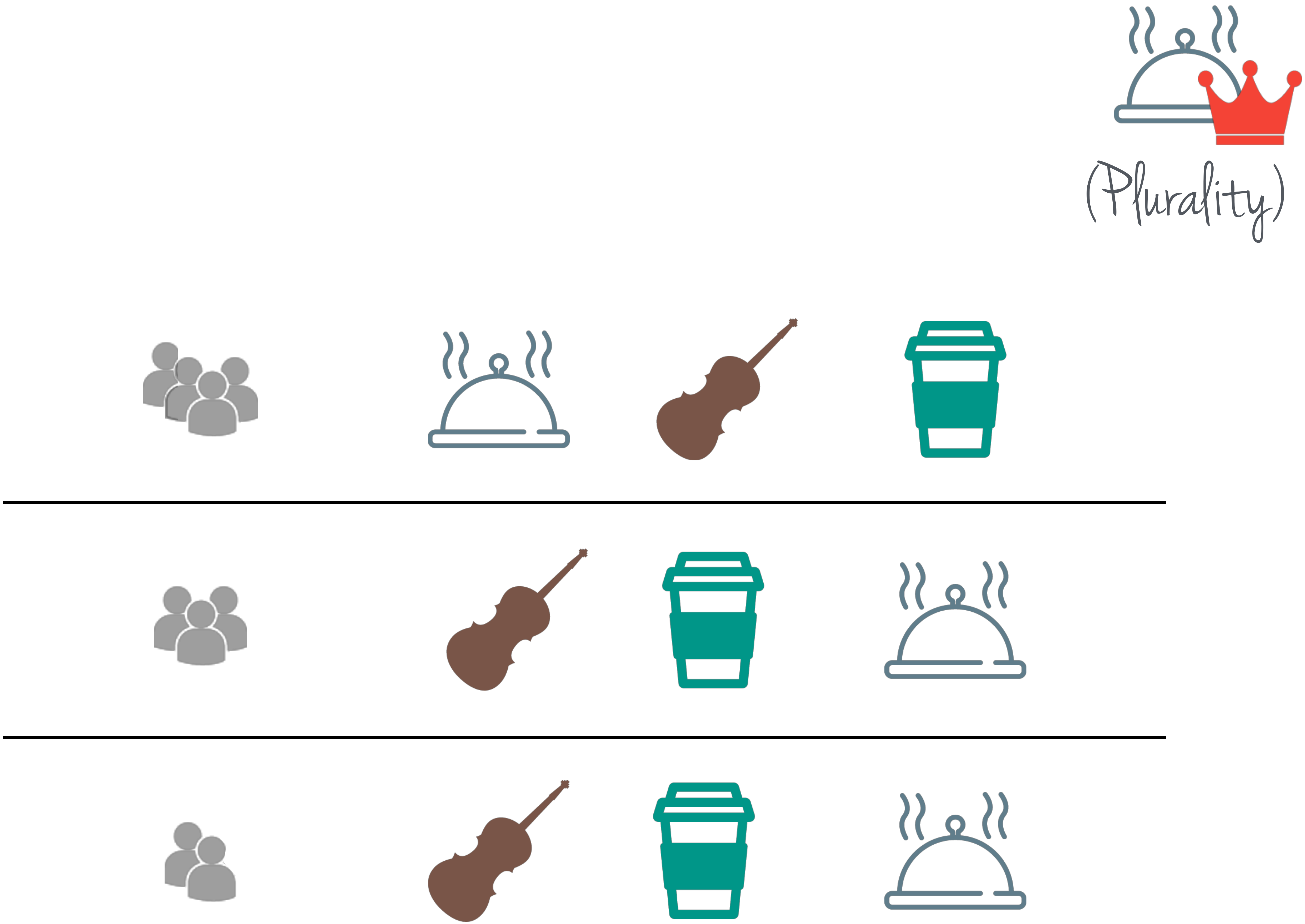
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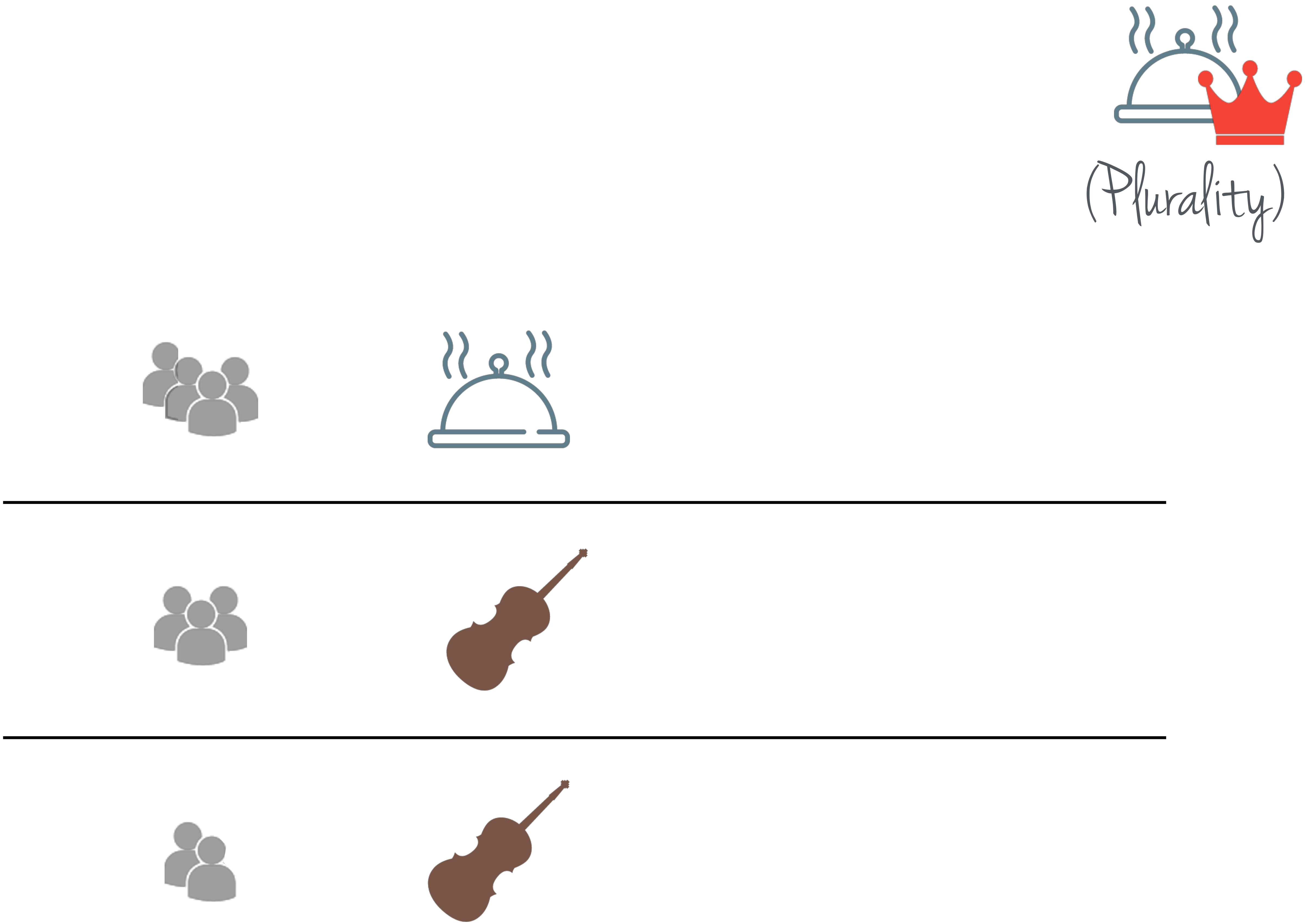
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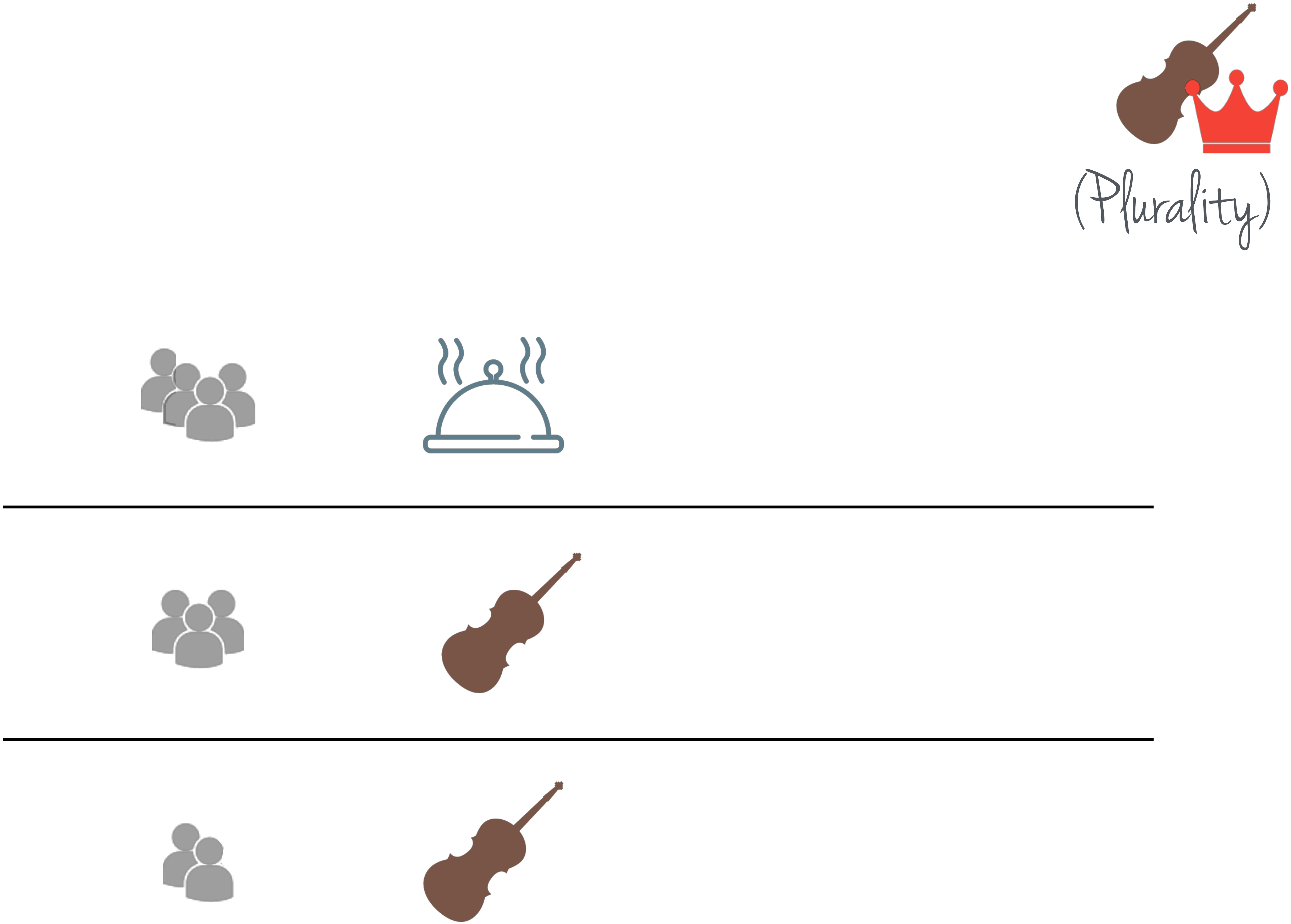

(Plurality)

This scheme is intended only for honest men.

Borda

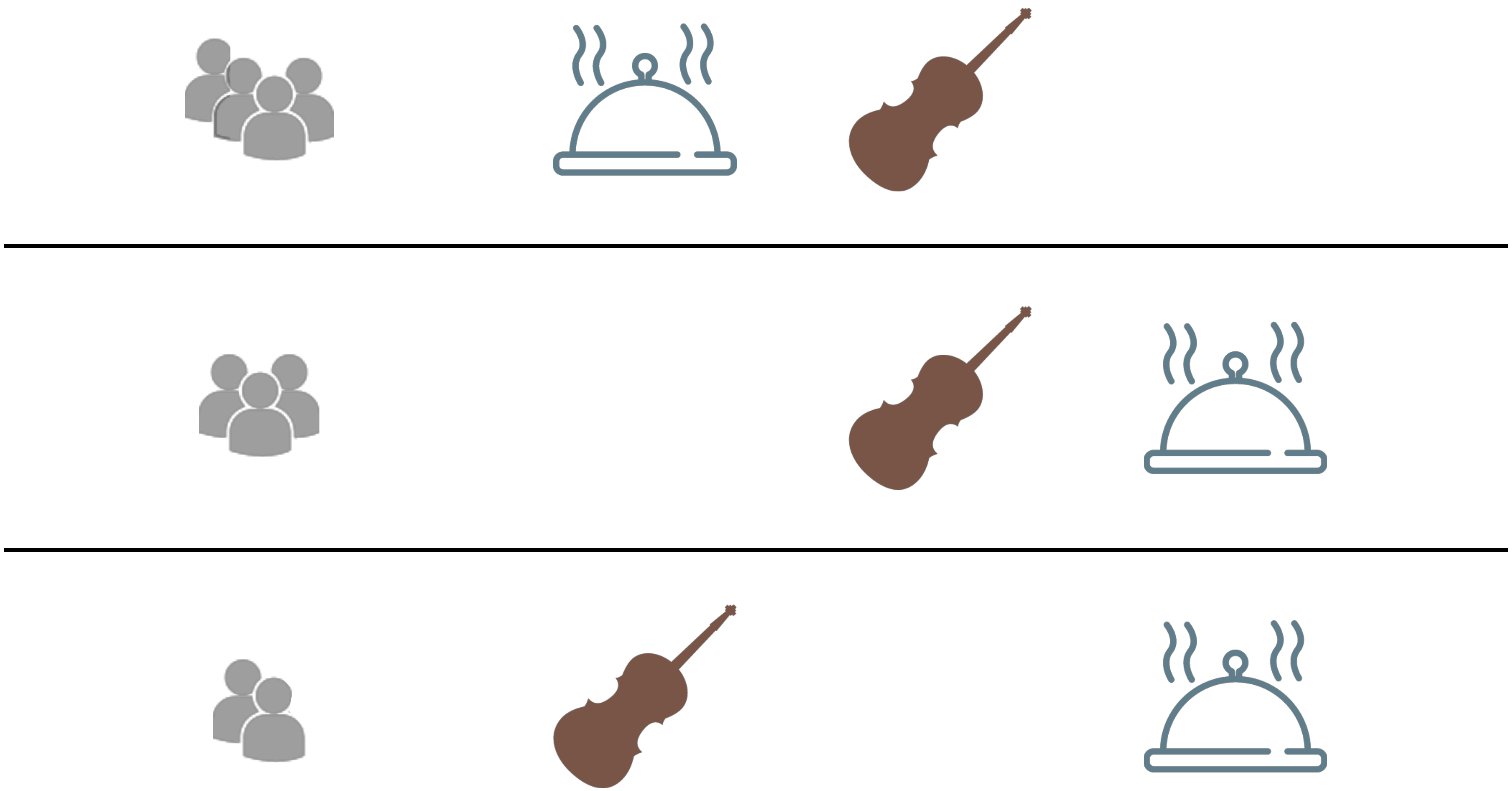
VOTING RULES

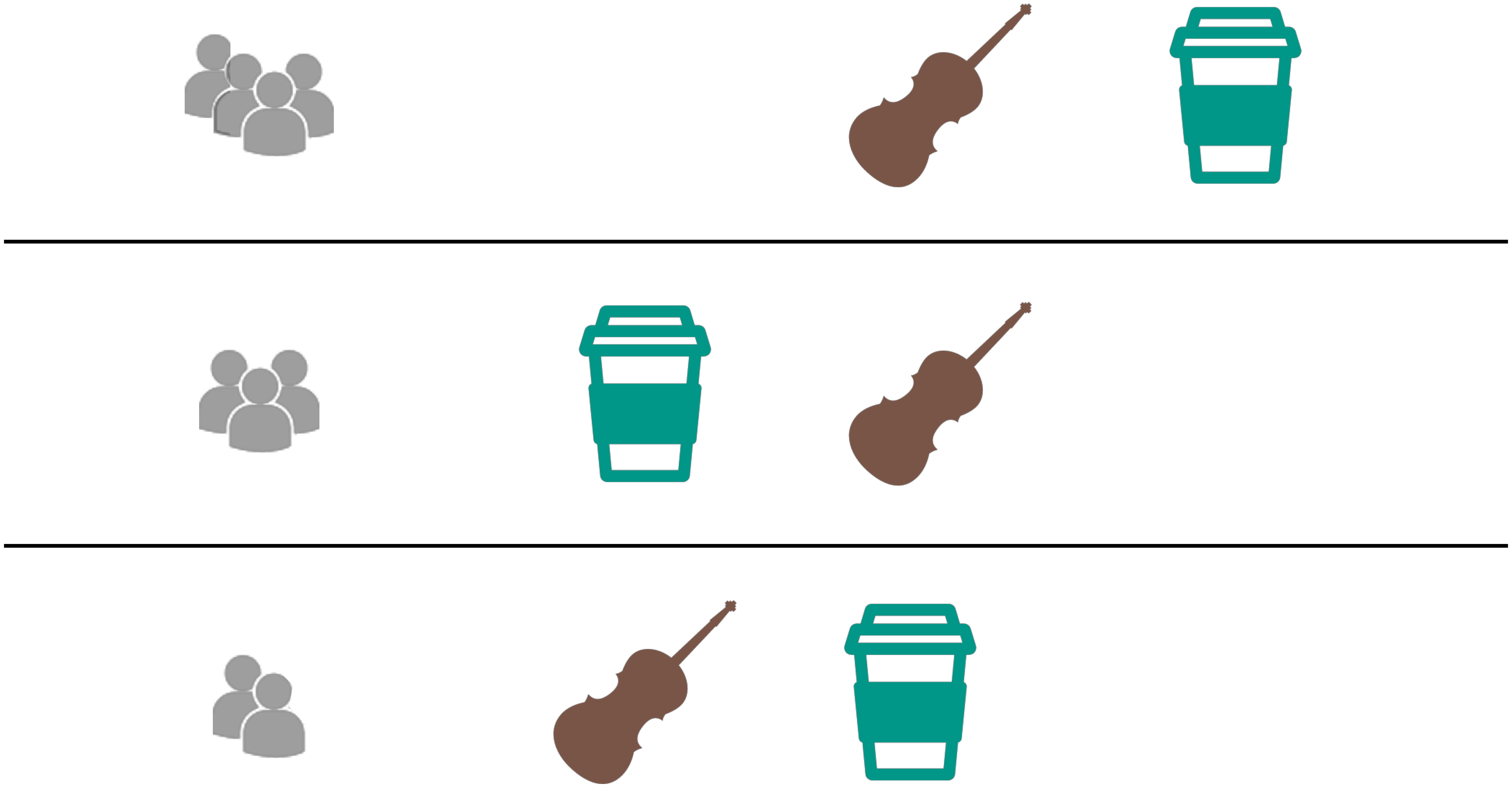
STV

(STV)

VOTING RULES

Condorcet





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(Condorcet)

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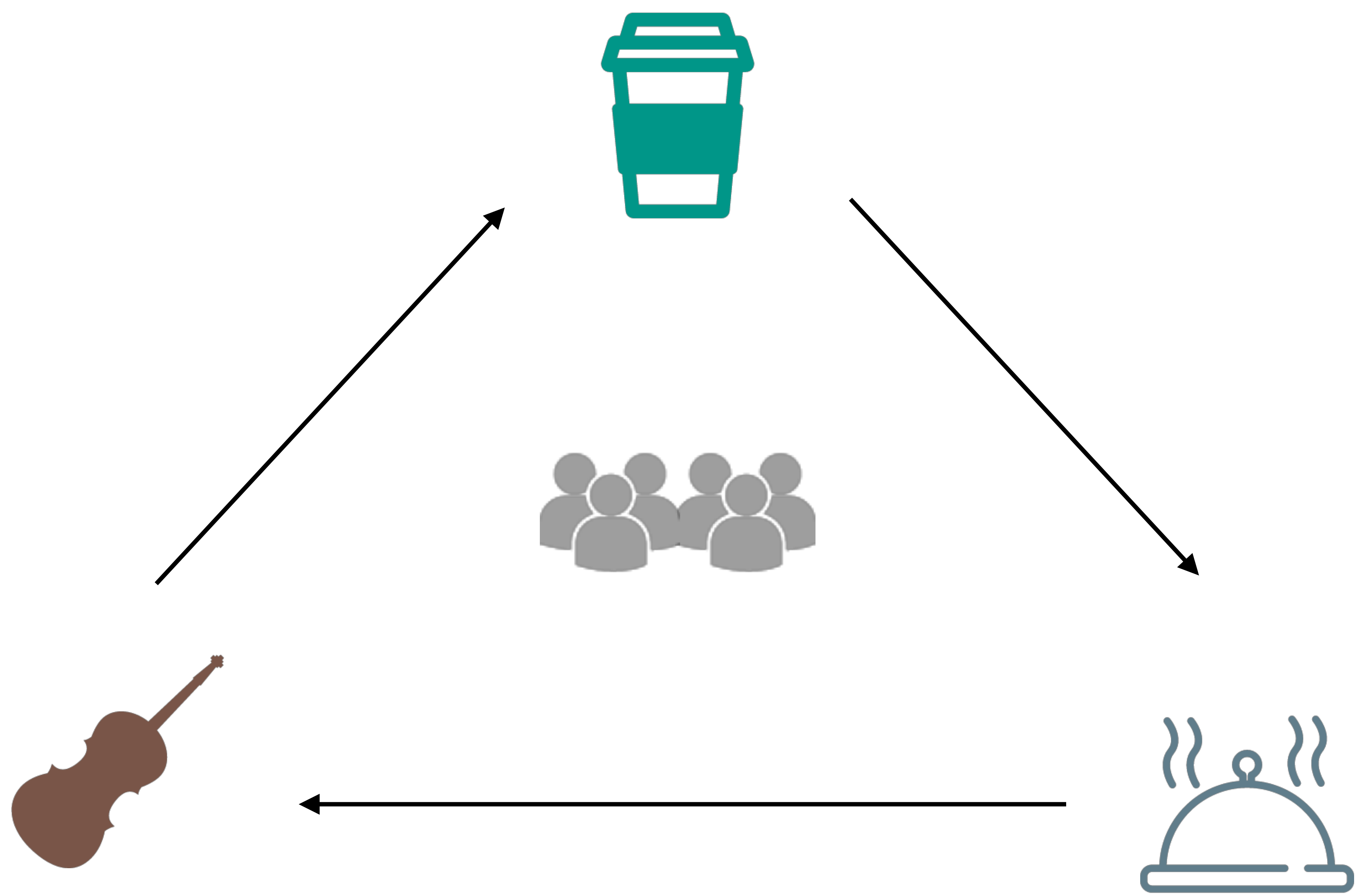

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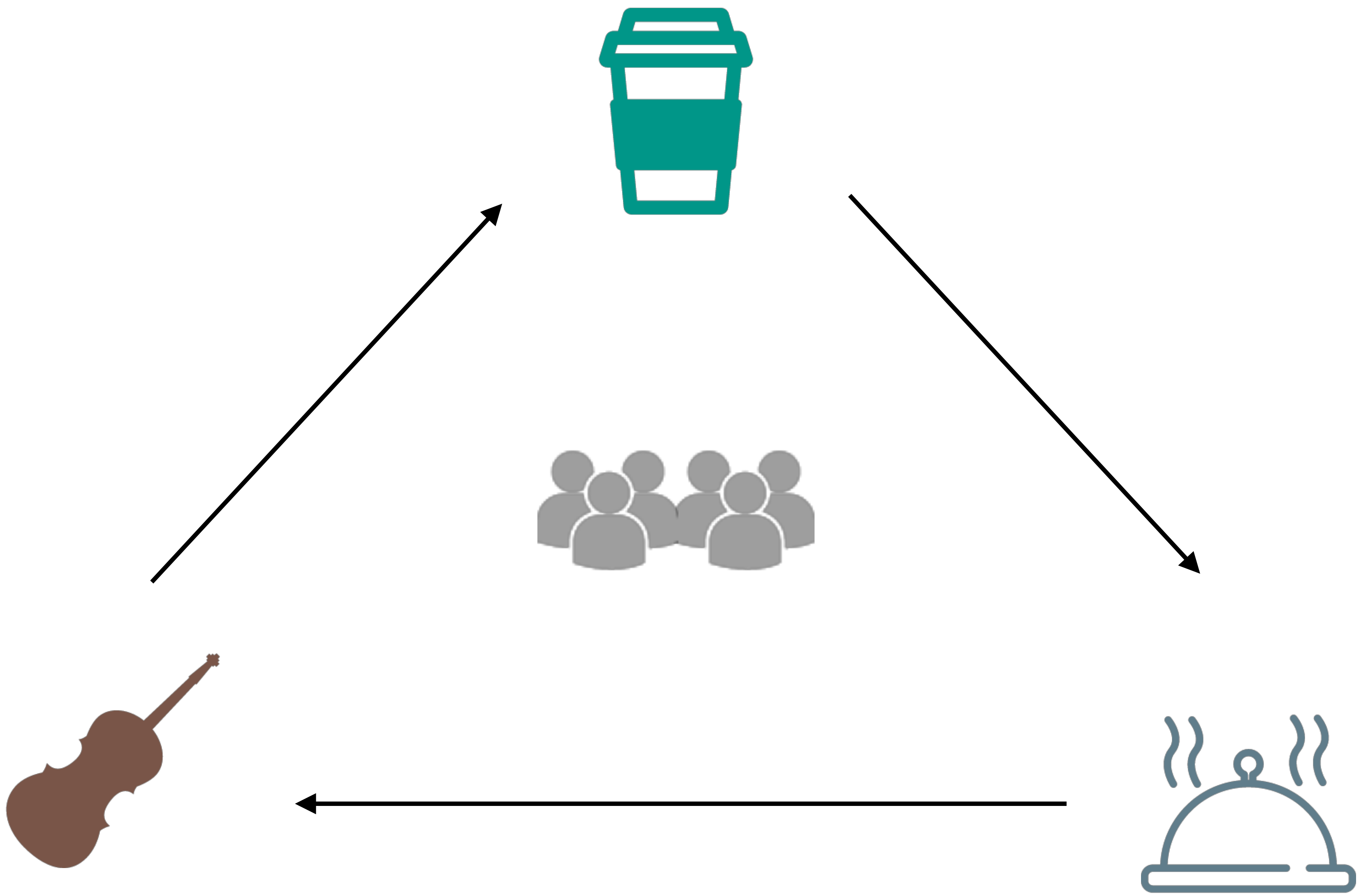

may not exist!

VOTING RULES

Dodgson

Dodgson

Dodgson

Dodgson

Dodgson score of c

Smallest #of swaps needed to **make c a Condorcet winner.**

Preference Aggregation

VOTING RULES

Kemeny

Kemeny

Kemeny

Kemeny score of a ranking

Sum of **pairwise agreements** across all votes.

Multiwinner

VOTING RULES

Chamberlin-Courant

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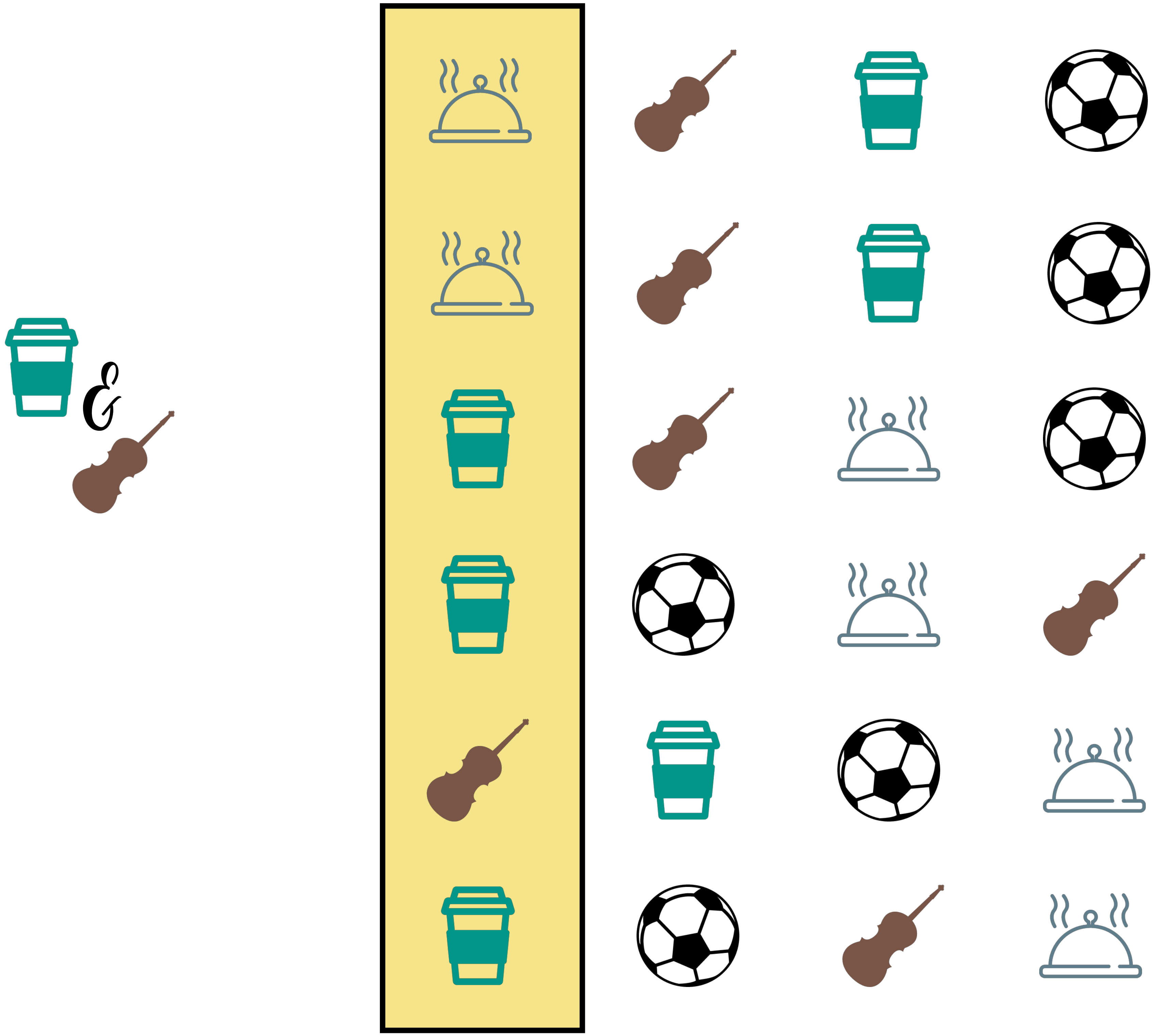
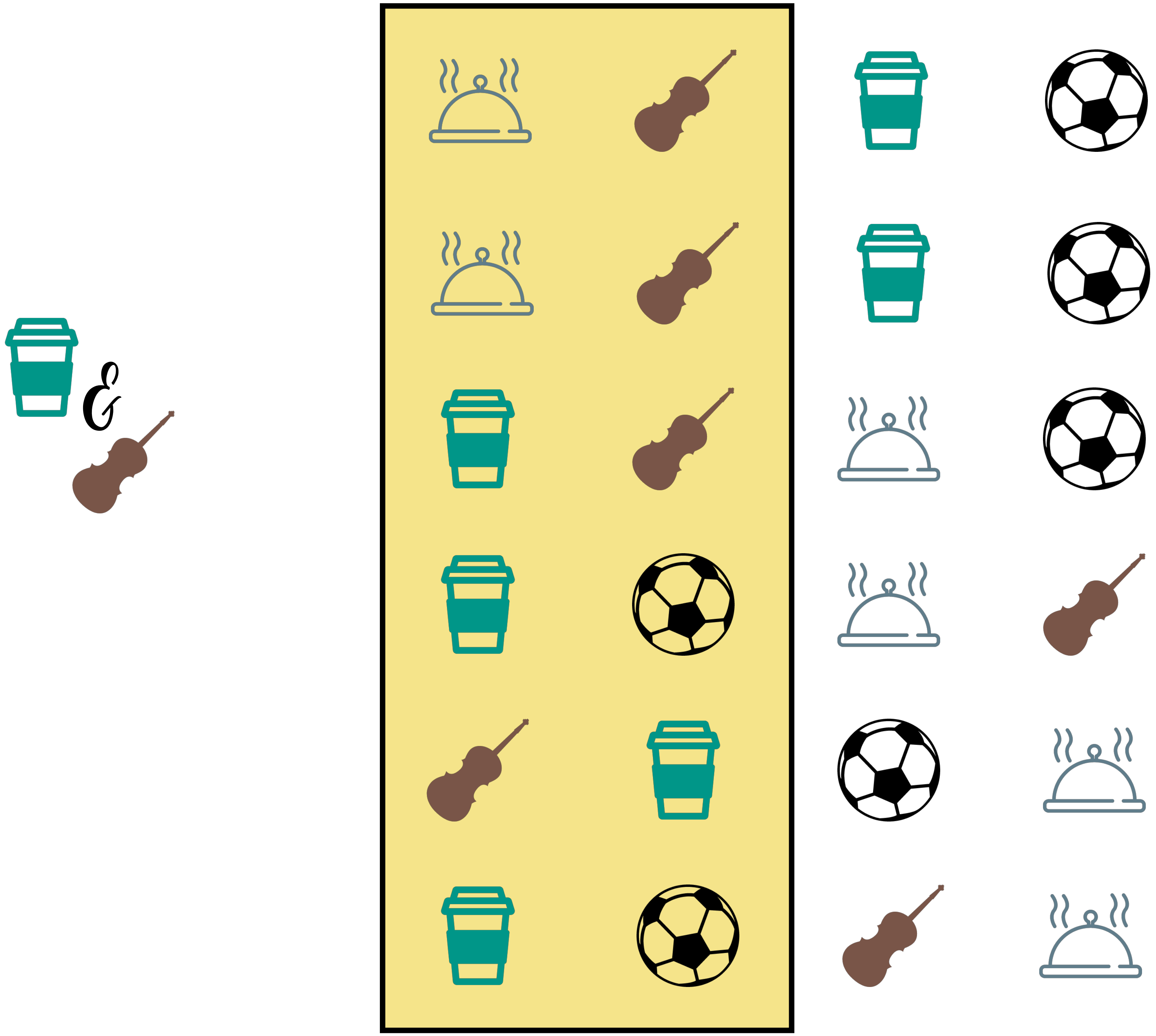


Chamberlin-Courant



CHAMBERLIN-COURANT

CC-score score of a committee:

maximum **dissatisfaction**
across all votes.

CHAMBERLIN-COURANT

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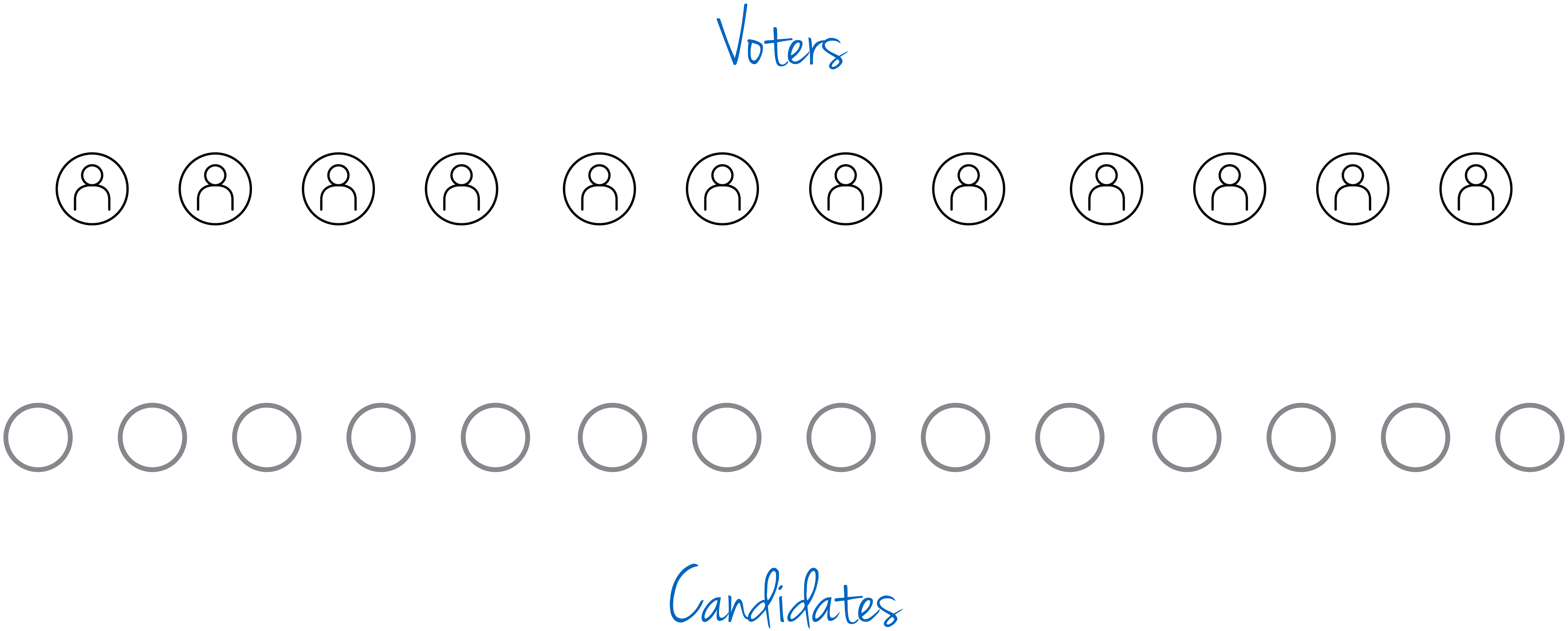
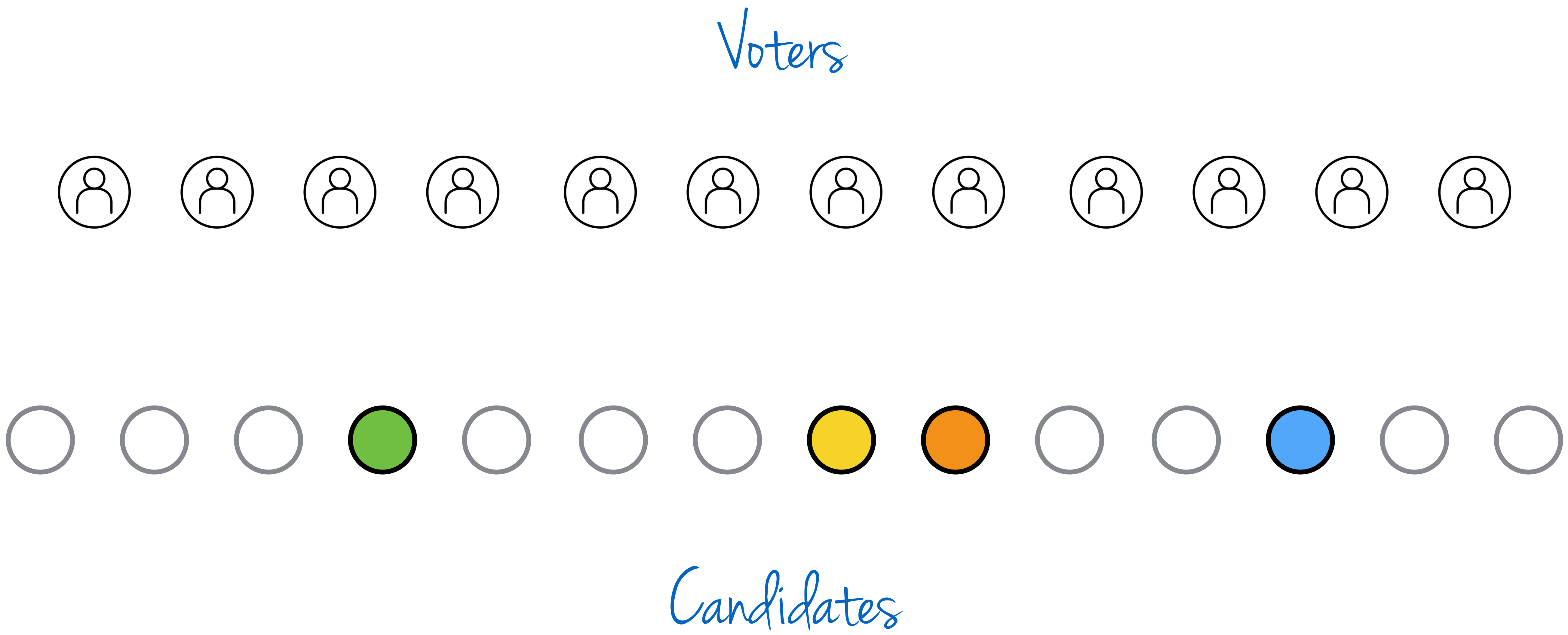
More precisely...


Voters
Candidates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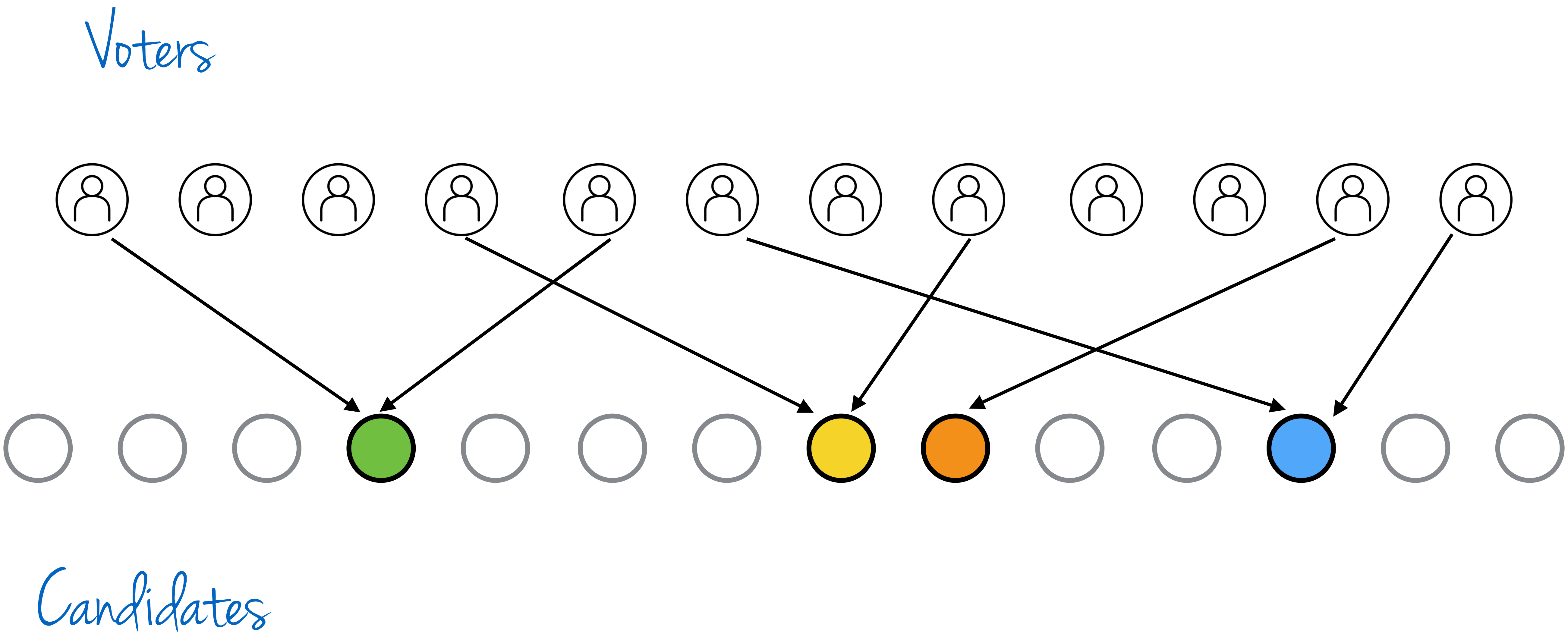
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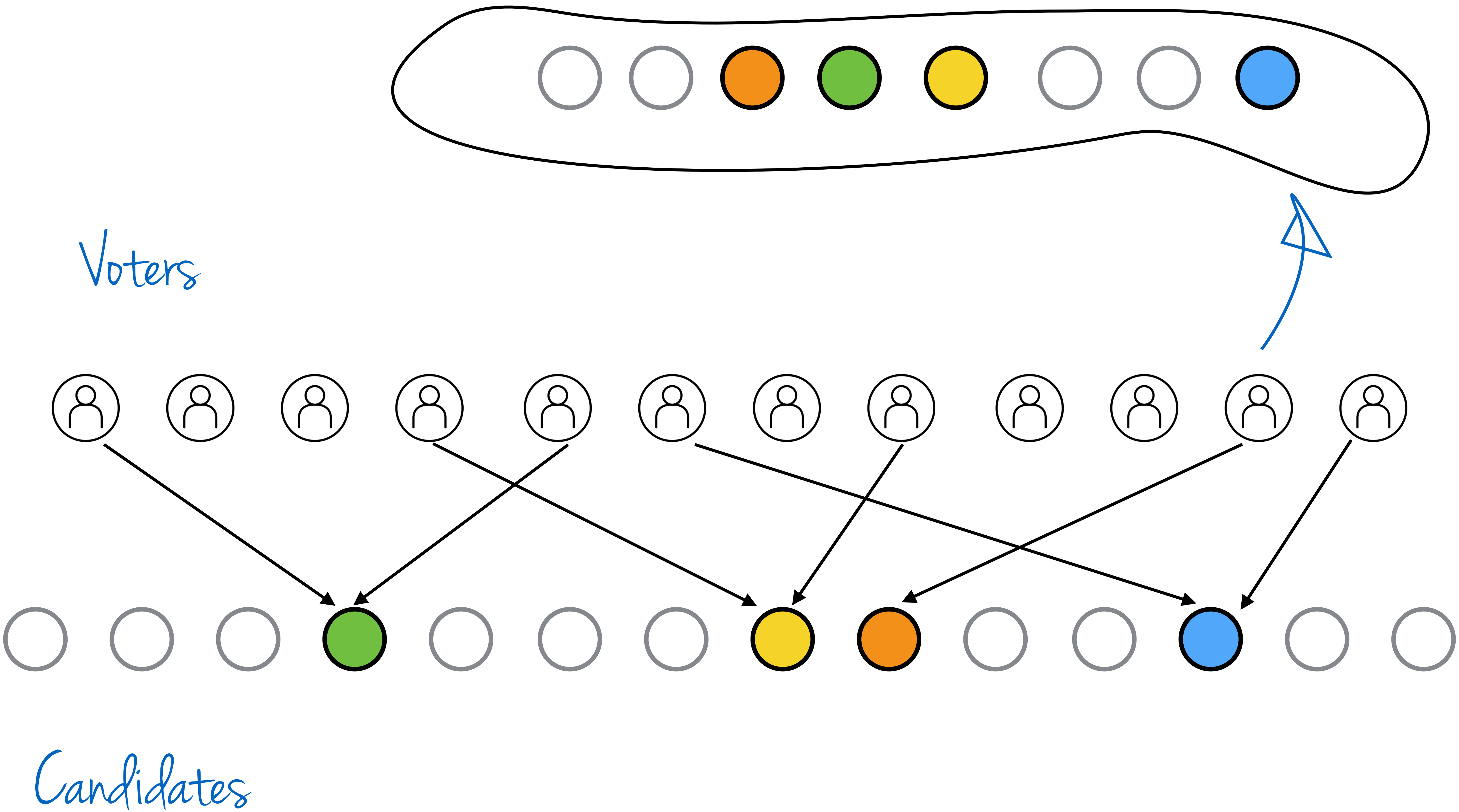
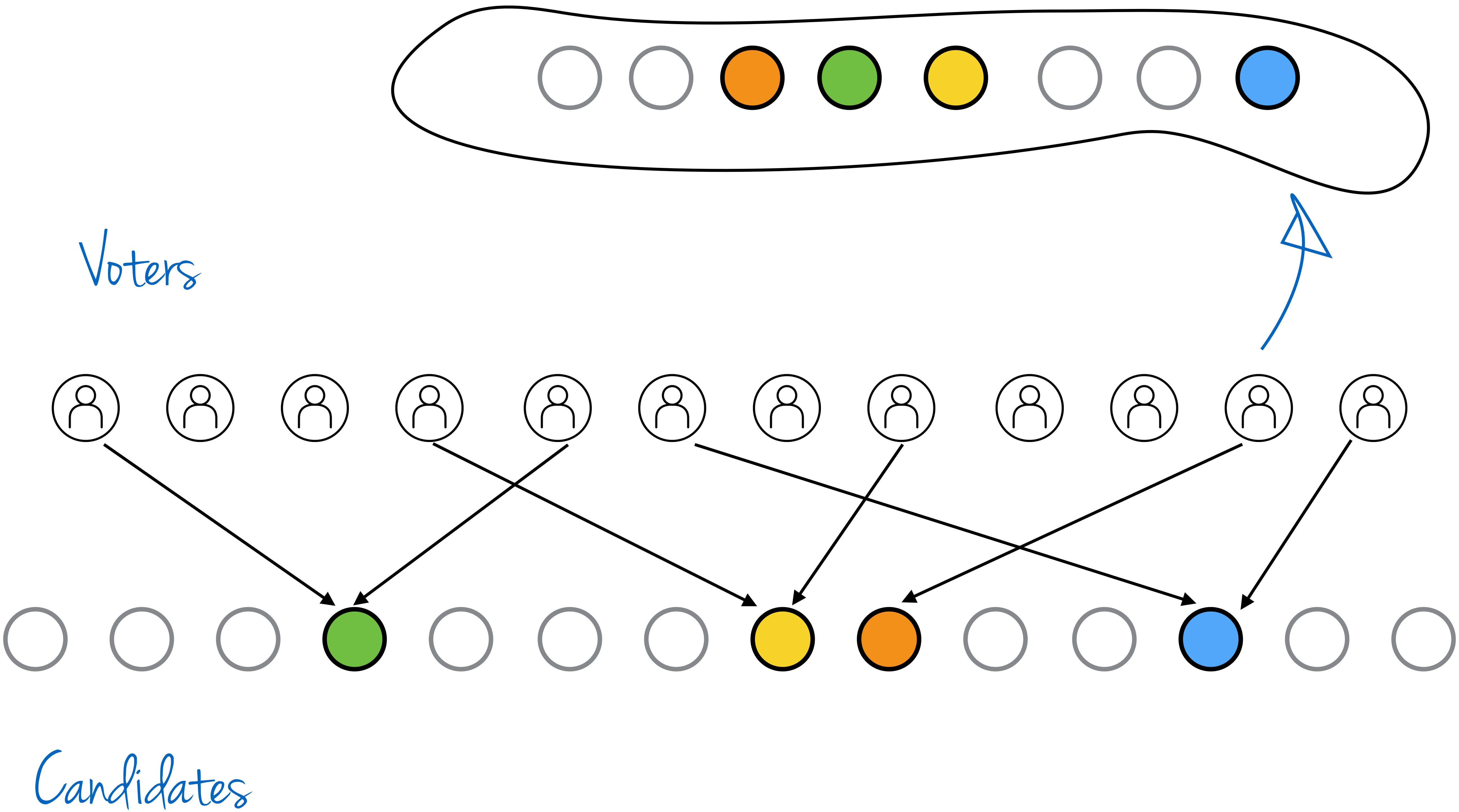


Voters
Candidates



dissatisfaction of voter v =
rank of best candidate from the committee in his vote

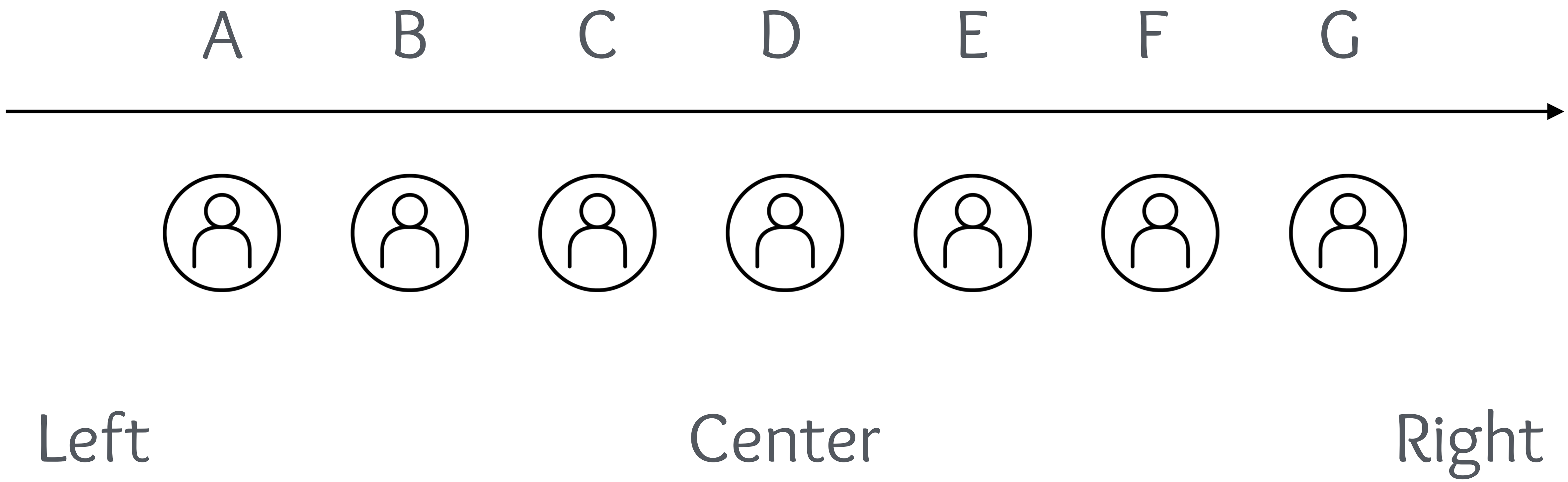
SINGLE-PEAKED & SINGLE-CROSSING PREFERENCES

…better winner determination, greater resilience to manipulation, etc.

SINGLE PEAKED PREFERENCES

Definition

The Theory of Committees and Elections.
Black, D., New York: Cambridge University Press, 1958



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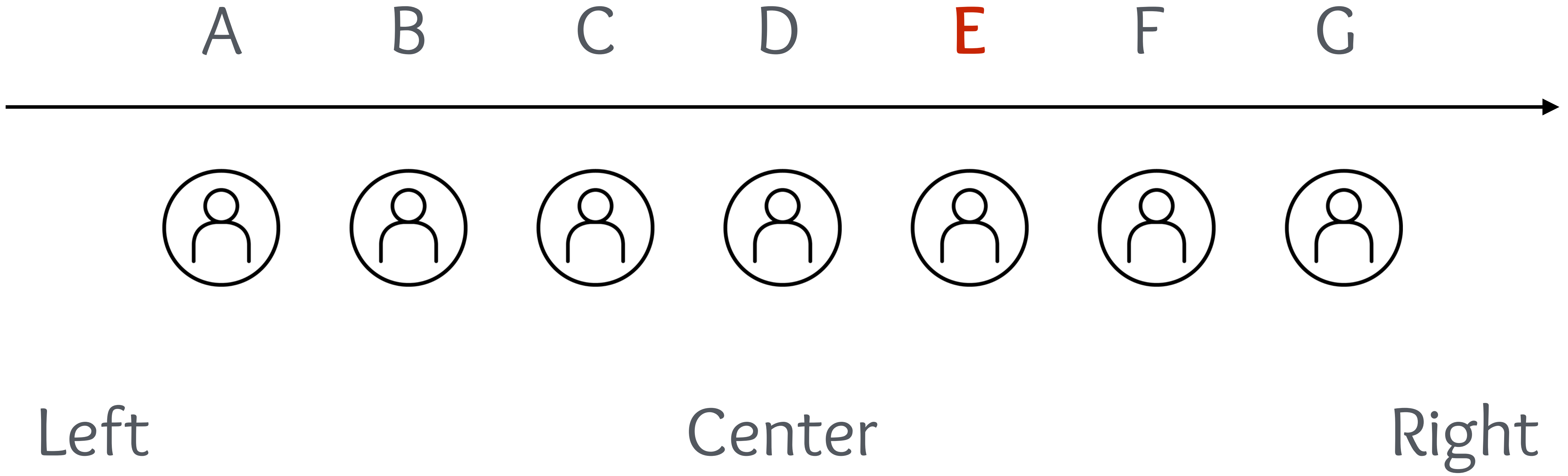
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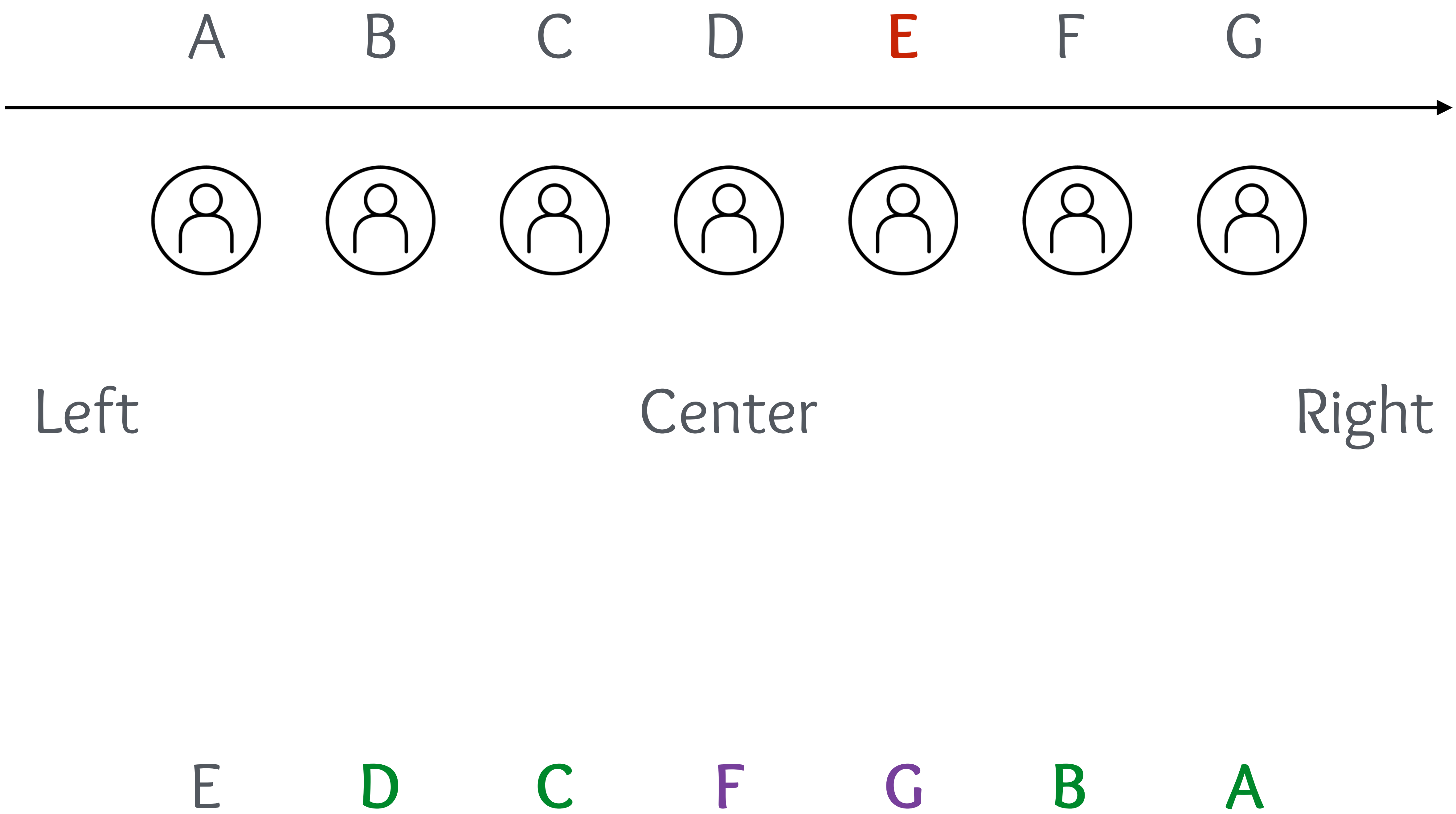
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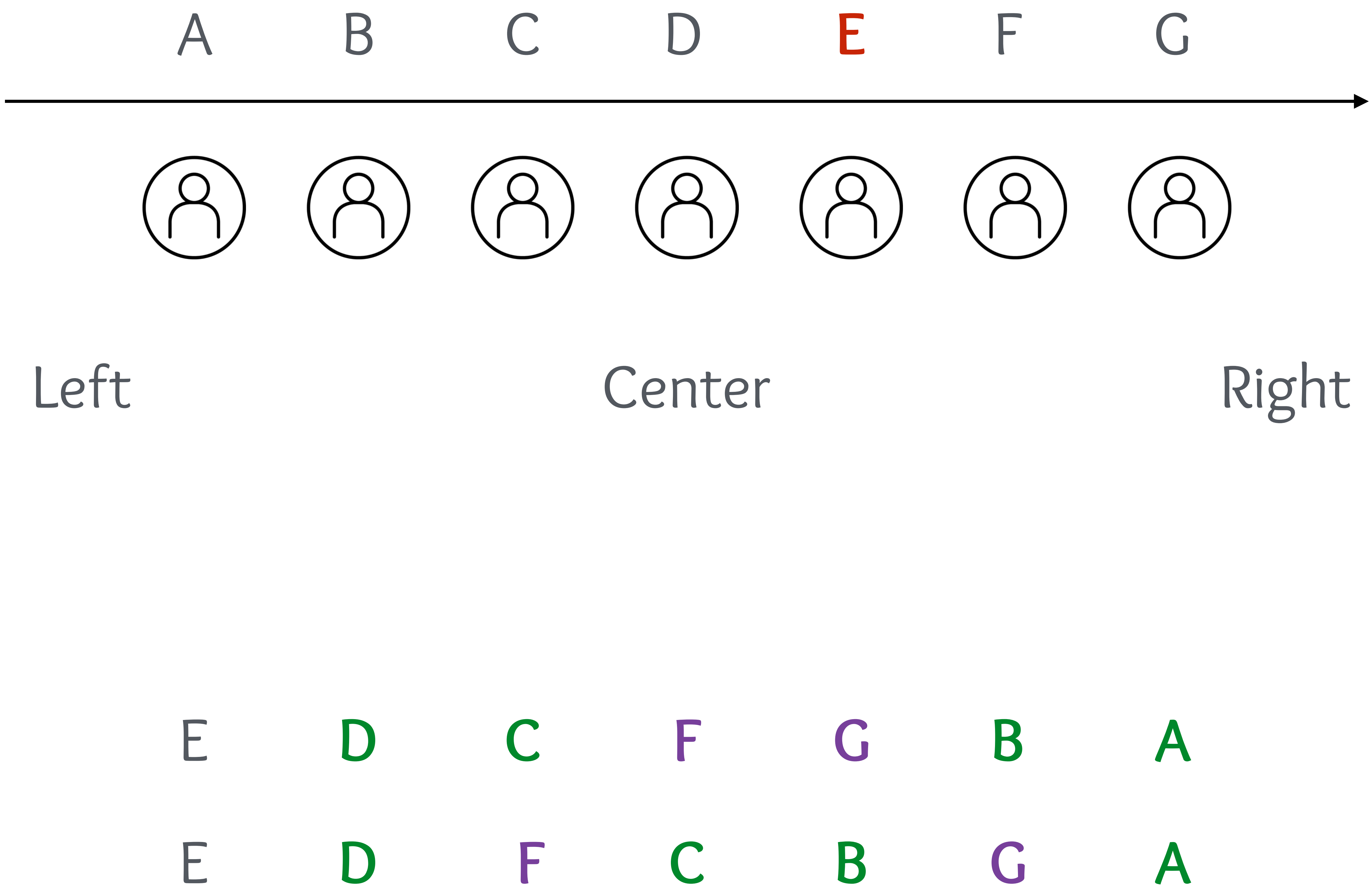
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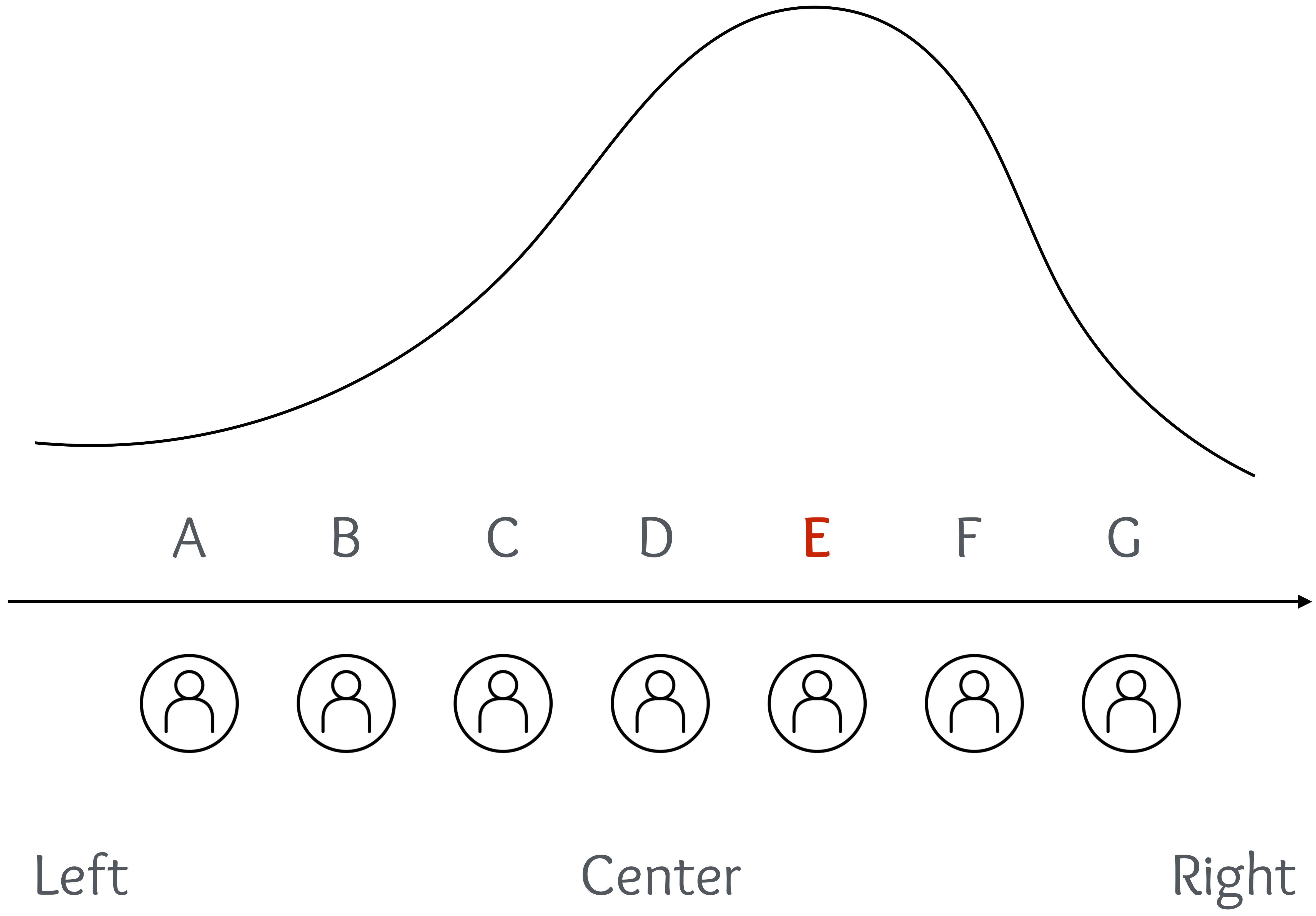
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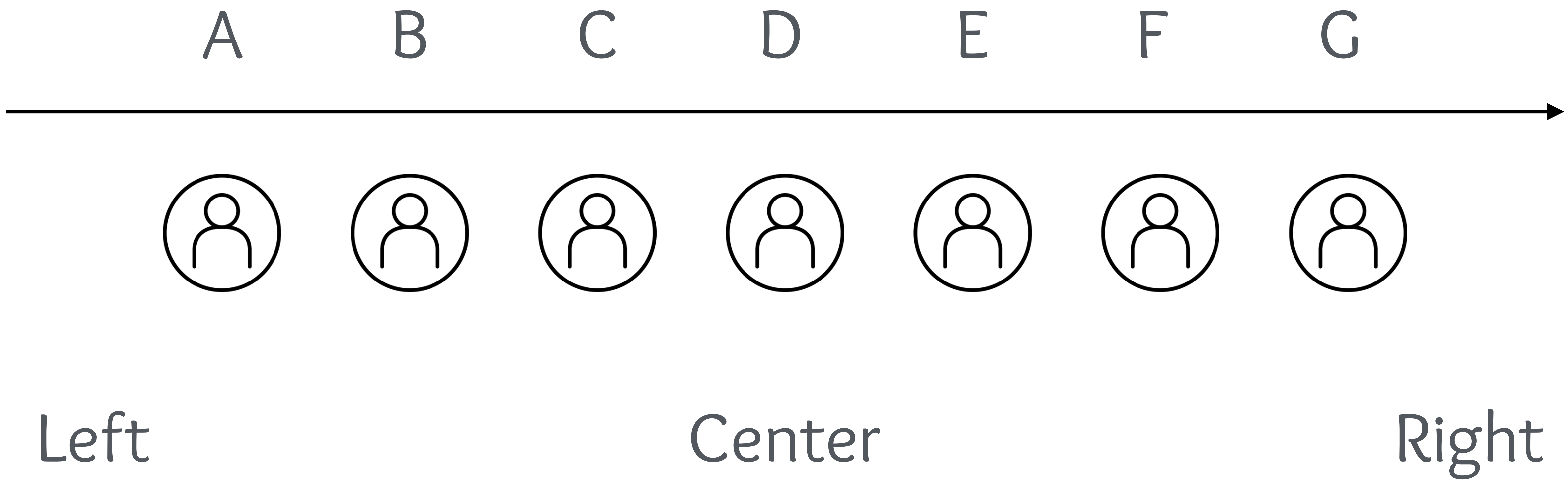
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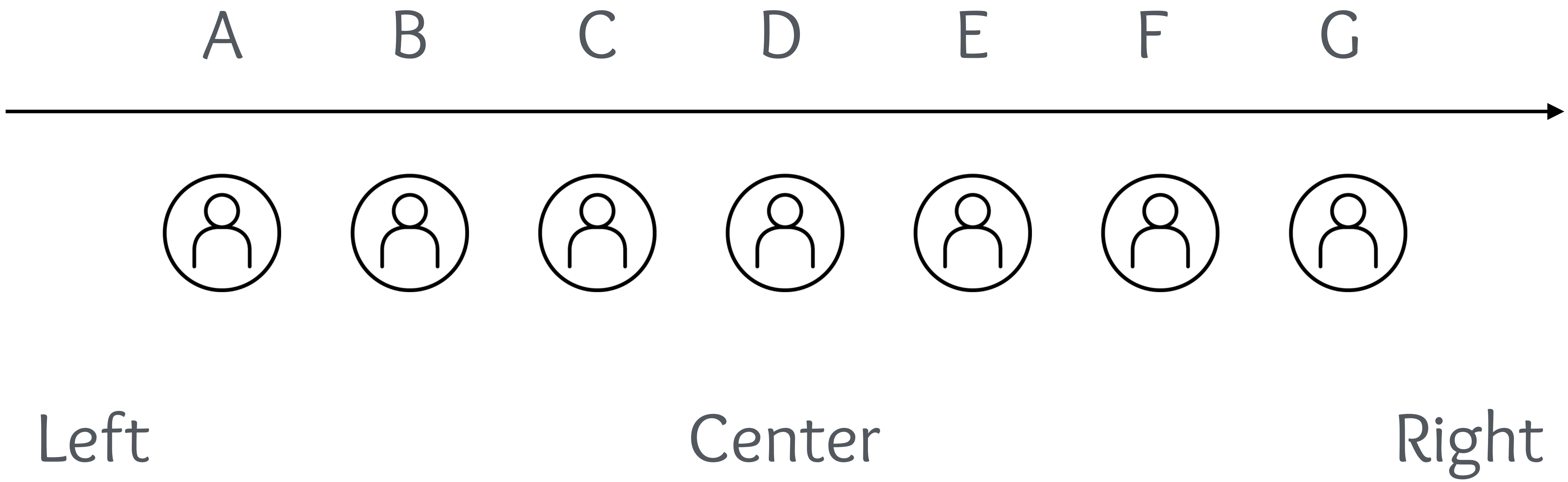
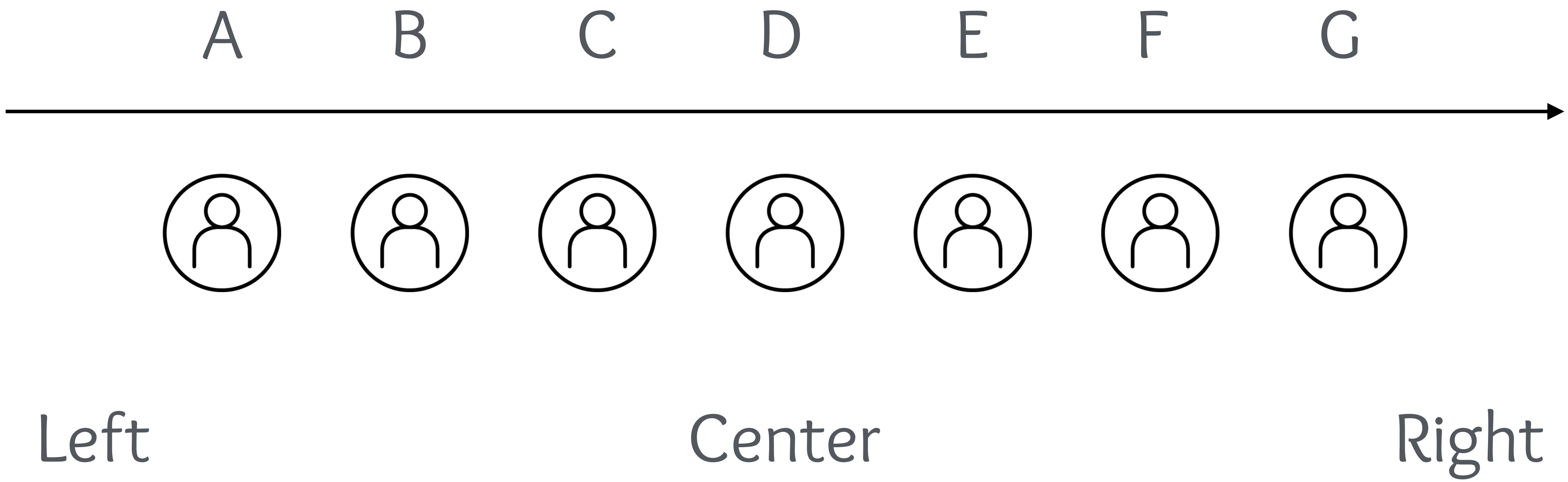


If an agent with single-peaked preferences prefers x to y, one of the following must be true:

- x is the agent's peak,
- x and y are on opposite sides of the agent's peak, or
- x is closer to the peak than y.



The notion is popular for several reasons:

- No Condorcet Cycles.
- No incentive for an agent to misreport its preferences.
- Identifiable in polynomial time.
- Reasonable (?) model of actual elections.

SINGLE PEAKED PREFERENCES

Strategyproofness

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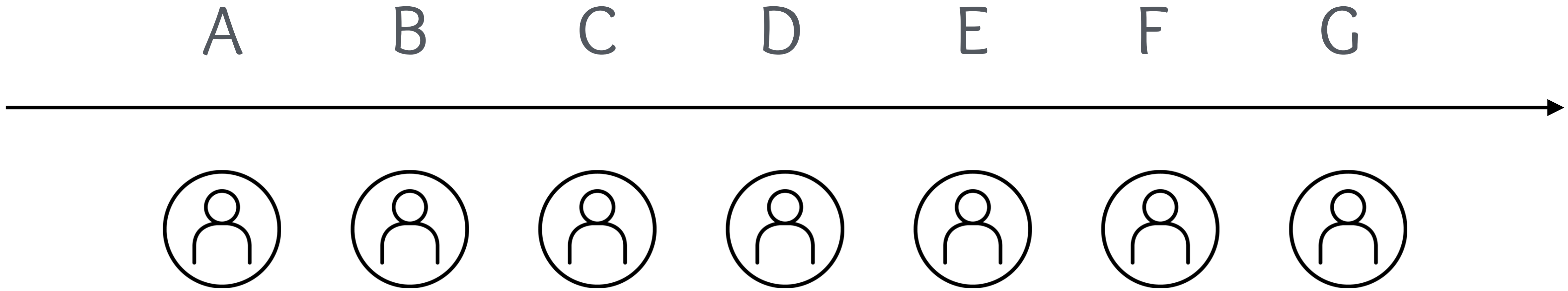
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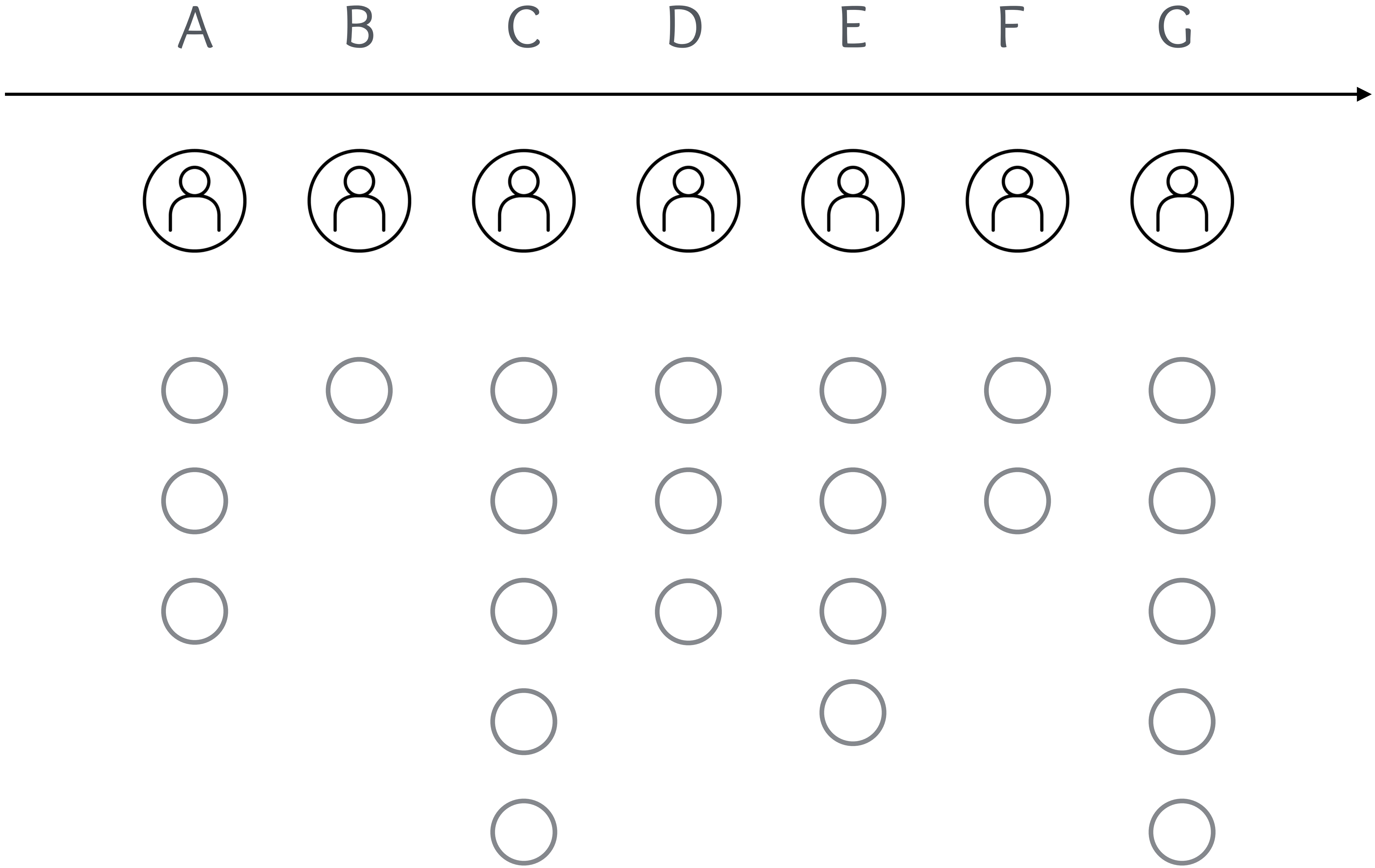
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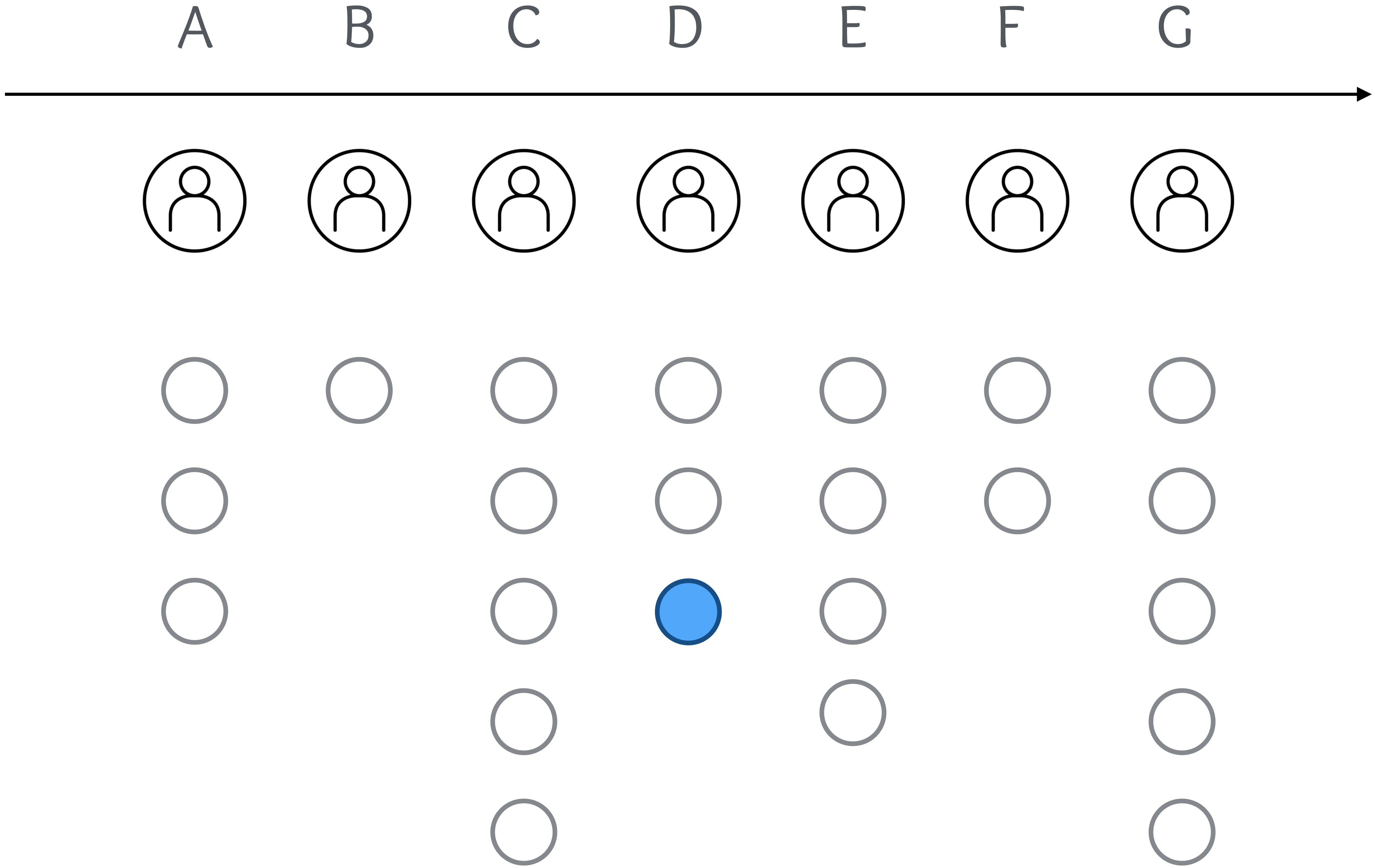
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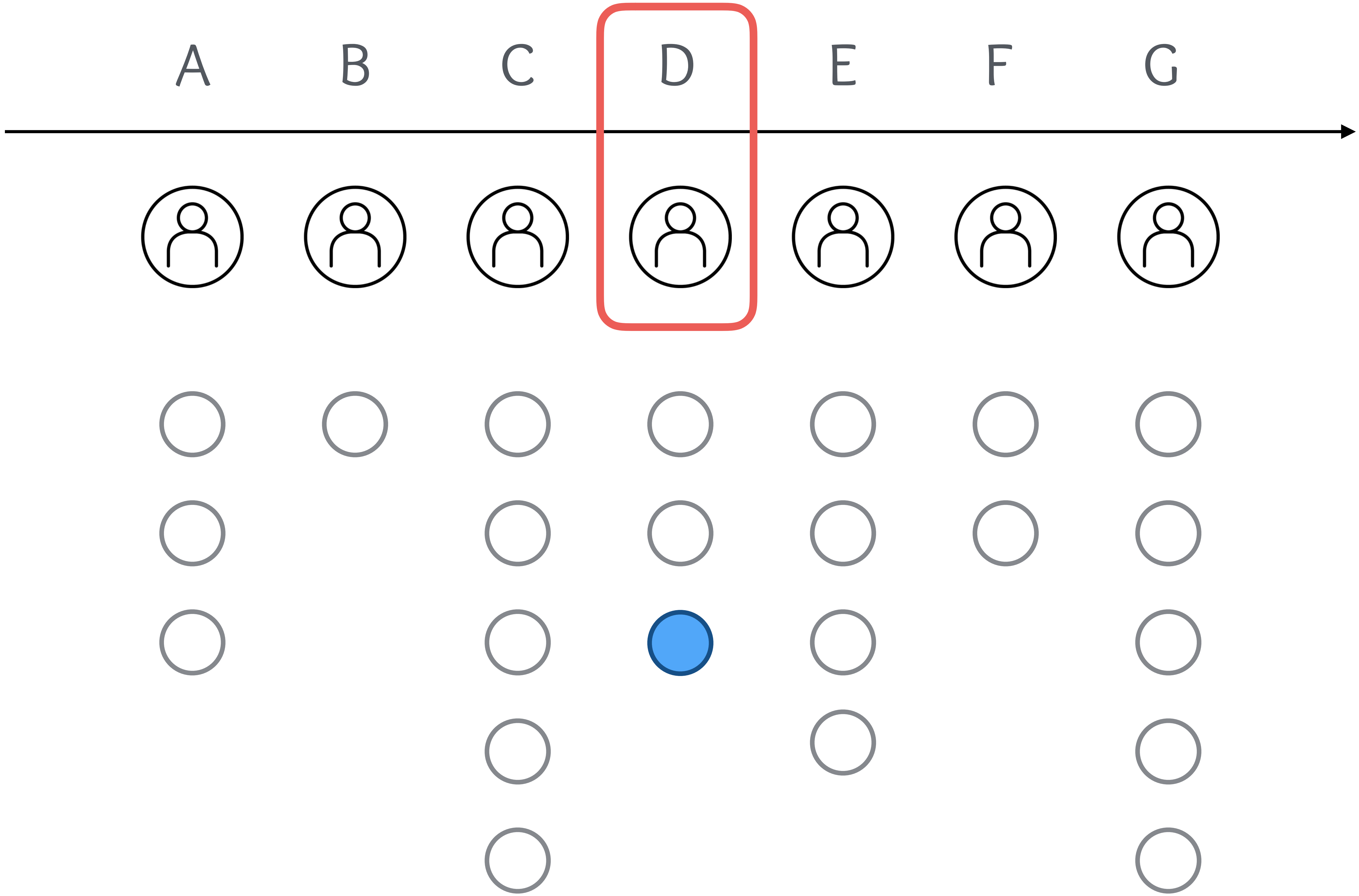
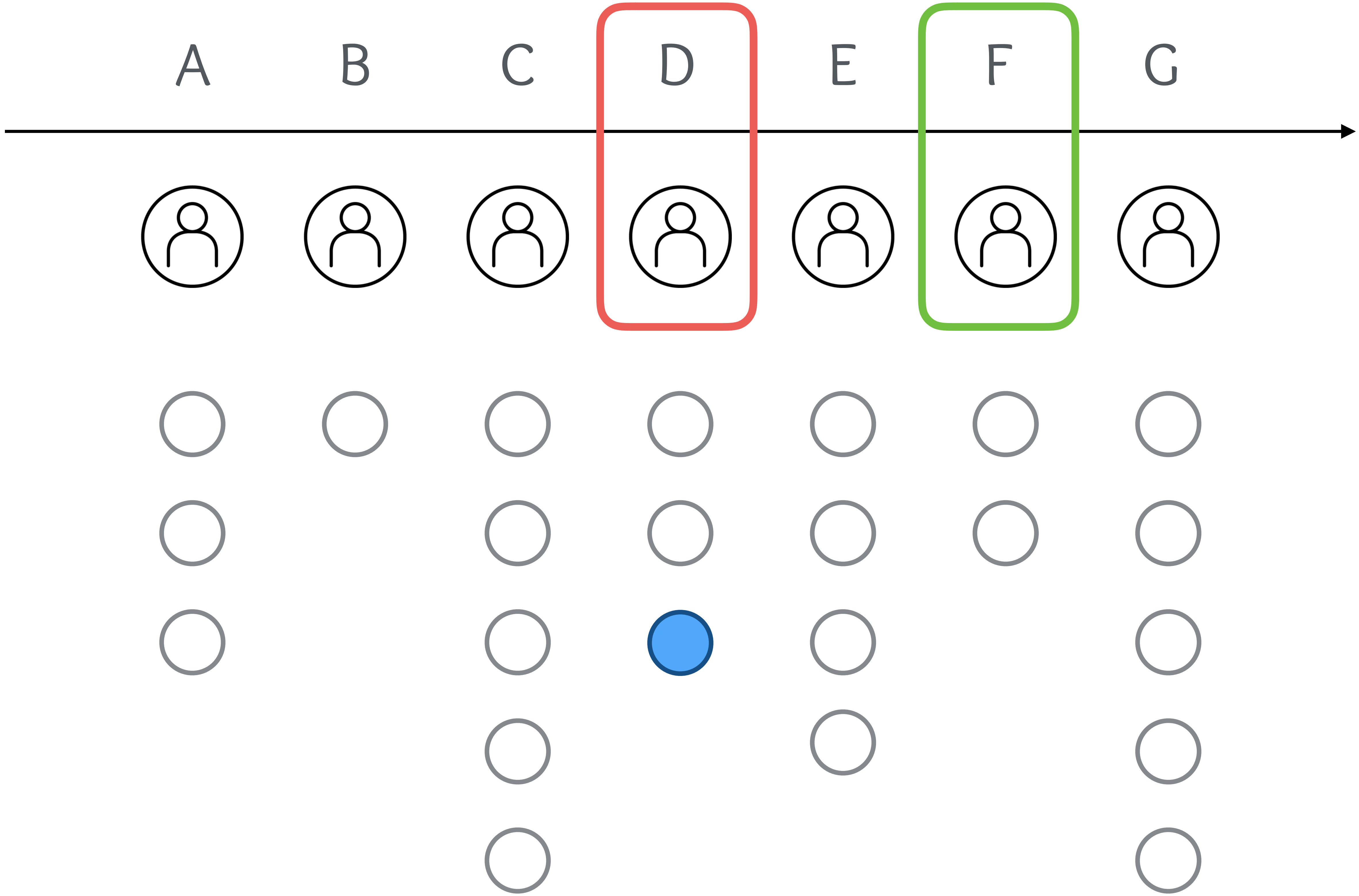


Claim: D beats all other candidates in pairwise elections.



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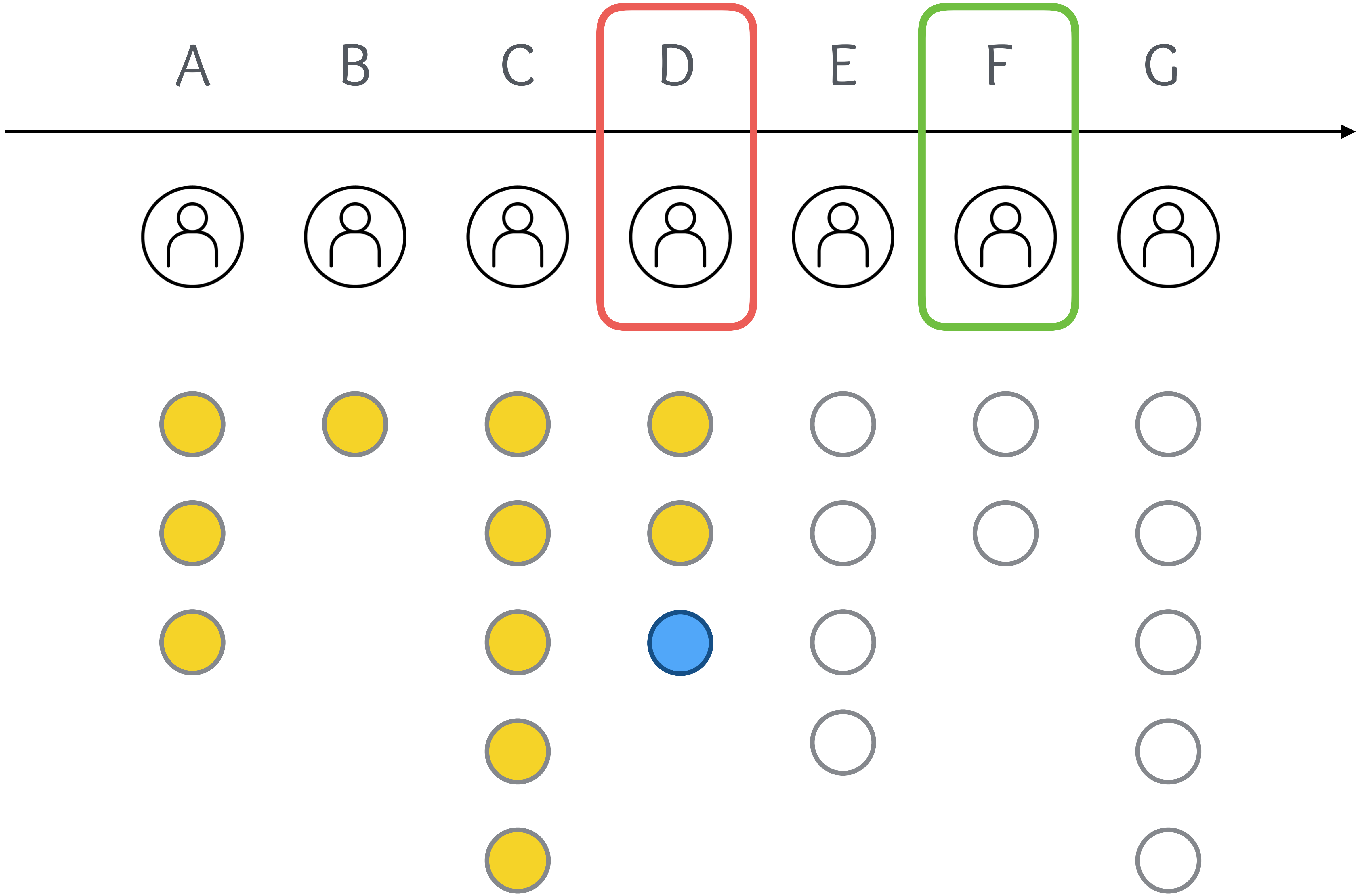

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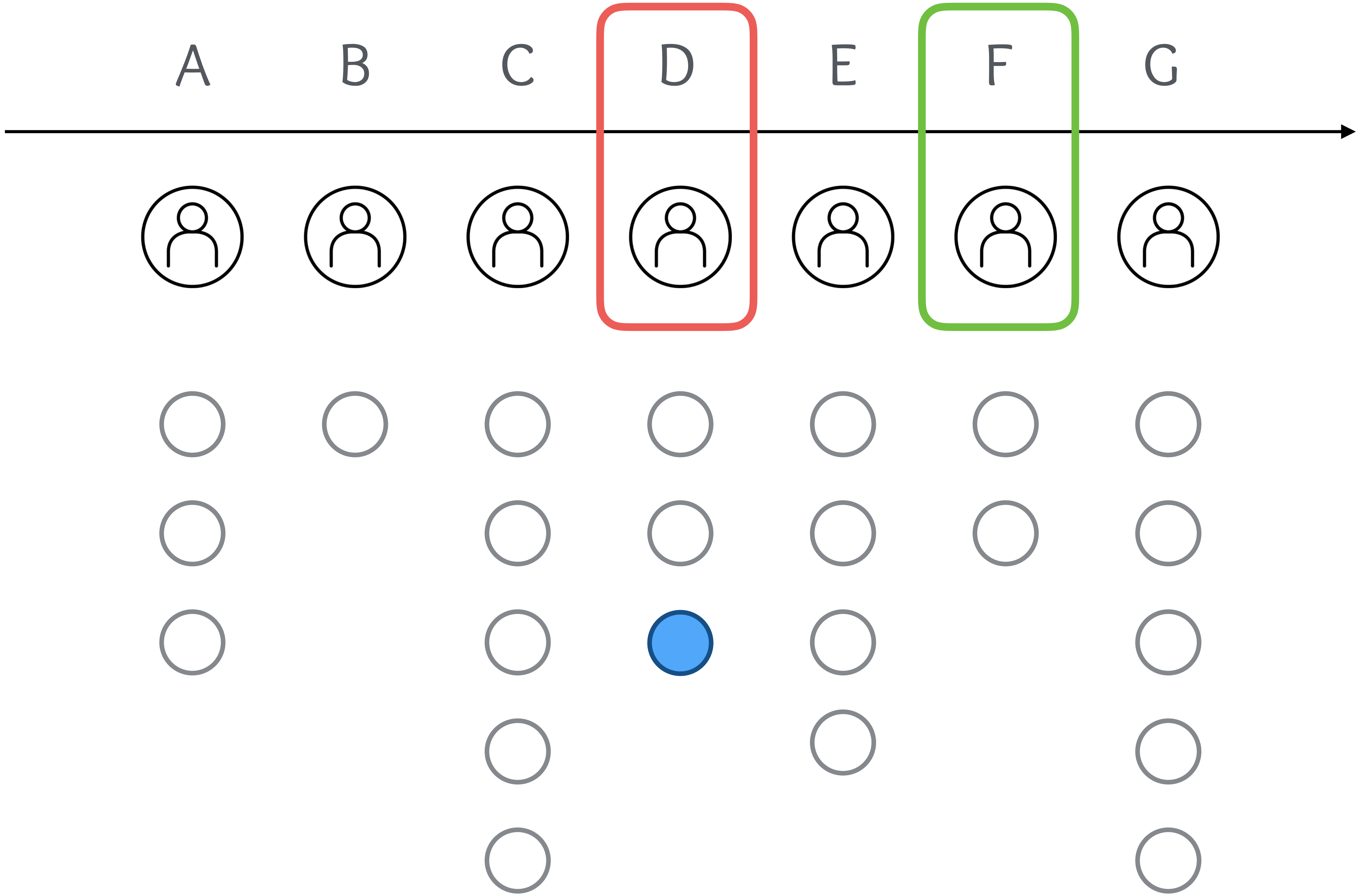

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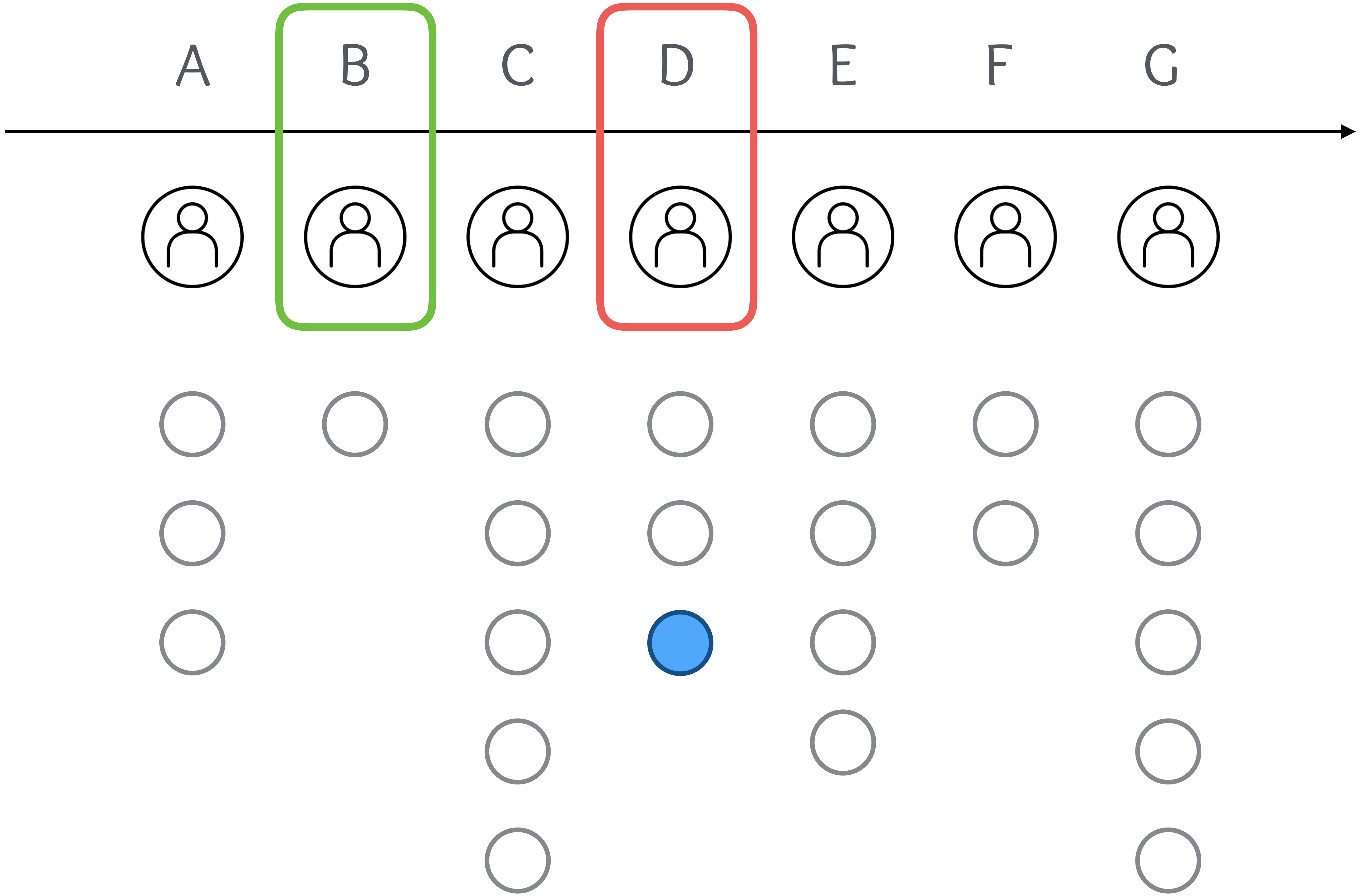

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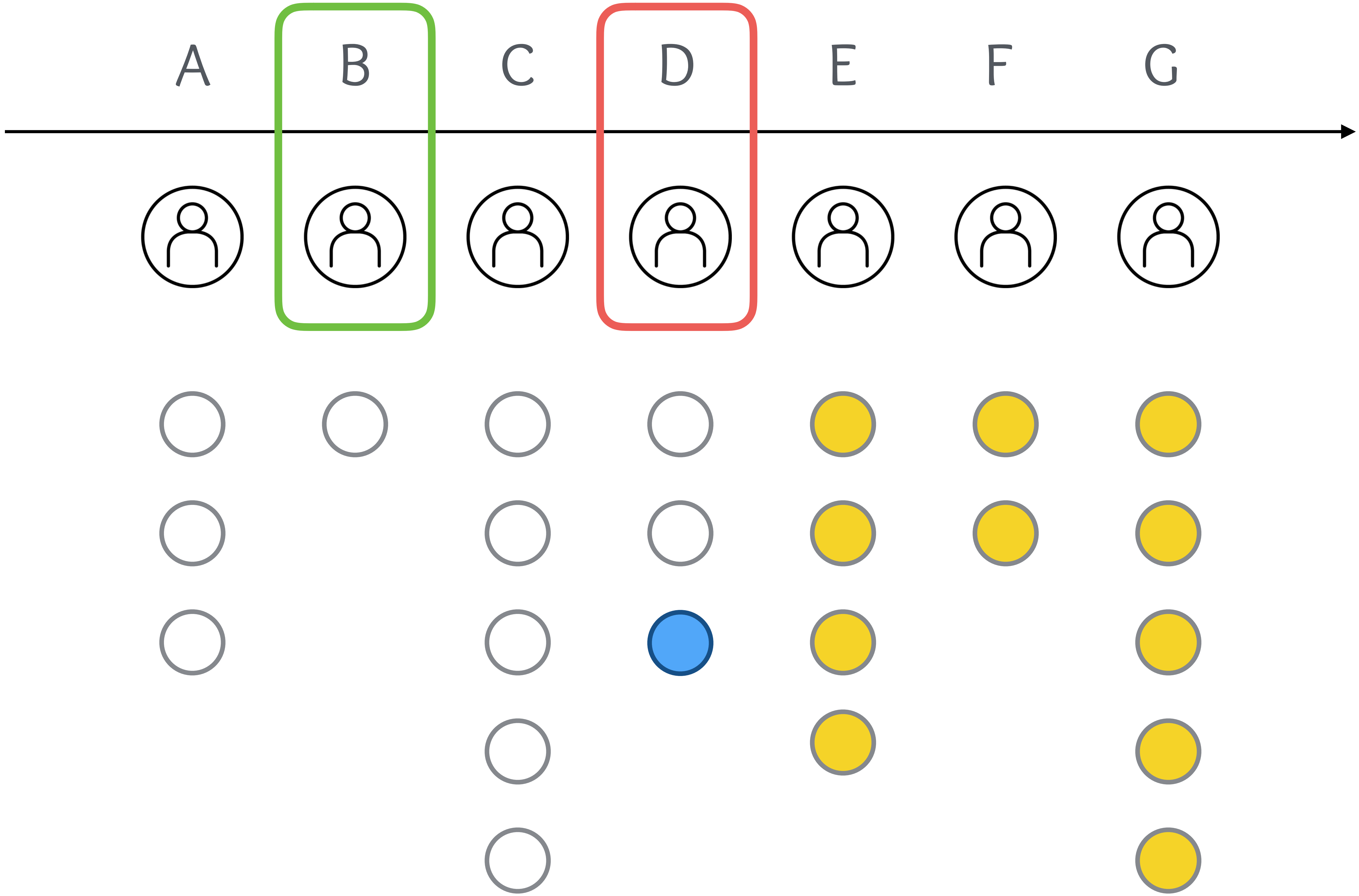

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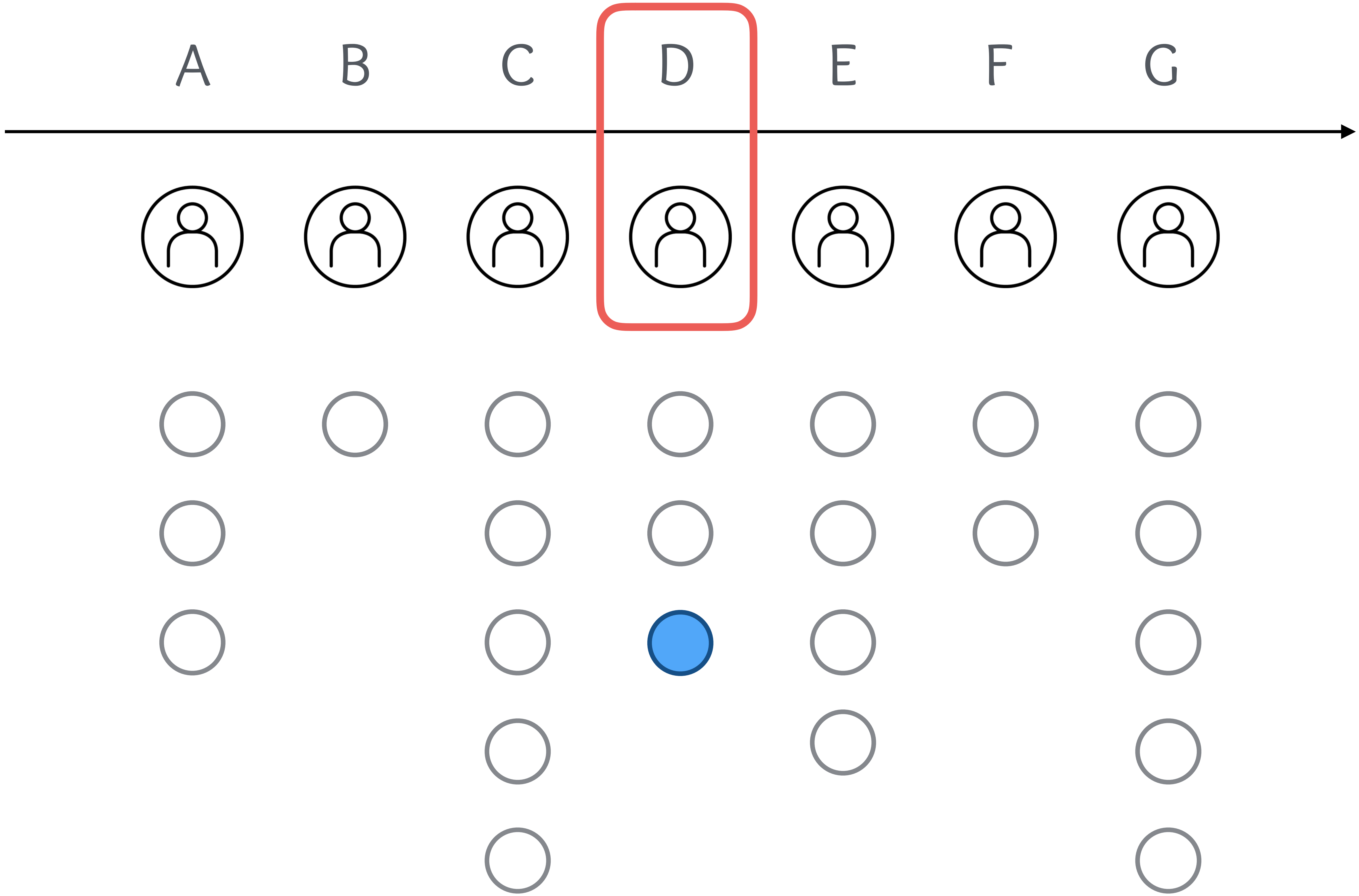

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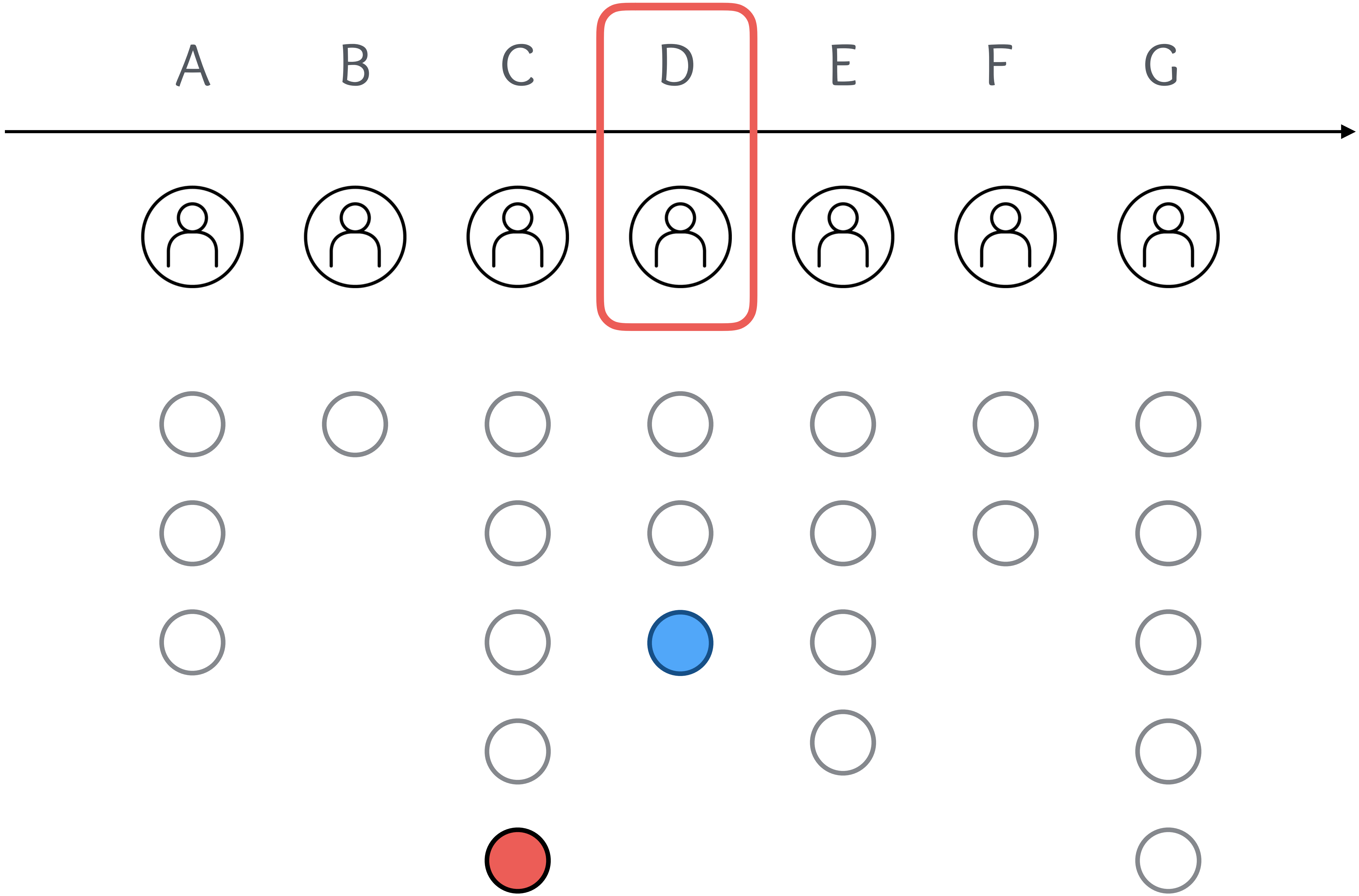

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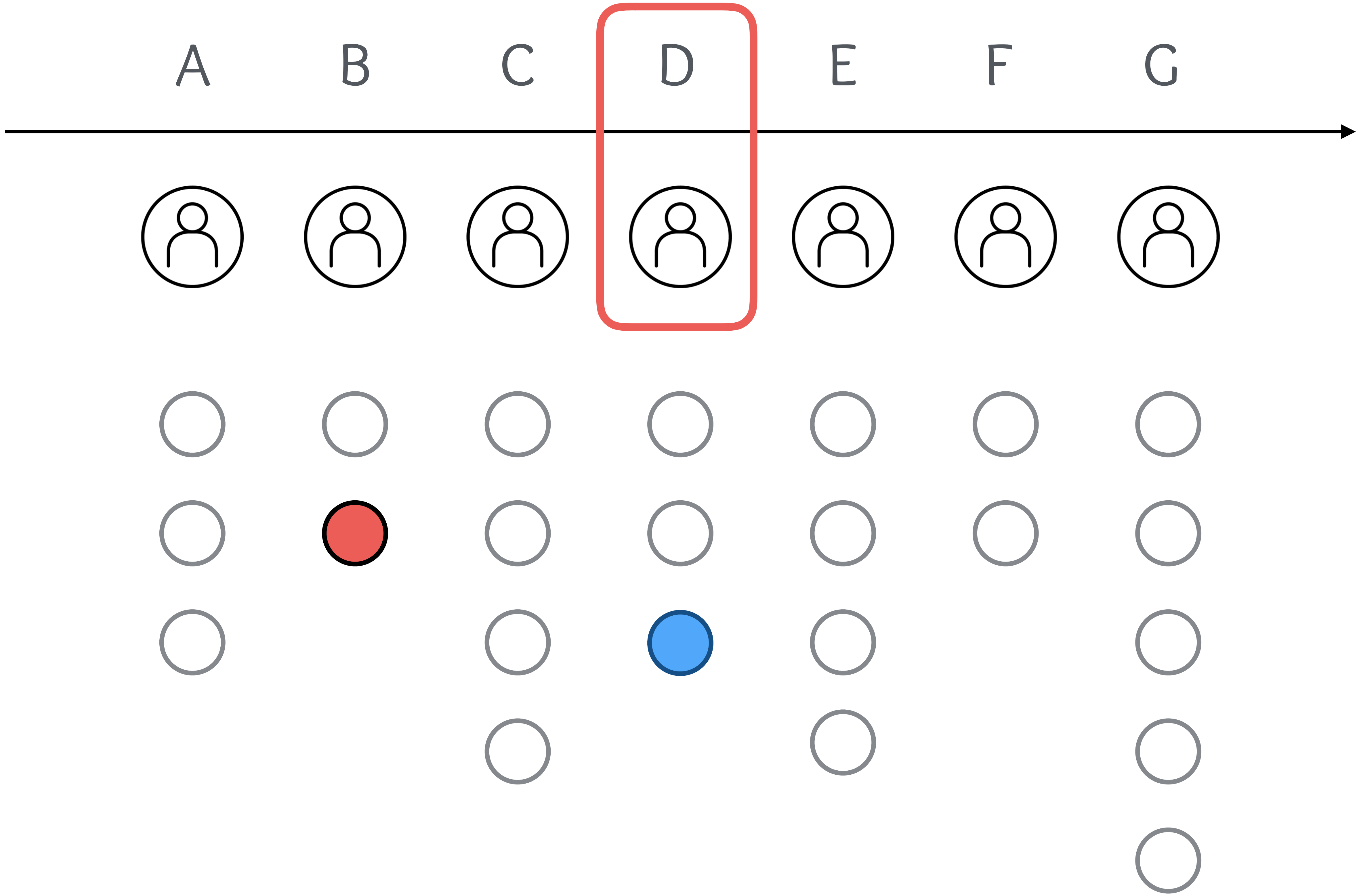

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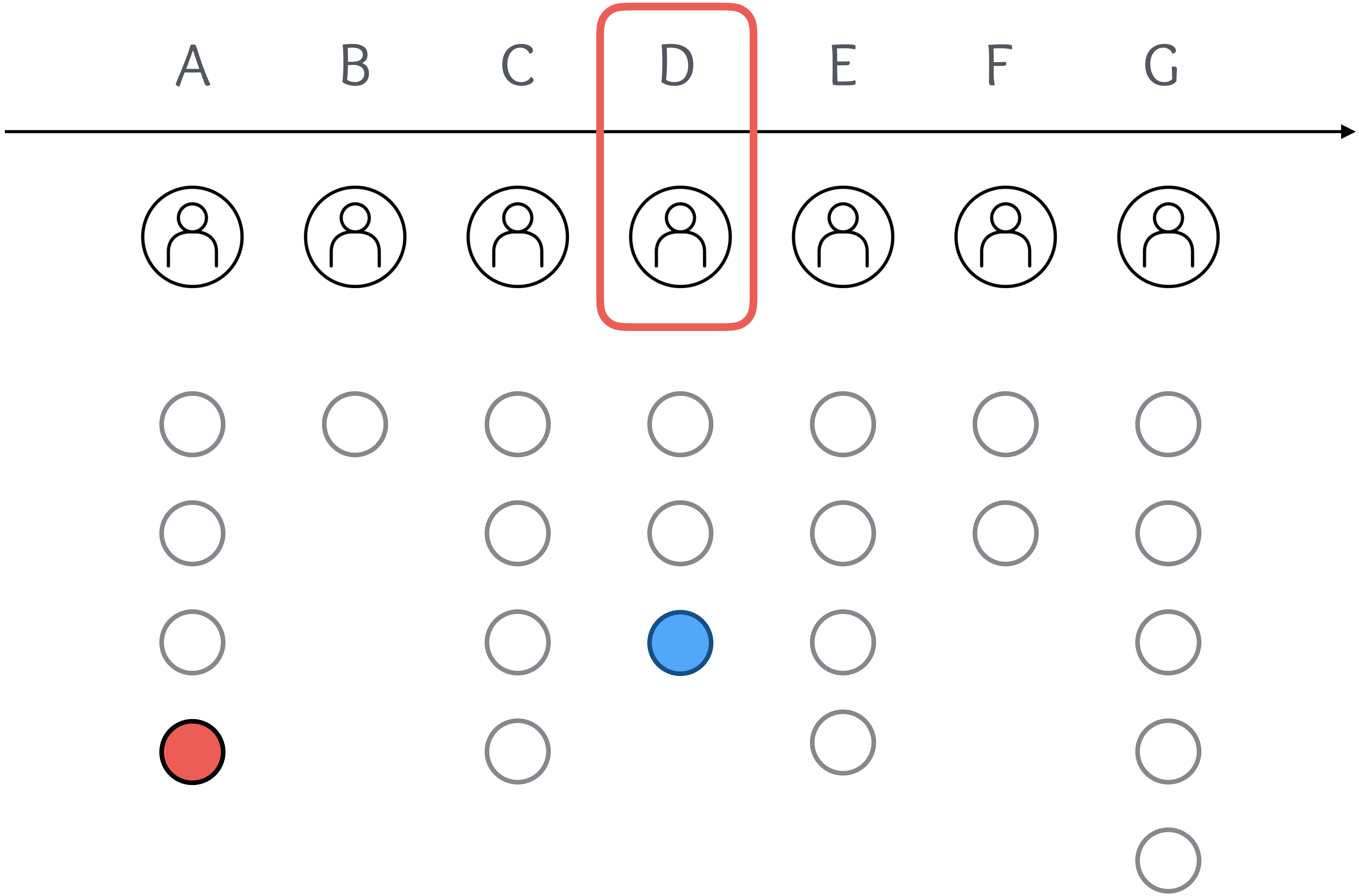

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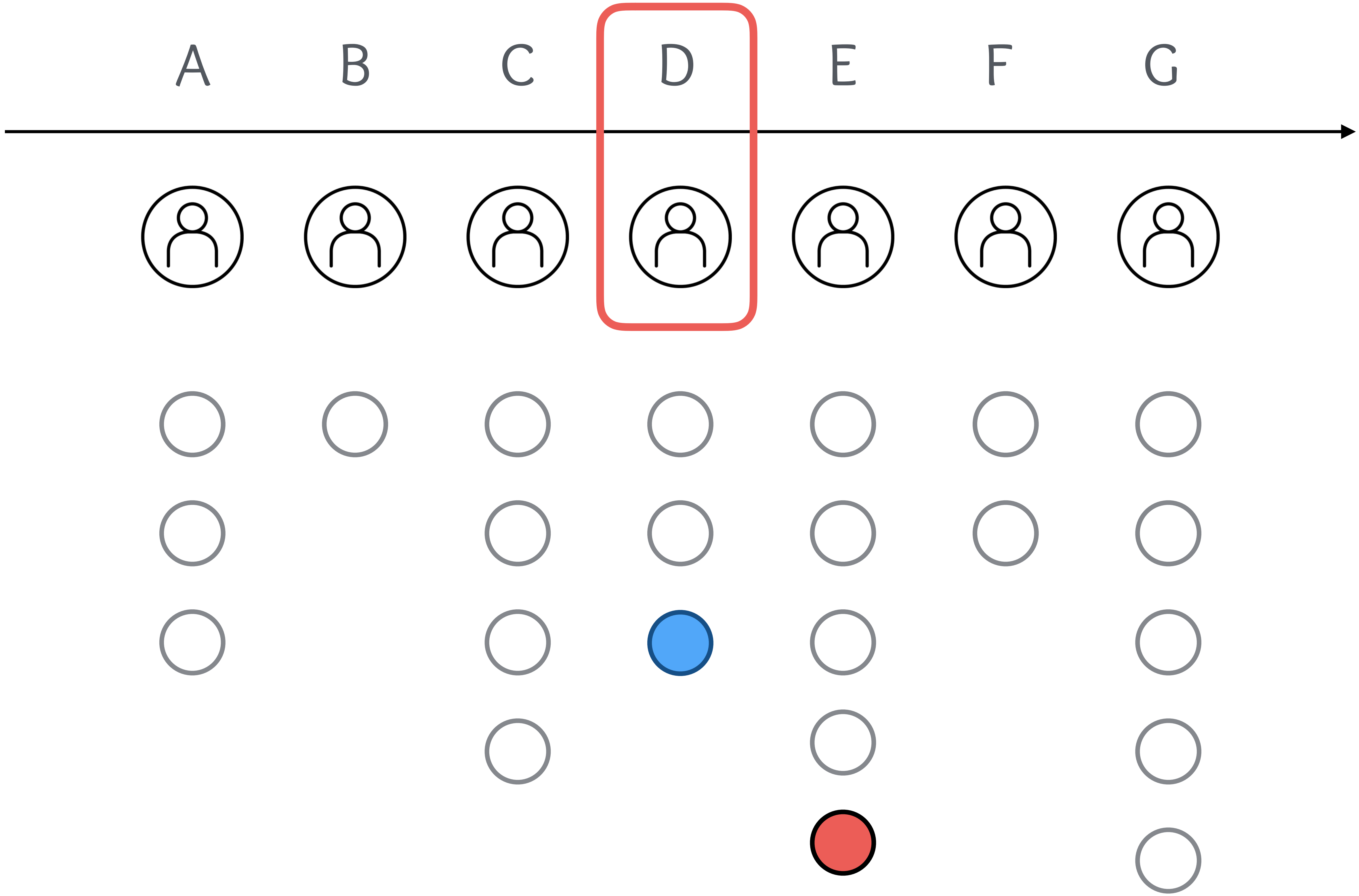

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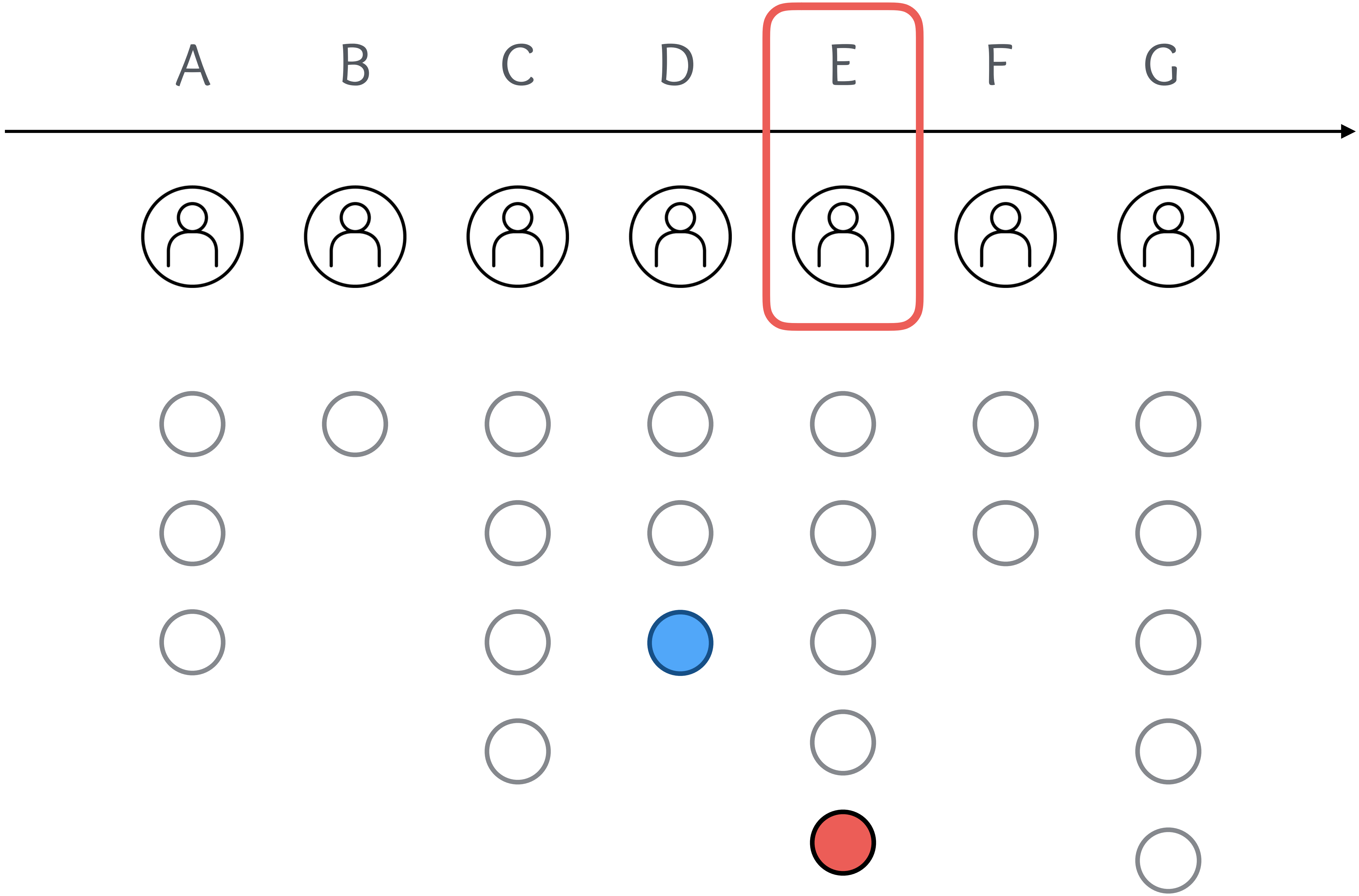

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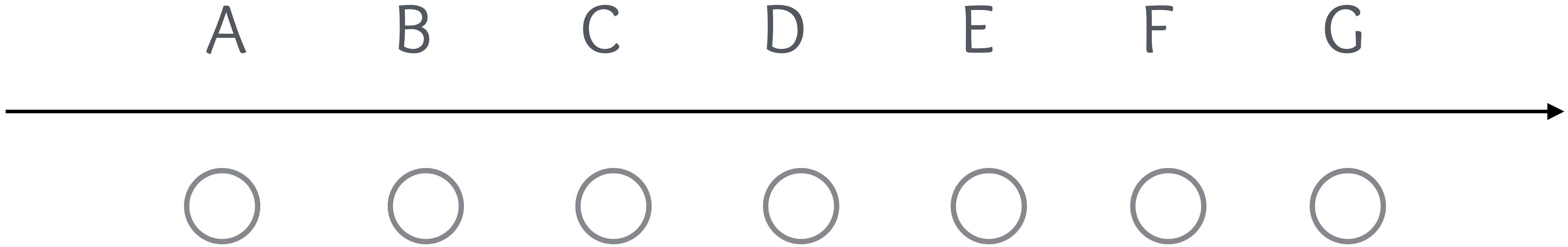
SINGLE PEAKED PREFERENCES

Chamberlin-Courant

N. Betzler, A. Slinko, and J. Uhlmann. On the computation of fully proportional representation. Journal of Artificial Intelligence Research, 47(1):475–519, 2013.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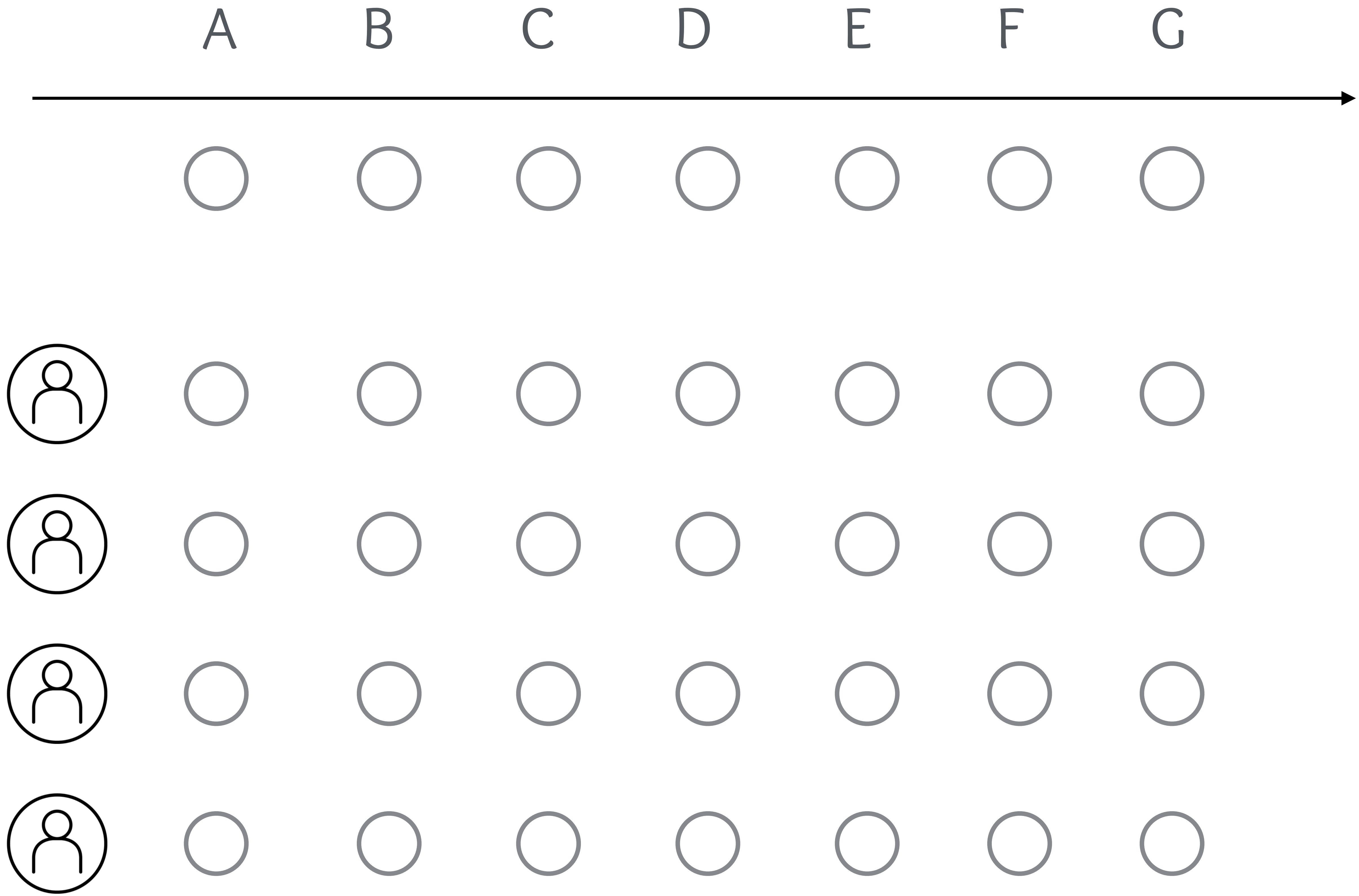
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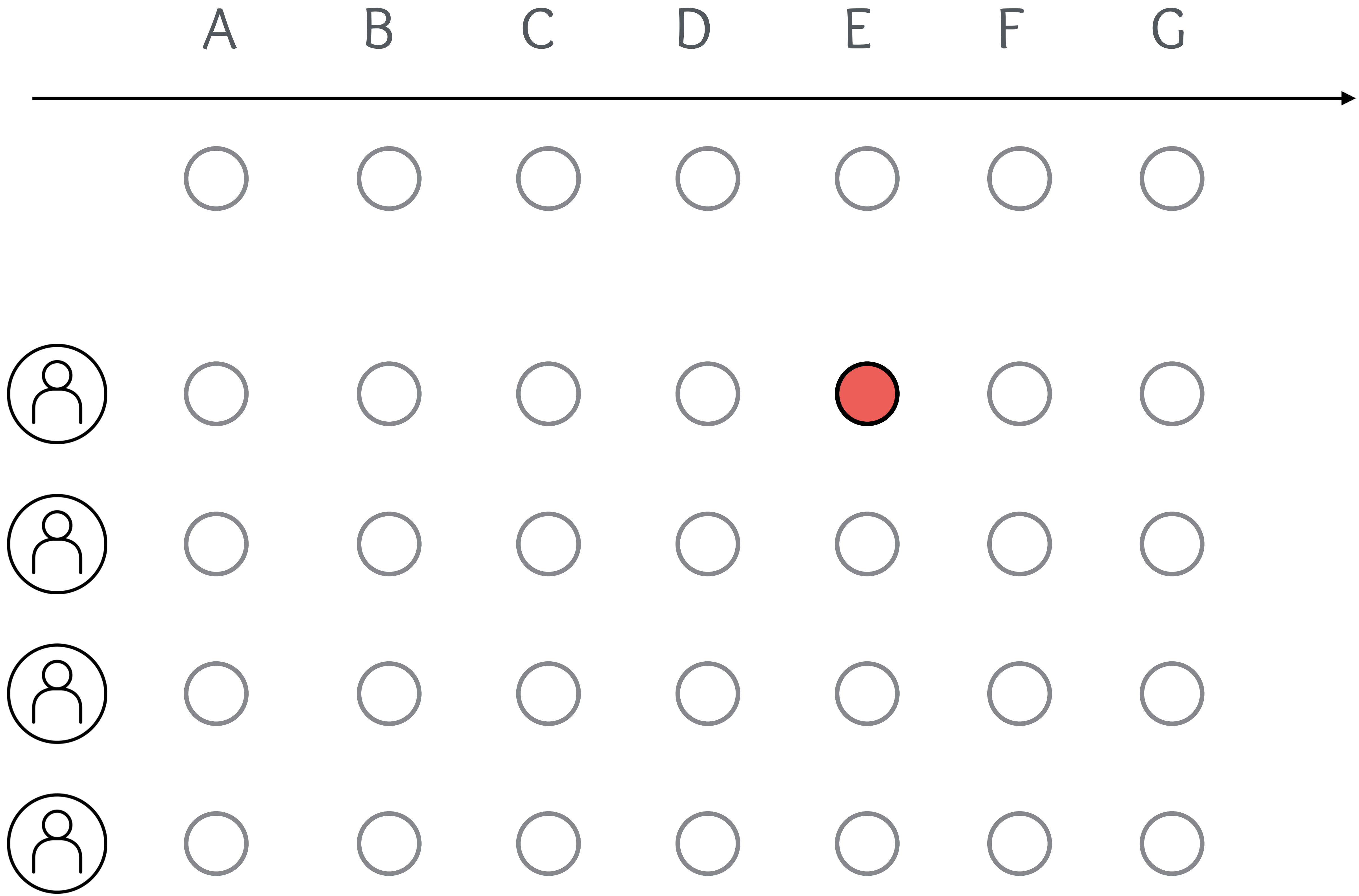
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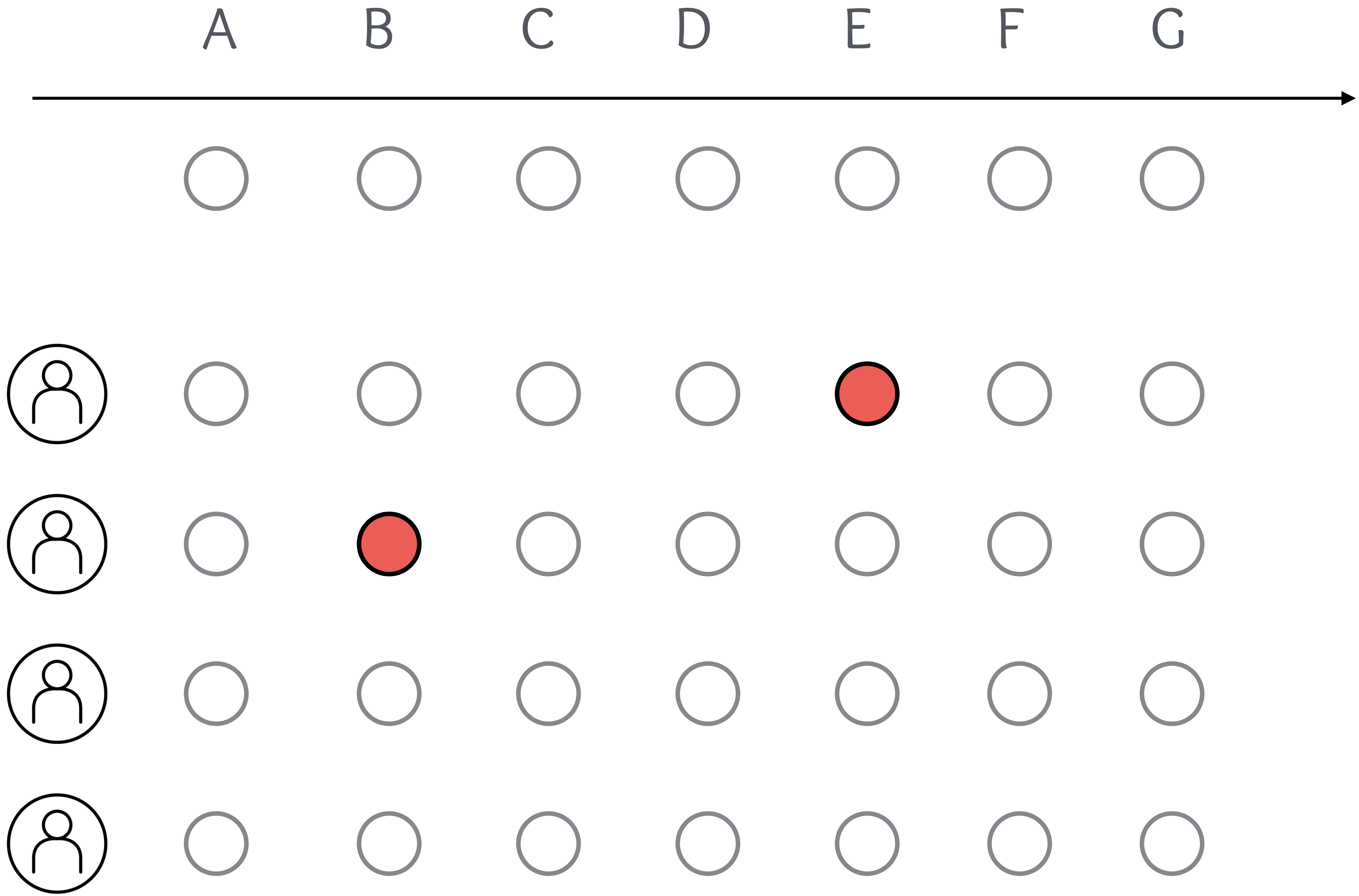
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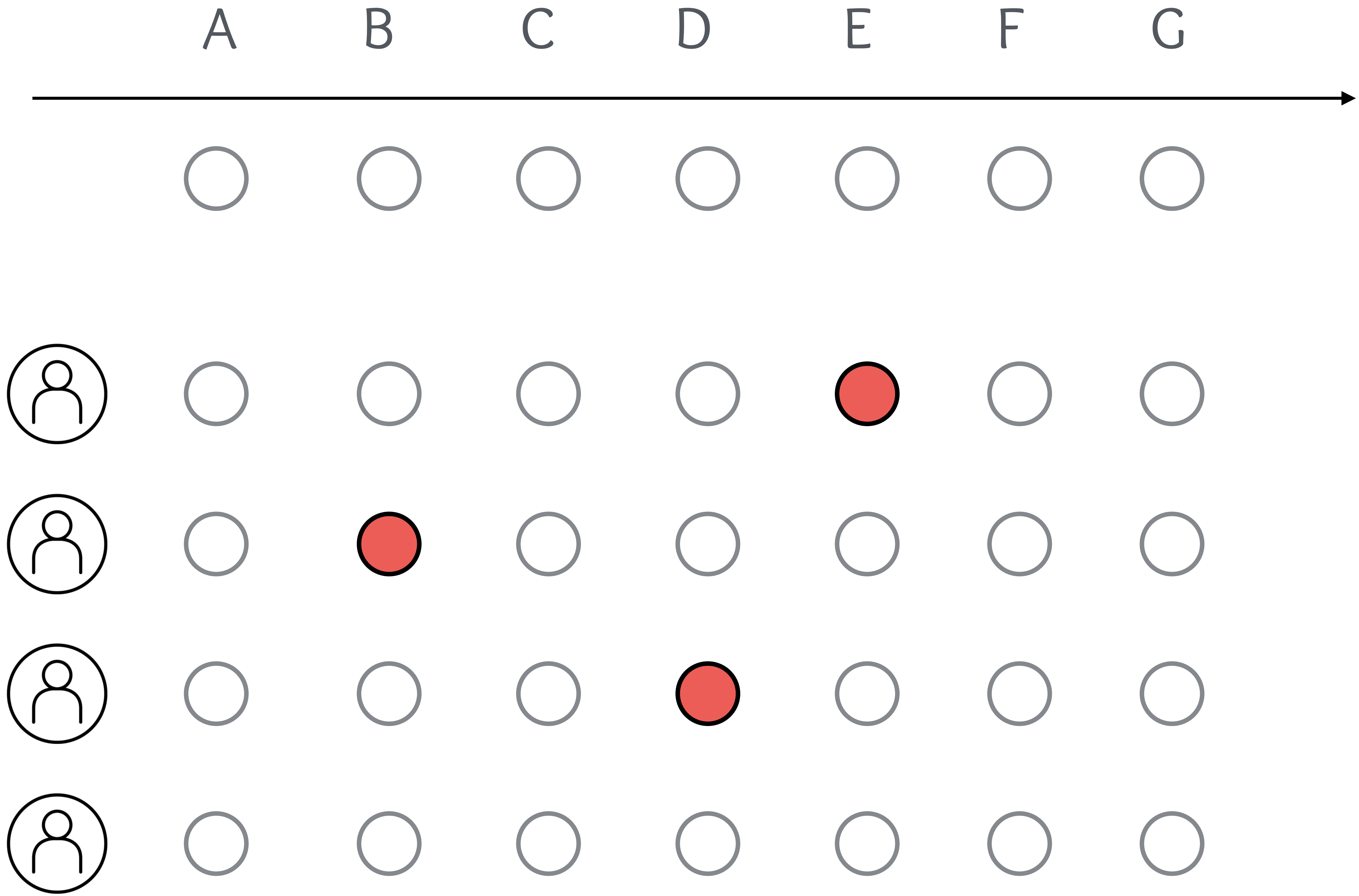
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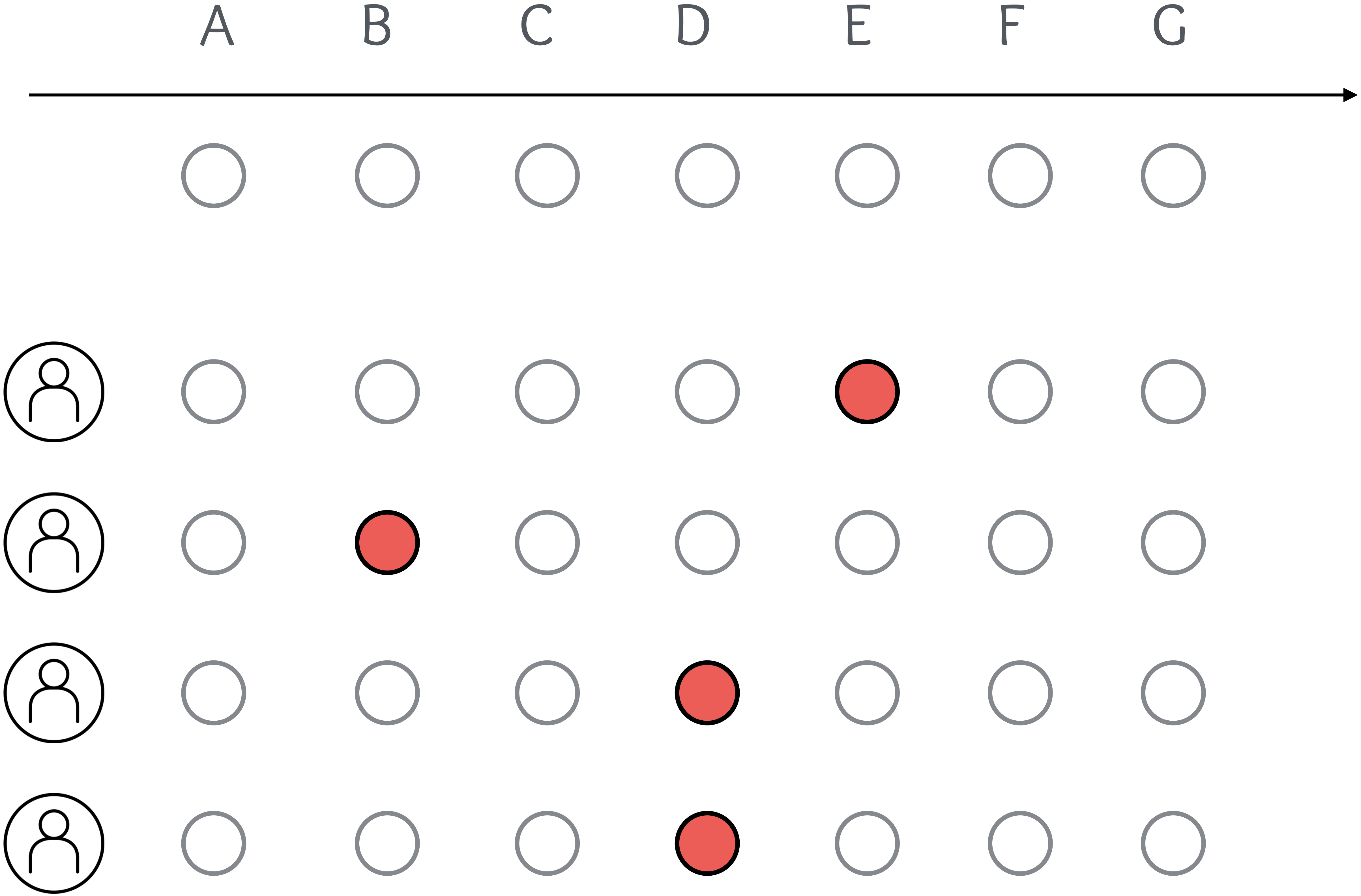
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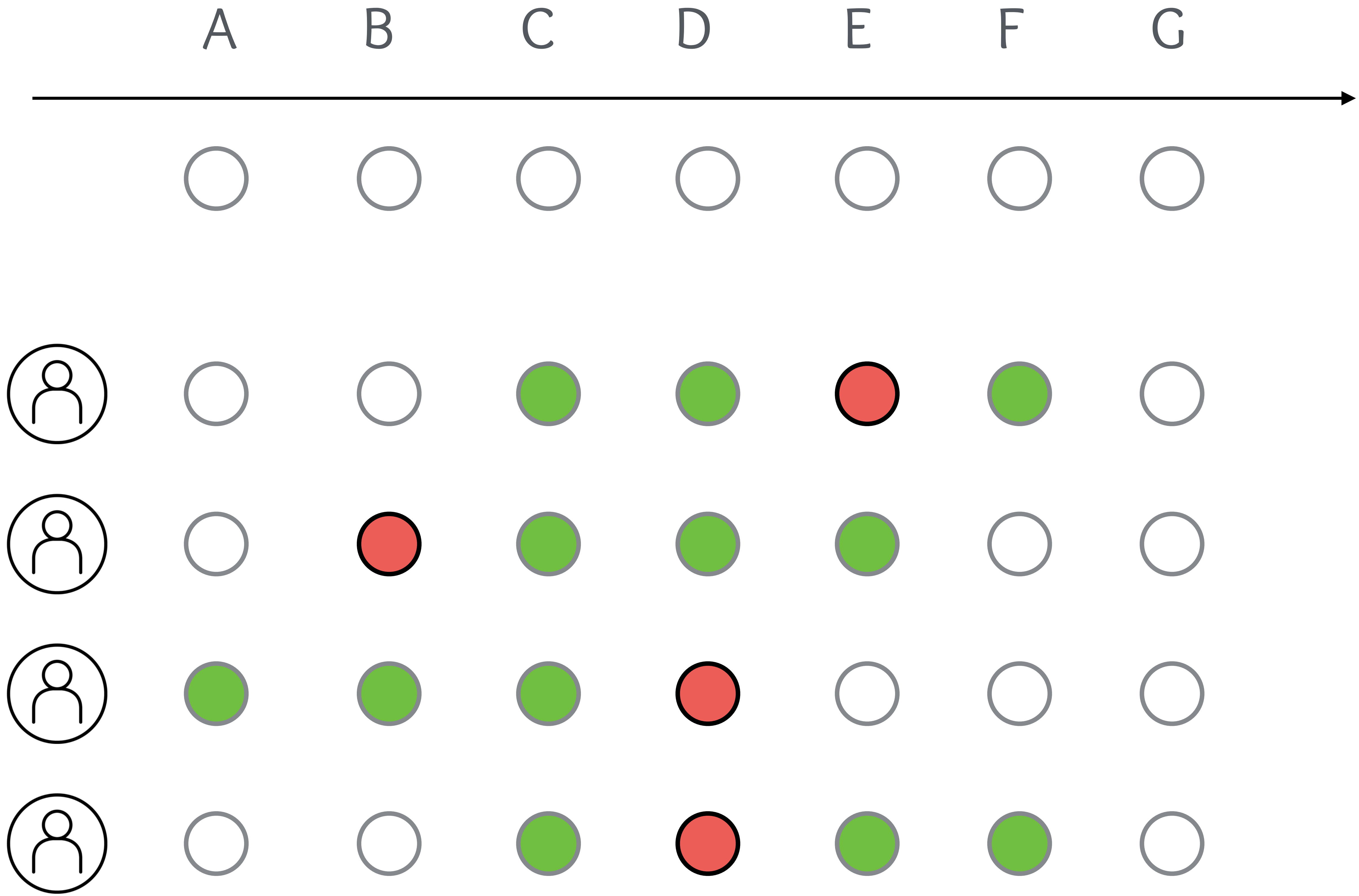
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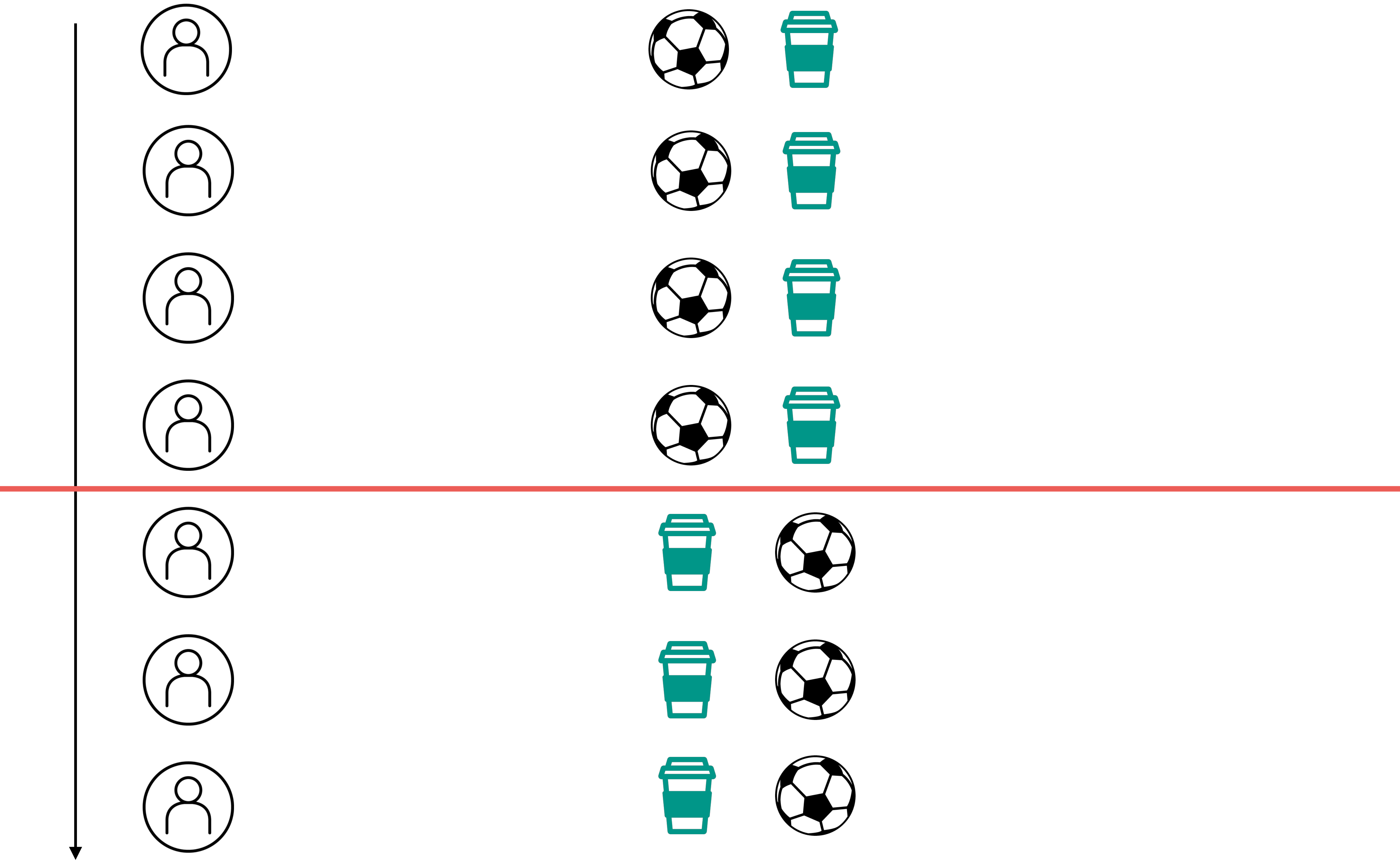
Determining the winner reduces to stabbing a set of intervals with k lines.

SINGLE CROSSING PREFERENCES

Definition

A profile is **single-crossing** if it admits an ordering of the voters such that for *every* pair of candidates (a,b), either:

a) all voters who prefer a over b appear before all voters who prefer b over a, or,
b) all voters who prefer a over b appear after all voters who prefer b over a, or,



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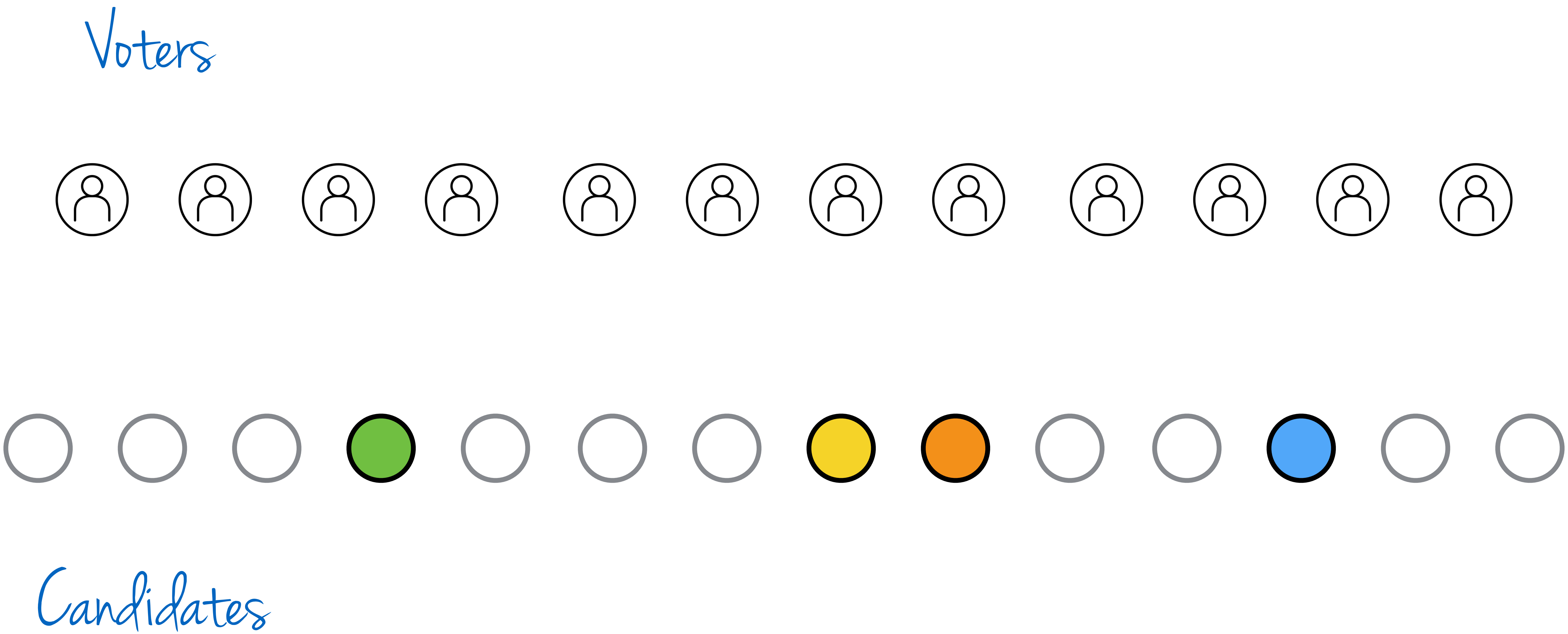
- Reasonable (?) model of actual elections.

SINGLE CROSSING PREFERENCES

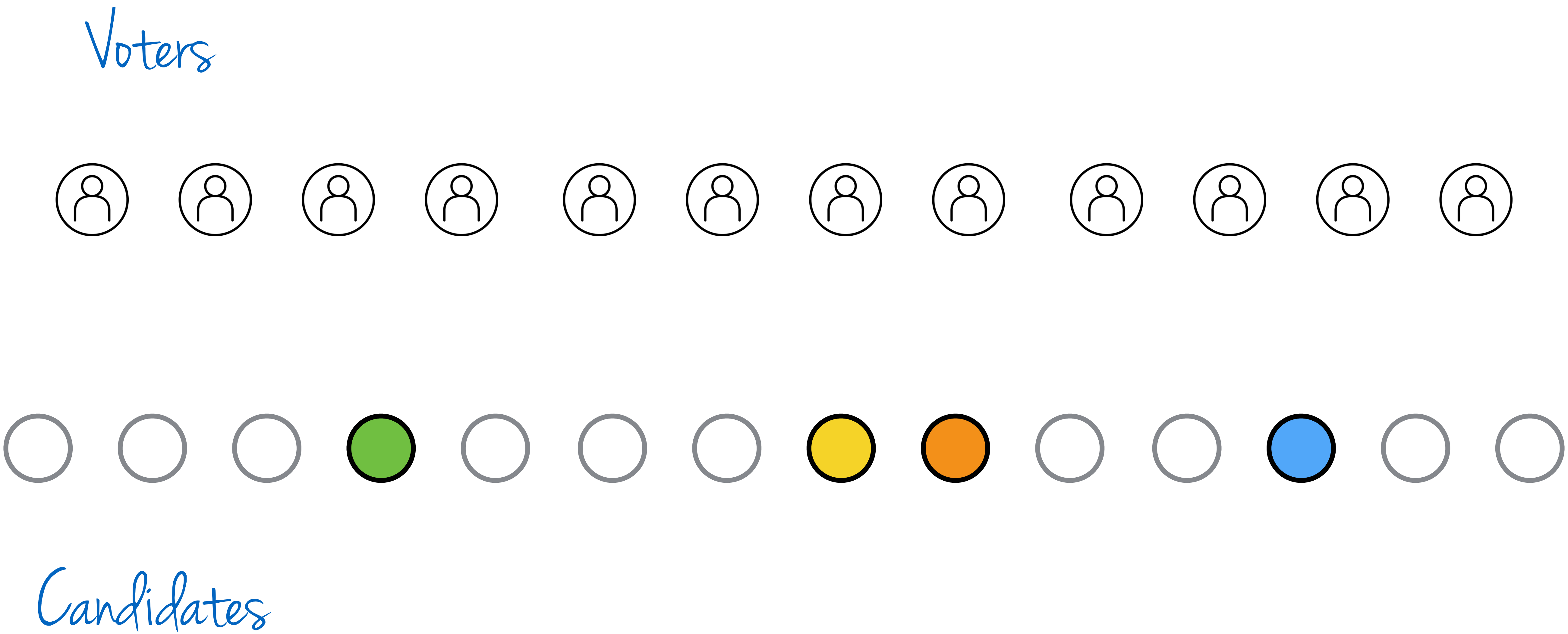
Chamberlin-Courant

The Complexity of Fully Proportional Representation for Single-Crossing Electorates
Skowron, SAGT, 2013



dissatisfaction of voter v =
rank of best candidate in the committee in his vote

On single-crossing profiles, optimal CC solutions exhibit a "contiguous blocks property".



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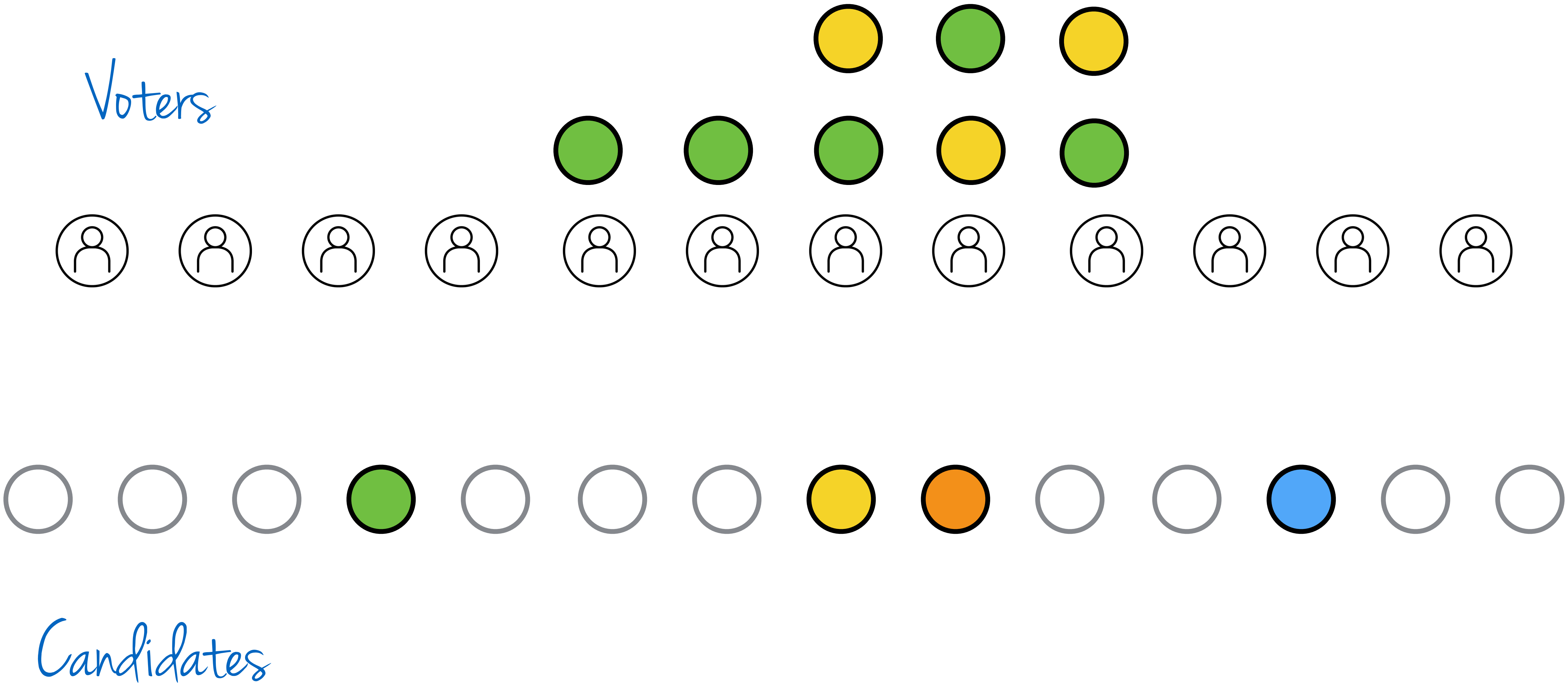

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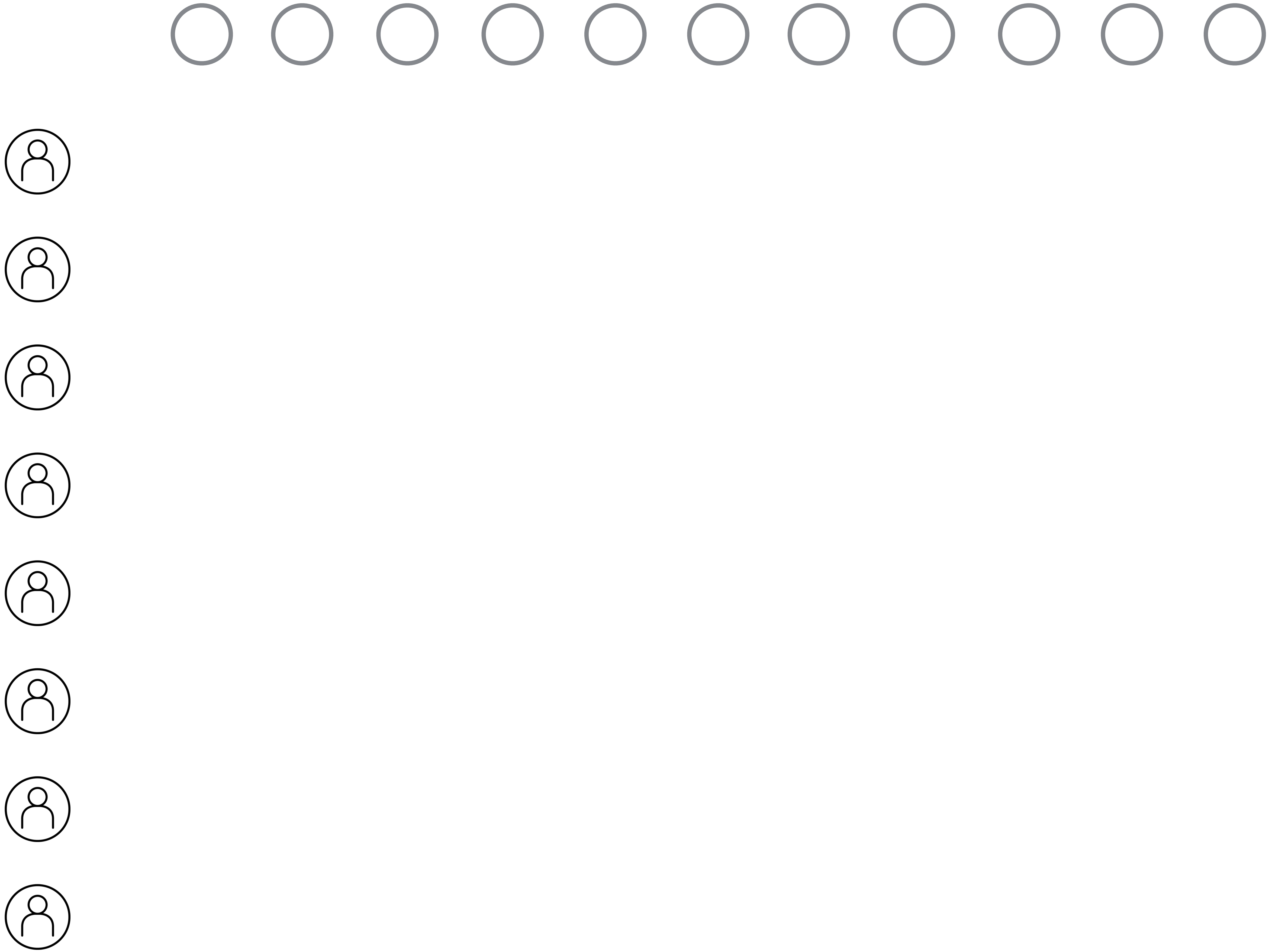

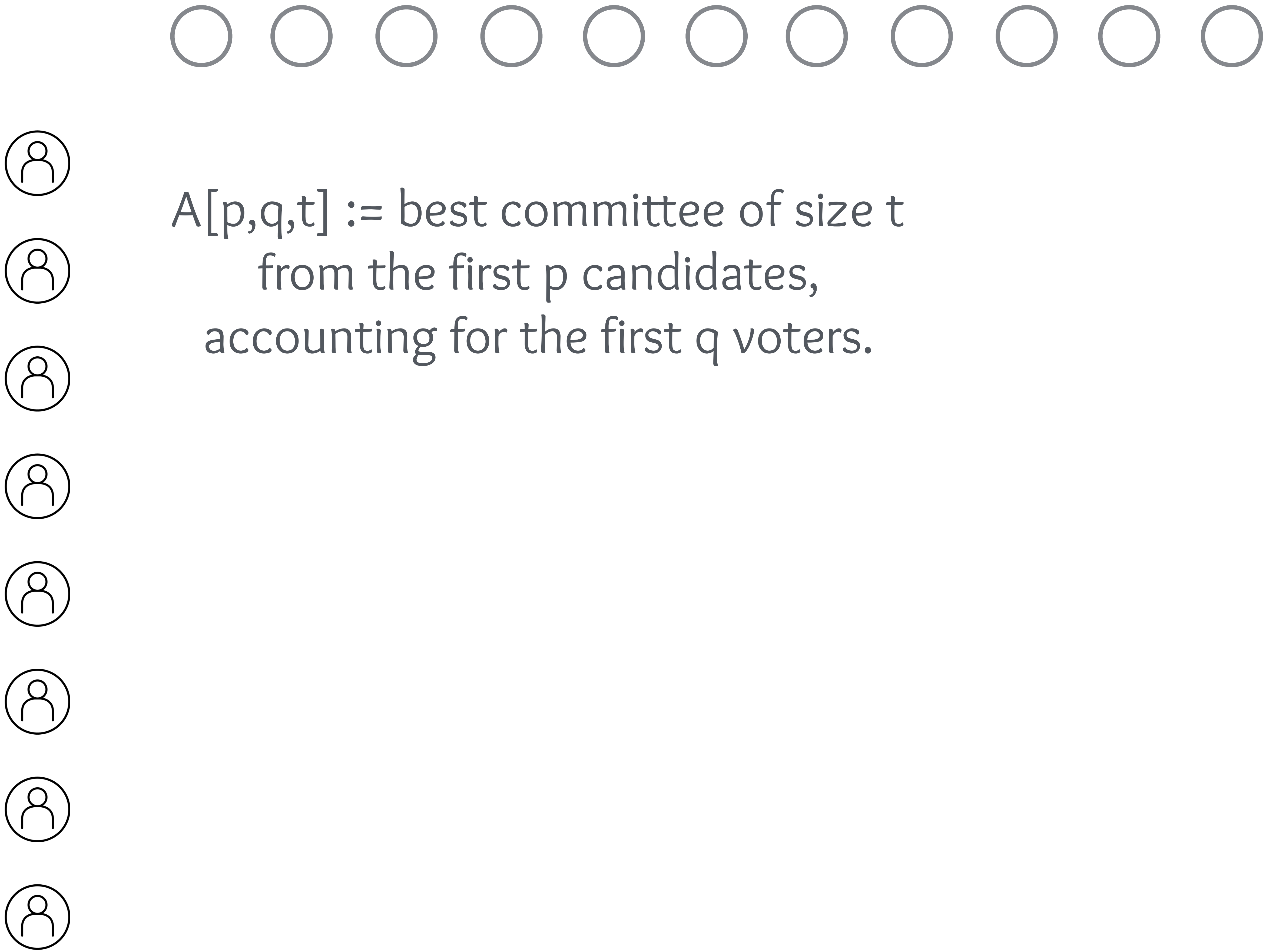
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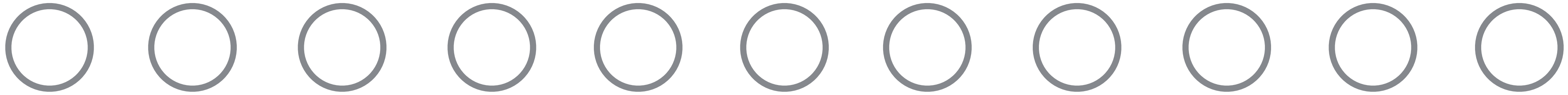

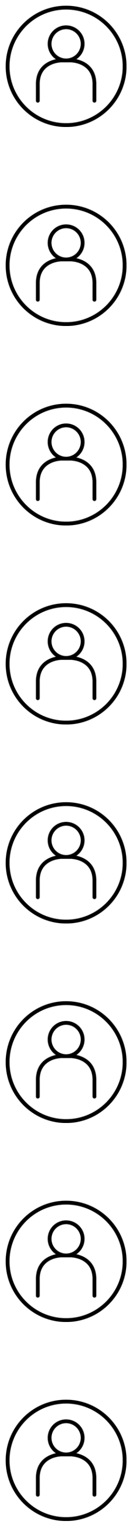

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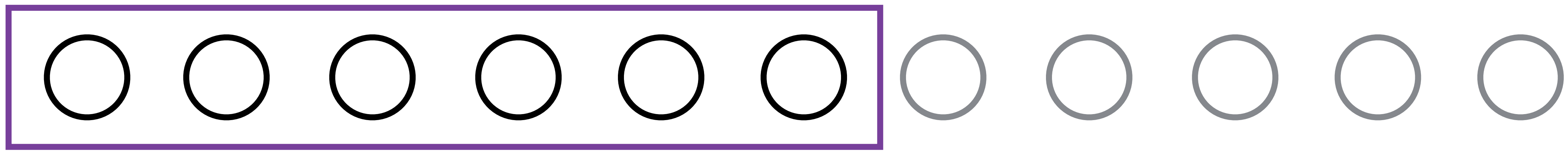

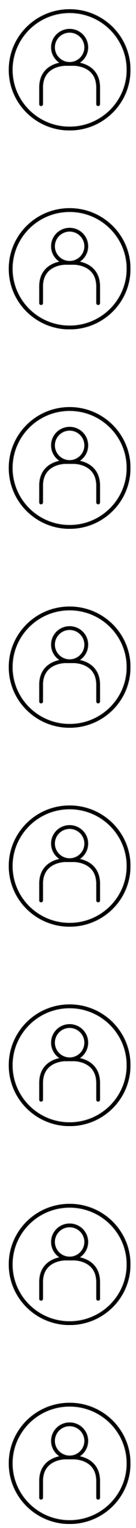

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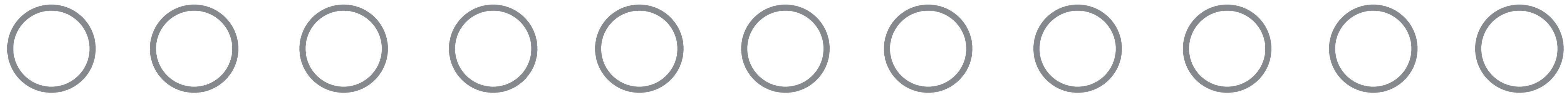

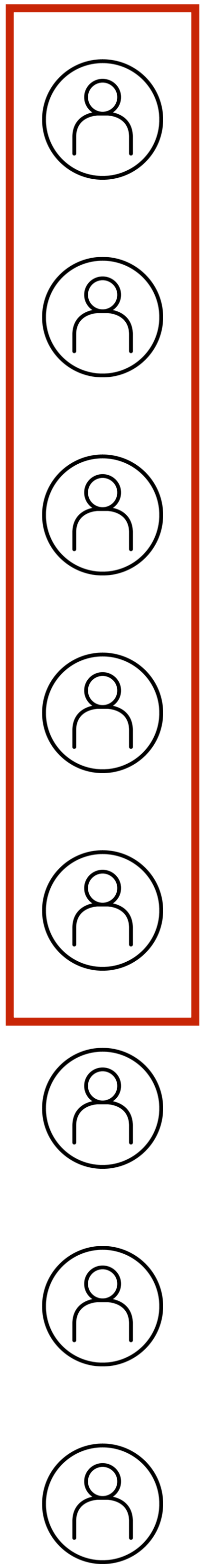

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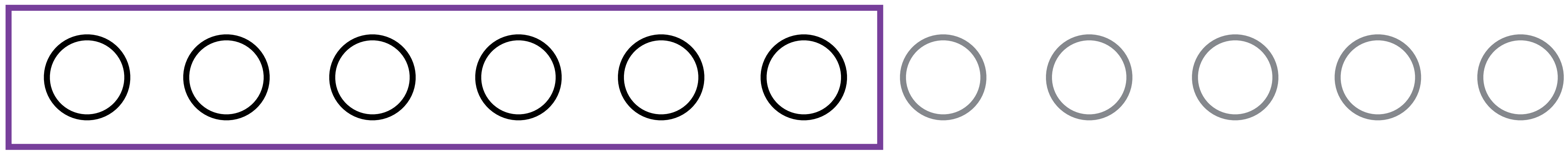

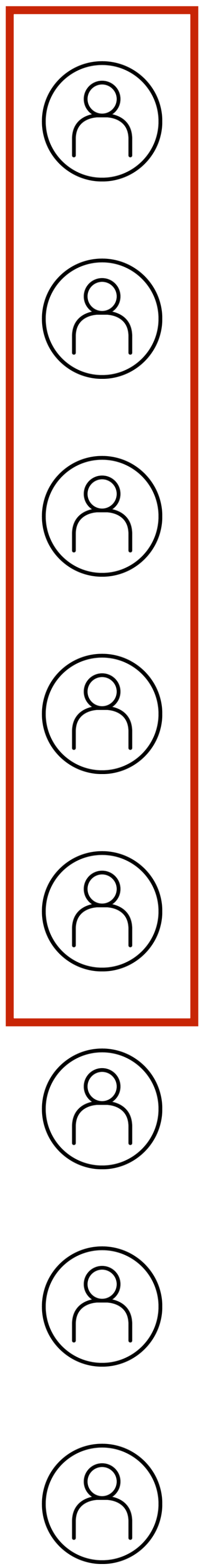

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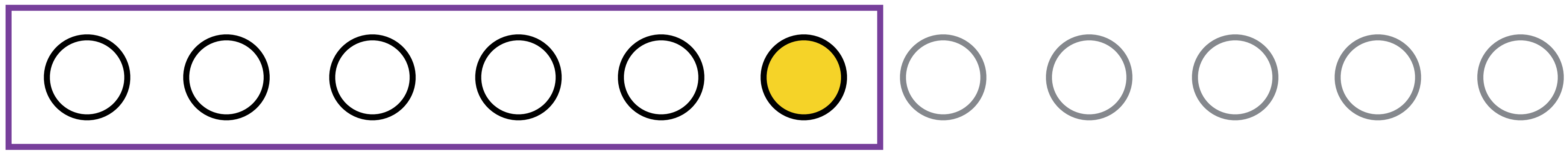

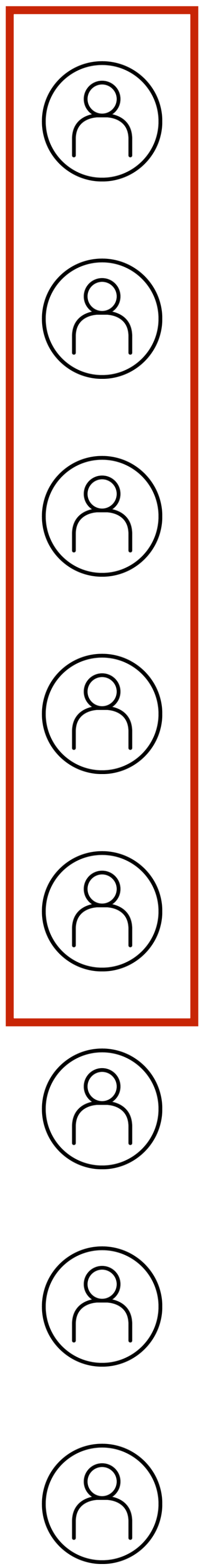

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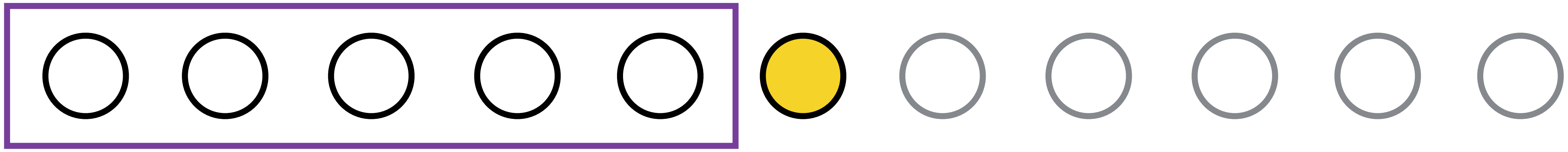

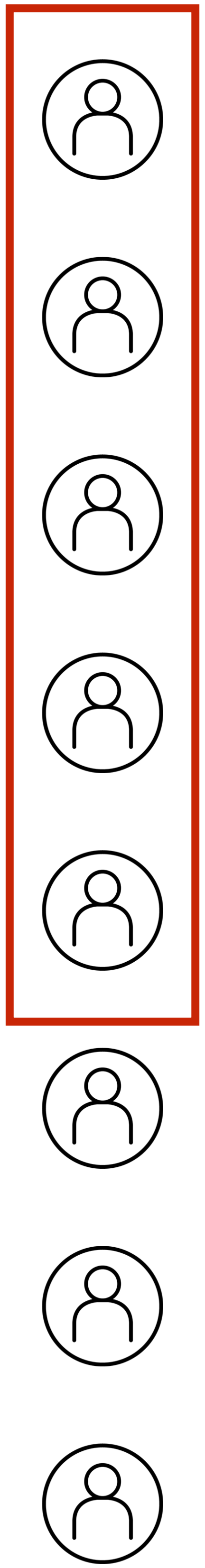

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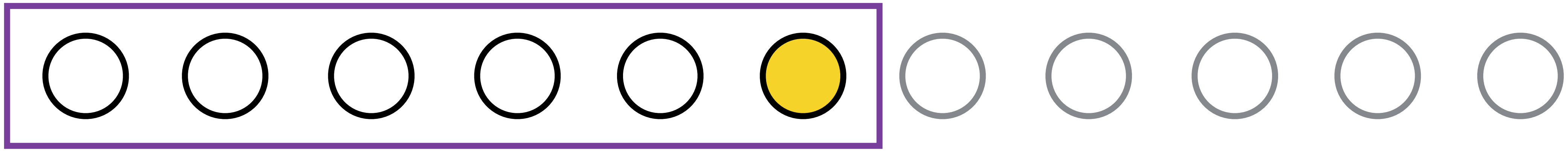

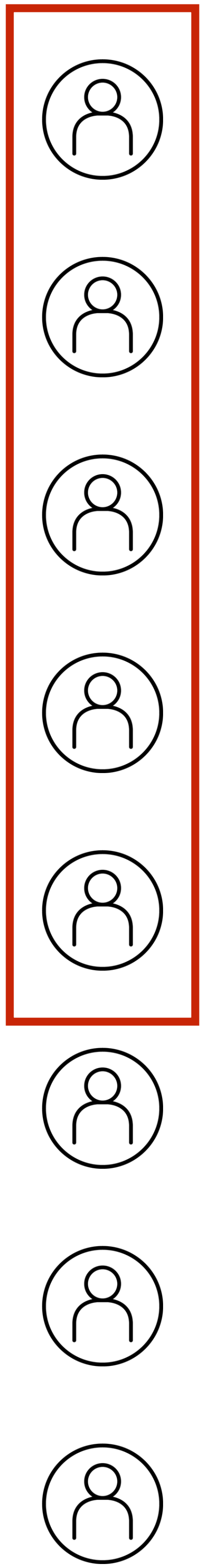

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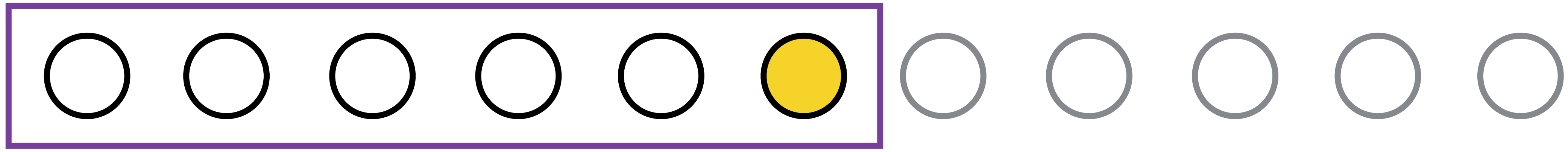

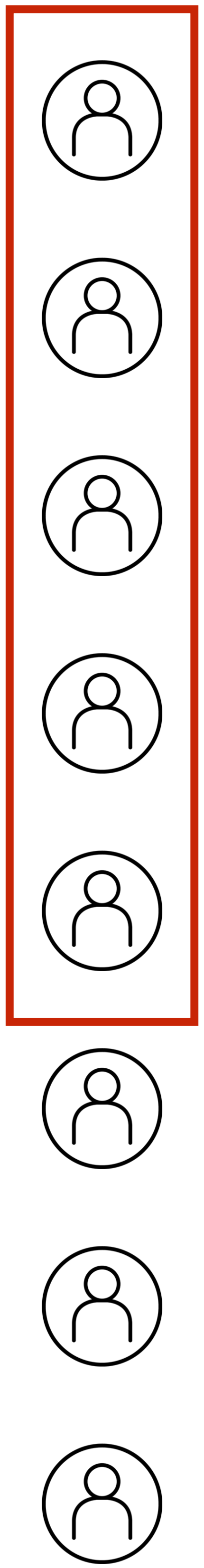

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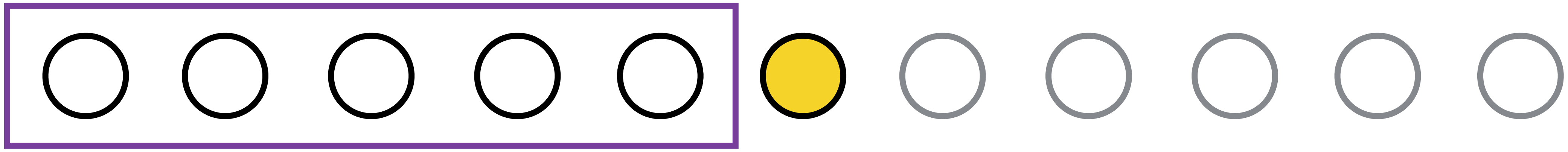

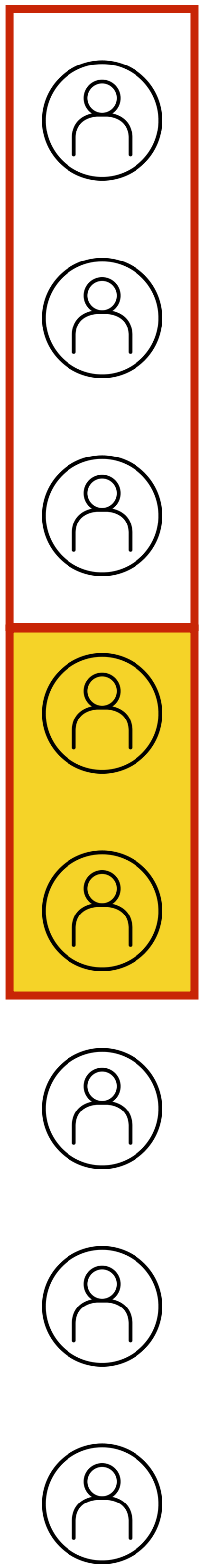

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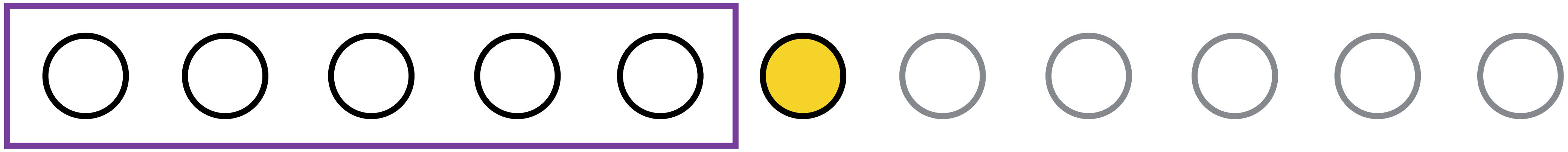
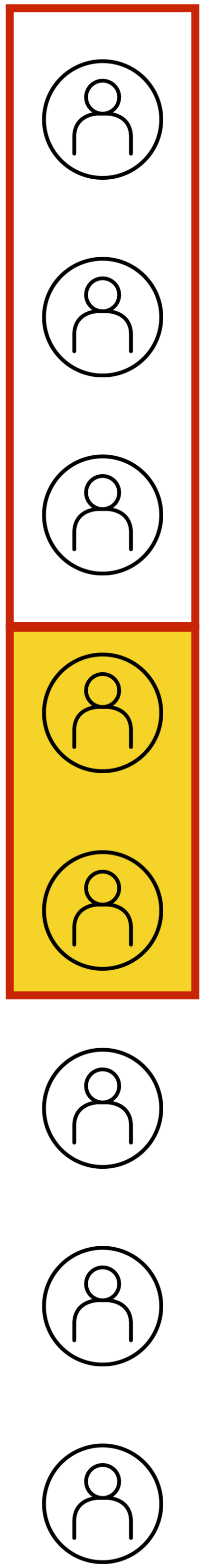
(Guess all possible choices for x.)





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ALMOST SPECIAL

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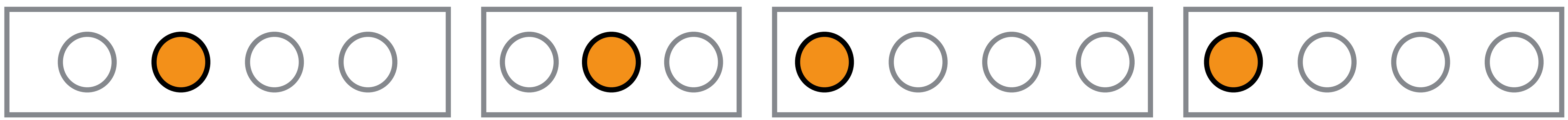
Getting realistic about domain restrictions.

The single-peaked and single-crossing domains have been generalised to notions of **single-peaked and single-crossing on trees**. The generalised domains continue to exhibit many of the nice properties we saw today.

Single-peaked orders on a tree, *Gabrielle Demange,*
Math. Soc. Sci, 3(4), 1982.

Generalizing the Single-Crossing Property on Lines and Trees to Intermediate Preferences on Median Graphs, *Clearwater, Puppe, and Slinko,* IJCAI 2015



The single-peaked and single-crossing domains have been generalised to notions of **single-peaked-width and single-crossing-width.**

Here, it is common that algorithms that work in the single-peaked or single-crossing settings can be generalised to profiles of width w at an expense that is exponential in w.

Kemeny Elections with Bounded Single-peaked or Single-crossing Width,
Cornaz, Galand, and Spanjaard, IJCAI 2013

Profiles that are “close” to being single-peaked or single-crossing (closeness measured usually in terms of candidate or voter deletion) have also been studied.

It’s typically NP-complete to determine the optimal distance, but FPT and approximation algorithms are known.

On Detecting Nearly Structured Preference Profiles
Elkind and Lackner, AAAI 2014

Computational aspects of nearly single-peaked electorates,
Erdélyi, Lackner, and Pfandler, AAAI 2013

Are There Any Nicely Structured Preference Profiles Nearby?
Bredereck, Chen, and Woeginger, AAAI 2013

Γ	VDEL		CDEL
	$k < n/2$	$k \geq n/2$	
Single-peaked / Single-caved	$\mathcal{O}^*(1.28^k)$	$\mathcal{O}^*(2.08^k)$	P
Single-crossing	P	P	$\mathcal{O}^*(5.07^k)$
Best-/Medium-/Worst-restricted	$\mathcal{O}^*(1.28^k)$	$\mathcal{O}^*(2.08^k)$	$\mathcal{O}^*(2.08^k)$
Value-restricted	$\mathcal{O}^*(2.08^k)$	$\mathcal{O}^*(2.08^k)$	$\mathcal{O}^*(2.08^k)$
Group-separable	$\mathcal{O}^*(1.28^k)$	$\mathcal{O}^*(2.08^k)$	$\mathcal{O}^*(3.15^k)$

Γ	VDEL	CDEL
Single-peaked / Single-caved	2	P
Single-crossing	P	6
Best-/Medium-/Worst-restricted	2	3
Value-restricted	3	3
Group-separable	2	4

Summary from:
On Detecting Nearly Structured Preference Profiles *Elkind and Lackner,* AAAI 2014

On profiles that are k candidates or k voters away from the single-peaked and single-crossing domains, CC admits efficient algorithms:

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For profiles that are **k candidates away** from being single-peaked or single-crossing, we have algorithms whose running time is **FPT in k**.

On profiles that are k candidates or k voters away from the single-peaked and single-crossing domains, CC admits efficient algorithms:

For profiles that are **k voters away** from being single- peaked or single-crossing, we have algorithms that are **XP in k**.

One could also generalize SP/SC notions to profiles with multiple peaks/crossings, instances that can partitioned into a small number of disjoint sub-instances which are themselves SP or SC, and so on.

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Checklist of questions to ask when broadening a domain:

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Checklist of questions to ask when broadening a domain:

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Checklist of questions to ask when broadening a domain:

(1) Efficient recognition.

(2) Algorithmic utility.

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Checklist of questions to ask when broadening a domain:

(1) Efficient recognition.

(2) Algorithmic utility.

(3) Preservation of nice axiomatic properties.

CONCLUDING REMARKS

Red flags and research directions.

The Dark Side: Domain restrictions also have some side-effects: problems like manipulation, bribery, and so forth also become easy!

The Shield that Never Was: Societies with Single-Peaked Preferences are More Open to Manipulation and Control, *Faliszewski et al;* TARK 2009

Bypassing Combinatorial Protections: Polynomial-Time Algorithms for Single-Peaked Electorates, *Brandt et al;* AAAI 2010

Directions for future work

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Parameterizing by “distance to tractability”.

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Generalize structure in dichotomous
preference domains to trichotomous and beyond.

Consider completely new domain restrictions.

Investigate the impact of structured preferences in other settings:
matchings and fair division.

THANK YOU!

The Handbook of Computational Social Choice,
Brandt, Conitzer, Endriss, Lang and Procaccia; 2016

Structured preferences.
Elkind, Lackner, and Peters — Trends in Computational Social Choice; (2017): 187-207.